Operational Quantum Gravity from Standard-Model Stress Response: Static Matching, Causal Tensor Dynamics, and Einstein-Equivalent Gravity

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Abstract

Operational Quantum Gravity (OQG) develops a matter-first framework in which quantum matter is fundamental and spacetime geometry is reconstructed as the long-wavelength covariant description of its causal response. Starting from the complete interacting Standard Model, the total conserved stress tensor defines a closed-time-path stress Hessian whose transverse-traceless sector supplies a matter-generated constitutive stiffness and inverse compliance. After the matter-only reduction, a fixed-measure Weyl-transverse diffeomorphism (WTDiff) tensor provides the massless two-helicity infrared representation; its minimal local two-derivative completion is Einstein-equivalent through conservation and the contracted Bianchi identity.

The static Standard-Model calculation is carried through complete perturbative order $O(g^2)$, including hard, electric, Higgs, Ward/contact and local-curvature matching. The analytic continuum result closes without an additional gravitational coefficient, leaving only the renormalized Higgs-curvature coupling and two finite pure-three-dimensional Yang-Mills observables whose flat-lattice extraction is explicitly defined. The causal finite-frequency and finite-wave-number response contains a massless long-range branch, a simple underdamped resonance on the two-particle-production sheet, and distinct Landau/scattering and two-particle-production continua; the physical retarded resonance is distinct from both the principal-sheet analytic double zero and the separate strong-field critical boundary.

The fixed-measure construction also removes an arbitrary homogeneous vacuum-energy offset from the local connected gravitational source while retaining vacuum polarization, boundary stresses, condensates, phase-transition differences, anomalies and fluctuations. Finite changes of material state remain compact perturbations of a common asymptotic massless response, preserving long-range universality. These results support a concrete microscopic route from Standard-Model quantum response to Einstein-equivalent gravity without treating spacetime geometry as an independently fundamental field.

Keywords: operational quantum gravity; Standard Model; stress-tensor response; closed-time-path formalism; emergent gravity; quantum foundations

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1. Introduction

Quantum mechanics and general relativity are often presented as fundamentally incompatible descriptions of nature. The calculations developed in this paper support a different diagnosis. Their successful domains do not contain a direct physical contradiction that must be repaired by forcing one theory to replace the other. The apparent conflict arises when both are required to be fundamental descriptions of the same microscopic degrees of freedom—quantum fields on one side and an independently dynamical spacetime geometry on the other. Operational Quantum Gravity (OQG) removes that additional assumption. It begins with the quantum matter already known to exist, described by the interacting Standard Model, and asks whether the gravitational geometry of general relativity can arise as the long-wavelength collective response of that matter. The result developed here is that, at the two-point and infrared level addressed by this work, the quantum and relativistic descriptions fit together naturally once their causal order is reversed.

The operational starting point is elementary. Time is what physical clocks measure and space is what physical rulers and signal paths measure. Clocks, rulers, atoms, resonators, interferometers and freely falling test bodies are all made of matter. Every empirical inference of a gravitational metric is therefore ultimately a comparison among matter-built standards. General relativity provides an extraordinarily successful covariant language for organizing those comparisons. OQG does not dispute that success and does not seek a modified Einstein equation in regimes where general relativity is established. It asks a more microscopic question: must the metric also be an independently fundamental substance, or can it be the macroscopic record of causal relations among quantum matter?

This distinction changes the quantum-gravity problem. Rather than quantizing a pre-existing gravitational geometry and then coupling quantum matter to it, OQG starts from the complete matter stress and its response. The metric, which records interval comparisons, or the equivalent vielbein, which supplies a local frame, first serves as a source coordinate for defining that response; it is not imposed as a second dynamical equation that constrains the Standard Model. A doubled closed-time-path, or in-in, construction orders both branches of the physical history so that the response is causal. It includes Ward contact terms, the local source-variation terms required by stress conservation. Constitutive inversion then reverses the stress-response relation to answer the matter-first question: when the physical matter stress changes, what collective geometrical response corresponds to it? Geometry is reconstructed only after that response has been obtained. The microscopic ordering is therefore

$$\text{Standard-Model matter} \;\rightarrow\; \text{causal stress response} \;\rightarrow\; \text{constitutive geometry,}$$

rather than matter evolving on an independently fundamental gravitational background.

Several established lines of research point toward pieces of this construction. Induced-gravity calculations show that Einstein-like two-derivative terms can be generated by quantum matter response [3,41,44]; composite-gravity models show that metric-like collective variables need not be fundamental [17–19]; closed-time-path and stochastic-response methods provide the causal, dispersive, absorptive and fluctuation structure of quantum matter [13,14,25,28,30,38]; and fixed-measure/Weyl-transverse diffeomorphism (WTDiff) formulations isolate a massless two-helicity infrared tensor while separating local dynamics from an arbitrary homogeneous stress offset [4,5,7,15]. The purpose of OQG is not to relabel these ingredients, but to connect them into one calculable Standard-Model constitutive chain and follow that chain far enough that its coefficients, singularities, matching conditions and failure modes can be tested.

The complete interacting Standard Model provides the microscopic source. Its Yang-Mills, Dirac/Yukawa and Higgs contributions belong to one conserved total stress tensor; gravity is not assigned separate electromagnetic, quantum-chromodynamic (QCD), Higgs or fermionic constants. The connected stress Hessian then determines both the static and causal response. Its transverse-traceless (TT) sector supplies the finite two-derivative stiffness $A_{2,E}$, while the inverse susceptibility supplies the corresponding macroscopic compliance. Newton's coupling is thereby read in the reverse direction from the usual fundamental-metric interpretation: it is the state-matched infrared compliance represented macroscopically by gravity, not a universal microscopic constant assigned in advance to spacetime.

The symmetry and the strength of the gravitational response play different roles. Matter supplies the TT response and its stiffness. After the matter-only constitutive reduction, a fixed-measure/WTDiff tensor provides the selected massless two-helicity infrared representation. The minimal local, two-derivative, gauge-preserving WTDiff completion of that derived two-point sector is Einstein-equivalent: conservation and the contracted Bianchi identity restore the trace and yield the Einstein equations with the cosmological term as an integration datum. Thus the Einstein equations are not discarded or phenomenologically modified. They reappear as the macroscopic infrared description of a response whose microscopic input is quantum matter. A derivation of every nonlinear gravitational vertex directly from the higher connected Standard-Model stress hierarchy—TTT for the connected three-stress kernel, TTTT for the connected four-stress kernel, and higher kernels—is a later microscopic question; it is not required to establish the Einstein-equivalent minimal infrared completion used here.

This tensor construction also clarifies the earlier engineering polarizable-vacuum program. Refractive-index and polarizable-vacuum descriptions developed by Dicke and Puthoff showed that static Schwarzschild comparisons of clock rate, ruler scale, energy, frequency and coordinate light propagation can be organized through a scalar comparison variable [23,37]. The Desiato-Storti engineering work developed that scalar description as a practical scaling language [21,39]. In OQG the scalar is no longer promoted to the fundamental gravitational field. Instead, the historical polarizable-vacuum/refractive-index model is recovered as the static, isotropic projection of the general matter-generated tensor response. What began as an engineering representation therefore acquires a place inside the covariant theory: it is the reduced comparison law seen when the full WTDiff tensor response is specialized to the static isotropic branch. The tensor theory supplies what the scalar engineering model could not—momentum flow, anisotropic stress, causal dispersion, radiation and the general spin-2 structure—while retaining the useful static clock-and-ruler interpretation that motivated the original program.

The static Standard-Model calculation is quantitative. Here $O(g^2)$ denotes terms through second order in the interaction couplings; k_B is Boltzmann's constant; T is the equilibrium temperature; and ξ_H is the renormalized Higgs-curvature coupling. The superscript NP means nonperturbative. The coefficients c_3^{NP} and c_2^{NP} are distinct pure-three-dimensional Yang-Mills observables for the magnetostatic $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ sectors, not fit parameters. The equilibrium TT Kubo coefficient is evaluated through the complete perturbative $O(g^2)$ hard/electric/Higgs/Ward/local-curvature matching problem. The hard two-loop operator sectors, Higgs variational contacts, dimensionally reduced electric and Higgs modes, Ward seagulls, the two renormalization scales and the local three-dimensional curvature coefficient combine into one renormalization-closed Hessian. The resulting high-state stiffness is

$$\frac{A_{2,E}}{(k_B T)^2} = 0.260056643133723 + 0.066623718018839\,\xi_H$$
$$+0.002999279573124\,c_3^{\mathrm{NP}} + 0.000799379975567\,c_2^{\mathrm{NP}}.$$

No additional perturbative gravitational coefficient remains hidden in this result. The two remaining magnetostatic constants are finite pure-three-dimensional Yang-Mills observables, not adjustable OQG parameters, and the flat-lattice shear observable, contact subtraction, order of limits and continuum conversion required to determine them have already been derived. The remaining ξ_H dependence is the ordinary renormalized Higgs-curvature coupling. In this sense the analytic continuum calculation ends at a conventional Standard-Model/nonperturbative boundary rather than at an undefined gravitational parameter.

The causal calculation reaches an equally informative endpoint and corrects an earlier intuition rather than merely confirming it. At finite frequency and wave number, the weak-coupling Standard-Model TT response contains a massless long-range inverse-response branch, a simple underdamped resonance on the two-particle-production sheet, and distinct two-particle-production and Landau/scattering continua. Let u_P denote the dimensionless complex pole coordinate, F the inverse-response eigenvalue whose zero defines the pole, $\partial_u F$ its derivative with respect to u, and i the imaginary unit. At the benchmark state the physical retarded-sheet pole is

$$u_P = 1.353089335 - 0.093439111\,i, \qquad |\partial_u F| = 5.39112,$$

so ordinary matter is not naturally driven toward a critically damped exceptional point. The exact principal-sheet Lambert-W double zero is retained as an analytic benchmark, but it is not the physical retarded resonance. This distinction is important because long-range attraction does not depend on the finite-frequency resonance becoming critical: the $1/k^2$ compliance is already present in the infrared. The calculation therefore separates ordinary causal matter response from the distinct strong-field critical boundary, rather than forcing one object to perform both roles.

The fixed-measure construction has a second consequence that is independent of the detailed value of the stiffness. A homogeneous shift of the stress tensor by an arbitrary metric-proportional constant does not change the local connected gravitational source. The same construction continues to respond to vacuum polarization, Casimir and boundary stresses, phase-transition differences, condensates, anomaly contributions, pressure and fluctuations. OQG therefore removes the local-source form of the traditional vacuum catastrophe without subtracting two enormous numbers: the arbitrary homogeneous zero of quantum-field energy is not local gravitational source information in the first place. The observed numerical cosmological constant is a different question; in the Einstein-equivalent infrared reconstruction it remains a macroscopic integration or boundary datum and is not predicted in this paper.

State dependence also survives without destroying gravitational universality. A homogeneous thermal branch can possess a state-dependent stiffness, but a finite body whose temperature, phase or composition differs from its surroundings is a compact perturbation of one common asymptotic inverse response. Outside the compact region the same massless pole controls the monopole field. The body does not acquire a private Newton constant. This result makes the matter-first interpretation compatible with the empirical universality demanded by precision redshift measurements, equivalence-principle tests and gravitational-wave observations while still allowing the microscopic response inside matter to carry nontrivial state dependence.

These results suggest that the familiar statement that quantum mechanics and general relativity are incompatible may be asking the wrong question. OQG does not attempt to quantize general relativity as a fundamental microscopic geometry. It starts with quantum matter, calculates the causal response that matter actually possesses, and asks what covariant long-wavelength description follows. The Standard Model remains quantum; the infrared description remains Einstein-equivalent; and the two are connected by a constitutive response rather than placed in competition. The tension is shifted from an assumed clash of foundational formalisms to a set of concrete calculations: the stress Hessian, its static stiffness, its causal spectrum, its nonperturbative matching data, and the higher connected kernels that determine the fully microscopic nonlinear completion.

The resulting framework also has consequences beyond reproducing the tested equations. Because physical histories are generated by retarded matter response, OQG admits only geometrical representations that possess a causal matter preimage; formal geometrical solutions are not automatically promoted to physical states. The same matter-first ordering reframes strong-field questions such as black-hole infall and singularity formation, and it suggests a cosmological program in which an evolving matter state and its operational standards are primary while cosmological geometry is reconstructed from their collective response. These extensions are not needed to establish the two-point results reported here, but they illustrate why the distinction between fundamental geometry and emergent geometry is physically consequential rather than semantic.

The paper is therefore organized as an argument with an explicit proof layer. Section 2 constructs the complete Standard-Model causal stress response and fixes its normalization. Section 3 performs the matter-first constitutive inversion and identifies the long-range TT compliance.

Sections 4 and 5 derive the static stiffness and carry its Standard-Model matching through $O(g^2)$ to the nonperturbative Yang-Mills boundary. Sections 6 and 7 determine the causal finite-(ω, k) topology and separate the ordinary underdamped resonance from the principal-sheet analytic benchmark and the strong-field boundary. Sections 8 and 9 reconstruct the minimal WTDiff/Einstein-equivalent infrared description, establish homogeneous-vacuum-shift blindness, and show how cross-state universality is preserved. Section 10 states the scope of the completed two-point program. Appendices A–H provide the normalization, projector algebra, Kubo derivations, hard two-loop reduction, dimensional-reduction matching, lattice construction, retarded-sheet analysis and infrared claim ledger in full detail.

The central proposal can therefore be stated compactly: quantum matter does not have to be reconciled with an independently fundamental spacetime geometry if that geometry is the infrared constitutive description of quantum matter itself. The calculations that follow test that statement using the Standard Model rather than analogy. If the matter-first ordering is correct, quantum mechanics and general relativity are not rival descriptions waiting to be forced into one theory; they are microscopic and macroscopic descriptions of the same causal physical system.

2. Complete Standard-Model causal stress response

This section has two objectives. First, it defines one causal stress-response kernel for the *complete interacting Standard Model*, so gauge fields, fermions, the Higgs sector, and their interactions all enter through the same conserved energy-momentum tensor. Second, it fixes the normalization used later for both the real-time response and the static stiffness. No separate gravitational charge is assigned to any particle species.

Let x denote a spacetime point and $q_{\mu\nu}(x)$ a symmetric geometrical probe coupled to the total stress $T^{\mu\nu}(x)$. To ask the causal question—how the expectation value of the stress changes *after* a perturbation—the calculation uses the closed-time-path (CTP), or in-in, formalism [14,28,38]. CTP introduces forward (+) and backward (-) bookkeeping copies of the same source and matter system; these are not two physical geometries. With $T_{+}^{\mu\nu}$ and $T_{-}^{\mu\nu}$ the corresponding copies of the complete Standard-Model stress tensor, the source action is

$$S_{\text{src}}^{\text{CTP}}=\frac{1}{2}\int d^4x \left(q_{+\,\mu\nu}T_{+}^{\mu\nu}-q_{-\,\mu\nu}T_{-}^{\mu\nu}\right). \tag{1}$$

This choice keeps the microscopic content entirely within known matter physics. The same total tensor carries gauge-field stress, fermionic stress, Higgs stress and the stresses generated by their interactions, and its conservation follows from the Standard-Model matter equations [10]. The geometrical source therefore plays the role of a probe coordinate: it asks how the complete matter state responds, but it does not introduce a second matter sector or impose an additional equation of motion on the Standard Model. This distinction is what allows the later constitutive inversion to remain matter-first without sacrificing the standard field-theoretic definition of stress.

For causal response it is convenient to replace the two contour sources by their average and difference combinations. The subscript r labels the contour average, while a labels the contour difference, or response, variable. Define

$$q_r \equiv \frac{q_+ + q_-}{2}, \qquad q_a \equiv q_+ - q_-, \tag{2}$$

where the subscript r denotes the average (or ``classical'') CTP variable and a denotes the difference (or response-generating) variable. The average variable carries the physical applied perturbation, while the difference variable is set to zero only after the required functional derivatives have been taken. This is the standard CTP organization of causality, specialized here to a symmetric tensor source.

Let $W[q_r, q_a]$ denote the connected closed-time-path (CTP) generating functional, whose derivatives produce connected stress responses. The central object is the completed retarded stress susceptibility. Capital labels A and B below abbreviate symmetric tensor-index pairs, for example $A = \mu\nu$ and $B = \rho\sigma$. Let θ denote the Heaviside step function and $\Pi_{AB}^{\text{contact}}$ the local second-metric-variation, or seagull, contribution required by the same Hessian. After differentiating the doubled functional and taking the physical limit, the response is

$$\Pi_{AB}^{R}(x,y) = -\frac{i}{2}\,\theta(x^0 - y^0)\langle[T_A(x), T_B(y)]\rangle + \Pi_{AB}^{\text{contact}}(x,y). \tag{3}$$

Here the superscript R denotes the retarded response: Eq. (3) says how a stress perturbation at y influences the stress measured later at x. The factor one-half is fixed by the symmetric source convention in Eq. (1); it normalizes the response coordinate and is not a fitted gravitational parameter. Keeping it explicit makes the real-time commutator, the Euclidean Kubo calculation, and the spectral normalization mutually consistent.

The local contact term is equally important. It is the part of the second metric variation that occurs at coincident points (often called a seagull term). Together with the separated-point correlator it enforces the Ward identities—the conservation relations inherited from the underlying Standard Model. In this manuscript, "Ward-completed" therefore means that both pieces have been assembled before the response is interpreted or inverted.

Conservation provides the first stringent internal check. Let p_μ denote the external four-momentum conjugate to x-y, and let $\Pi_{\text{phys}}^{\mu\nu,\rho\sigma}(p)$ denote the completed physical response after the Ward-required local terms are included. In a local tangent-state representation it obeys

$$p_\mu \Pi_{\text{phys}}^{\mu\nu,\rho\sigma}(p) = 0. \tag{4}$$

Equation (4) is the response-theory expression of energy-momentum conservation. It is obtained only for the completed Hessian, with the Ward-related local terms included together with the separated-point correlator. This is useful physically: gauge fields, fermions, scalars and their interactions are not combined by an external universality rule after the calculation. They enter one conserved tensor response from the outset. Transversality does not by itself imply a Weyl identity for the full Standard Model, whose trace contains masses, running couplings, spontaneous symmetry breaking and the trace anomaly; the trace and volume sectors are therefore kept conceptually distinct from the long-range transverse-traceless (TT) sector developed below.

The same organization also clarifies renormalization. A calculation may move finite local pieces between a loop contribution and a local counterterm or Wilson coefficient—the coefficient multiplying a local operator in an effective description—but such bookkeeping does not change the completed susceptibility. What is physically meaningful is the Ward-consistent sum. This point becomes decisive in the static $O(g^2)$ calculation, where $O(g^2)$ denotes terms second order in a generic Standard-Model interaction coupling g. Hard, electric and magnetostatic contributions carry scale dependence that cancels only after the contact and matching terms are assembled. Individual gauge, Yukawa and Higgs couplings are introduced explicitly when they first enter the later calculations. The causal kernel and the static stiffness are therefore two projections of one renormalized matter response rather than independently normalized constructions.

The equilibrium stiffness used later is obtained from the Euclidean continuation of this same response. The Euclidean and retarded kernels answer different experimental questions and must not be conflated, but they are analytically connected projections of the same matter dynamics. Near equilibrium, fluctuation-dissipation relations connect stress fluctuations to the absorptive response, while dispersion relations connect absorption to the reactive part [13,25,30]. OQG can therefore contain a nondissipative static gravitational stiffness together with finite-frequency damping, pair-production thresholds, Landau continua and memory without introducing separate microscopic mechanisms for the static and dynamical regimes.

Nothing in the CTP construction introduces a physical backward time. The two contour branches are bookkeeping copies used to compute causal expectation values; the physical susceptibility in Eq. (3) is retarded, with response following perturbation. This preserves the operational causal ordering used throughout OQG while allowing the full analytic structure of the Standard-Model response to be treated with the usual real-time quantum-field-theory machinery.

The result of this section is therefore a single, Ward-completed causal matter kernel built from the complete Standard Model. Its short-distance content may depend on particle thresholds, state, composition and interaction scale, but the infrared TT projection is taken from this common total-stress response. The massless long-range branch identified later is consequently a collective property of the same matter system, not a sector-by-sector force added to it. This completed susceptibility is the object inverted in Sec. 3, where the description is reorganized from “geometry probes matter” to the matter-first constitutive form “changed matter determines the corresponding operational geometry.” Supplementary Appendix A gives the detailed CTP normalization and constitutive inversion, while Supplementary Appendix B gives the TT projection and physical spectral normalization used by the later static and causal calculations.

3. Matter-first constitutive inversion and emergent Weyl-transverse-diffeomorphism (WTDiff) representation

This section reorganizes the response of Sec. 2 into the matter-first form used by Operational Quantum Gravity (OQG). It has three steps: invert the causal constitutive relation, interpret that inverse as the operational tensor response to changed matter, and only then introduce the fixed-measure Weyl-transverse-diffeomorphism (WTDiff) representation used in the infrared. No new microscopic field is added in any of these steps.

In the probe description, changing the geometrical source changes the total Standard-Model stress. OQG reads the same constitutive information in the opposite direction: a physical change of the matter state changes the total stress, and the inverse susceptibility gives the tensor response associated with that change. The distinction is the same one familiar from ordinary constitutive physics—stiffness versus compliance—not a change in the underlying quantum dynamics.

The doubled closed-time-path (CTP) functional must remain doubled while this inversion is performed. Let $W[q_r, q_a]$ denote the connected CTP generating functional introduced in Sec. 2, where q_r and q_a are the average and difference source variables. Define the conjugate average stress $T_r^{\mu\nu}$ and difference stress $T_a^{\mu\nu}$ by

$$T_r^{\mu\nu}(x)=2\,\frac{\delta W}{\delta q_{a\,\mu\nu}(x)},\qquad T_a^{\mu\nu}(x)=2\,\frac{\delta W}{\delta q_{r\,\mu\nu}(x)}. \tag{5}$$

The factors of two follow from the symmetric one-half source coupling and are therefore fixed by the same normalization that produced the retarded susceptibility in Sec. 2. With these conjugate variables, define the doubled stress-space Legendre functional $\Gamma[T_r, T_a]$ by

$$\Gamma[T_r,T_a]=W[q_r,q_a]-\frac{1}{2}\int d^4x\,\left(q_{a\,\mu\nu}T_r^{\mu\nu}+q_{r\,\mu\nu}T_a^{\mu\nu}\right), \tag{6}$$

so that

$$\frac{\delta\Gamma}{\delta T_r^{\mu\nu}}=-\frac{1}{2}\,q_{a\,\mu\nu},\qquad \frac{\delta\Gamma}{\delta T_a^{\mu\nu}}=-\frac{1}{2}\,q_{r\,\mu\nu}.$$

The physical limit is taken only after the required derivatives have been evaluated. On that branch $q_a = T_a = 0$, while q_r and T_r carry the physical source-response relation. The non-gauge physical tensor subspace consists of the tensor components that remain after redundant gauge directions are removed. Linearizing on that subspace gives the retarded constitutive relation

$$\delta T_r=\Pi^R\,\delta q_r,\qquad \delta q_r=D^R\,\delta T_r,\qquad D^R\equiv(\Pi^R)^{-1}. \tag{7}$$

Stress-space Legendre transformations and inverse stress-correlator constructions have independent field-theoretic precedents [8]. Here Eq. (7) has a particularly direct physical meaning. It is the tensor analogue of writing a spring with force F, displacement x and stiffness k either as $F=kx$ or as $x=F/k$, or a dielectric either as polarization in response to an applied field or as the conjugate field required for a specified polarization. The inversion does not create new dynamics and does not change the Hilbert space. All microscopic information remains in the completed Standard-Model susceptibility Π^R; D^R is its compliance.

That distinction is essential to the causal ordering. The probe tensor introduced in Sec. 2 is useful because stress is defined by response to a geometrical source, but OQG does not elevate that probe into an independently fundamental gravitational substance. Once the constitutive relation has been obtained, the source coordinate can be eliminated from the working description in favor of the stress variables to which it is conjugate. A physical perturbation of the Standard-Model state changes the complete stress first. Equation (7) then assigns the collective tensor response associated with that changed stress. In this precise sense the microscopic order is

$$\text{matter state} \;\rightarrow\; \delta T_{\mathrm{SM}} \;\rightarrow\; D^R \;\rightarrow\; \delta q_{\mathrm{op}}.$$

Here q_{op} denotes the physical operational tensor response obtained from q_r after the CTP physical limit is taken. It is therefore a response coordinate: it records how matter-built clocks, rulers, signal paths and trajectories compare after the matter state has changed. Geometry is introduced as the macroscopic language of this collective relation, not as an additional microscopic matter field placed beside the Standard Model.

This constitutive inversion is already enough to expose the long-range matter-matter structure. A localized change of one matter system perturbs the common stress response; the inverse kernel carries that disturbance to another matter system. On the physical tensor block the same kernel that relates δT to δq_{op} therefore mediates the reciprocal cross-response between separated stress perturbations. The long-range branch identified in later sections is consequently a property of the common Standard-Model response rather than a force assigned independently to electromagnetic, fermionic, Higgs or quantum-chromodynamic (QCD) matter. Universality is not inserted by giving every sector the same phenomenological constant; it is a condition on the infrared residue—the coefficient multiplying the massless long-range pole—and on the tensor structure of this one completed response.

Only after this matter-response reduction is the fixed-measure geometrical representative introduced. Let $q_{\mu\nu}$ denote a nonsingular representative of the operational tensor response, write $q \equiv \det q_{\mu\nu}$, and let $\varpi(x)$ be a fixed, nowhere-vanishing scalar density that defines the nondynamical volume measure. In four dimensions define

$$\hat{g}_{\mu\nu} = \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2} q_{\mu\nu}, \qquad \sqrt{-\hat{g}} = \varpi. \tag{8}$$

For a local scalar function $\sigma(x)$, the rescaling $q_{\mu\nu} \rightarrow e^{2\sigma(x)} q_{\mu\nu}$ leaves $\hat{g}_{\mu\nu}$ unchanged. The determinant of the operational metric is therefore not an independent local degree of freedom in this representation. At the linearized level, let $h_{\mu\nu}$ denote the perturbation of the fixed-measure operational metric, let $\xi_\mu(x)$ be an infinitesimal transverse vector satisfying $\partial_\mu \xi^\mu = 0$, and let $\eta_{\mu\nu}$

be the Lorentz metric of an infinitesimal tangent frame. The corresponding WTDiff redundancy may then be written [4,5,7,15] as

$$\delta h_{\mu\nu}=\partial_\mu\xi_\nu+\partial_\nu\xi_\mu+2\sigma\,\eta_{\mu\nu}, \qquad \partial_\mu\xi^\mu=0. \tag{9}$$

Here the same $\sigma(x)$ acts as the local Weyl parameter. Its appearance does not postulate a fundamental global Minkowski spacetime. Local Lorentz frames are required to represent spinors and to evaluate local response coefficients; the global operational tensor is reconstructed from the matter response only after the constitutive reduction.

The fixed-measure step has one specific job: it chooses the infrared tensor representation without rewriting the microscopic Standard Model. Particle masses, spontaneous symmetry breaking, running couplings, and the trace anomaly remain part of the matter theory and continue to contribute to its completed response. WTDiff is instead a redundancy of the *long-wavelength geometrical description*. By removing an independent local volume mode from that representative, the linearized transverse-traceless sector carries the two physical helicities of a massless spin-two field [4,5,7,15].

This separation is visible directly in the quadratic infrared kernel. Let $h_{\mu\nu}^{TT}$ denote the transverse-traceless projection of $h_{\mu\nu}$, let p^μ denote the local four-momentum, and let $A_{2,E}$ denote the Euclidean two-derivative TT stiffness calculated in Secs. 4 and 5. Writing $\Gamma_{\mathrm{IR,TT}}^{(2)}$ for the quadratic TT part of the infrared effective action, an algebraic mass term for the physical TT field is incompatible with the WTDiff redundancy, so after projection the long-wavelength inverse response begins at two derivatives,

$$\Gamma_{\mathrm{IR,TT}}^{(2)}=\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\,h_{\mu\nu}^{TT}(\text{-}p)\left[A_{2,E}\,p^2+O(p^4)\right]h_{\mathrm{TT}}^{\mu\nu}(p). \tag{10}$$

The coefficient $A_{2,E}$ is not fixed by WTDiff symmetry; it is the constitutive stiffness calculated from Standard-Model matter in Secs. 4 and 5. Equation (10) therefore cleanly separates representation from strength. WTDiff supplies the massless two-helicity infrared form, while the microscopic matter response determines how stiff that form is. Its reciprocal becomes the macroscopic gravitational compliance.

The relation to the Weinberg-Witten theorem must be assessed against this actual construction rather than against a different composite-graviton model. In plain terms, the theorem restricts conventional local composite massless particles that carry Lorentz-covariant conserved stress. OQG does not begin with such a particle propagating on a postulated global Minkowski background. Its microscopic object is a causal, generally nonlocal Standard-Model stress response, and the fixed-measure/WTDiff tensor is introduced only after the constitutive inversion. A direct identification with the theorem's standard target is therefore not automatic [17–19,42]. At the same time, neither the Legendre transformation nor nonlocality by itself proves theorem-level evasion: an invertible change of response variables does not erase the Standard Model's translation charges. The precise claim is consequently bounded. The present construction establishes the matter-first constitutive inversion and the WTDiff infrared representation; a theorem-level statement requires the complete reduced chiral-Standard-Model Noether/Ward and one-particle-state structure to satisfy the corresponding consistency conditions.

The accomplishment of this section is therefore the causal and representational bridge required by OQG. The complete Standard-Model Hessian of Sec. 2 can be inverted without changing its physics, so matter stress becomes the physical input and the operational tensor becomes the response. Fixed measure and WTDiff are then introduced only at the emergent infrared stage, where they isolate a massless two-helicity TT representation whose strength is left to the Standard-Model stiffness calculation. The next section determines that strength from the state-dependent Euclidean response. The detailed CTP normalization and inversion underlying Eqs. (5)-(7) are collected in Supplementary Appendix A, and Supplementary Appendix H later collects the fixed-measure infrared completion and its claim boundaries.

4. TT constitutive stiffness and state-generated local response

The objective of this section is to turn the long-wavelength transverse-traceless (TT) response into a calculable material property. The relevant quantity is the coefficient of the two-derivative TT kernel, denoted $A_{2,E}$. WTDiff fixes the allowed infrared tensor *form* but not this coefficient, and the calculation does not import it from the measured value of Newton's constant. Instead, $A_{2,E}$ is obtained as an equilibrium susceptibility of the Standard-Model matter state.

The strategy is concrete: apply a weak static shear, use the Kubo equilibrium-response relation to measure the quadratic change of the Euclidean stress response with spatial wave number, and identify that slope with the TT stiffness. Supplementary Appendix B supplies the physical TT projection and spectral weights; Supplementary Appendix C supplies the Euclidean Kubo derivation, the Higgs variational contact, and the leading soft thermal correction.

Consider a stationary isotropic state and apply a static off-diagonal shear $h_{xy}(z)$ carrying spatial momentum k along the z direction. This perturbation is transverse and traceless, so it isolates the tensor response without mixing in the scalar volume mode. Let $G_E^{xy,xy}(0,k)$ denote the conventional Euclidean metric-variational stress correlator. Its hydrostatic curvature coefficient is the k^2 slope of the equilibrium shear response and is defined by the Kubo relation developed by Moore and Sohrabi and by the metric-variational formulation of Kovtun and Shukla [29,34]

$$\kappa(T)=\frac{\partial^2}{\partial k^2}G_E^{xy,xy}(0,k)\bigg|_{k=0}. \tag{11}$$

Equivalently,

$$G_E^{xy,xy}(0,k)=G_E^{xy,xy}(0,0)+\frac{\kappa(T)}{2}k^2+O(k^4).$$

The CTP source normalization derived in Supplementary Appendix A converts this conventional correlator into the OQG Euclidean susceptibility according to $\Pi_{TT,E}$=-$G_E/2$. Therefore the coefficient of the two-derivative term appearing in Eq. (10) is

$$A_{2,E}(T)=-\frac{\partial\Pi_{TT,E}}{\partial k^2}\bigg|_{k=0}=\frac{\kappa(T)}{4}. \tag{12}$$

Equation (12) is the central normalization result of the static calculation. In physical terms, $A_{2,E}$ measures the curvature of the completed Standard-Model free energy with respect to a slowly varying TT shear. It is fixed by that response measurement alone—without a gravitational-radius identification, a critical-pole assumption, a physical ultraviolet cutoff, or an external dimensional ruler.

This is also where the role of state dependence becomes concrete. In a scale-free four-dimensional vacuum, the nonlocal stress response naturally contains higher-derivative structures such as $p^4 \ln p^2$. A physical equilibrium state contains additional finite scales supplied by the state itself, so a two-derivative contribution proportional to a dimension-two state quantity is allowed. For a Kubo-Martin-Schwinger (KMS) equilibrium state the simplest such quantity is $(k_B T)^2$, where k_B is Boltzmann's constant and T is the physical temperature. The resulting $T^2 p^2$ term is not produced by cutting off a divergent vacuum integral at $k_B T$; it is a finite state-dependent susceptibility obtained directly from the thermal functional. The subtraction scale used in intermediate renormalization remains an internal coordinate and does not become a second physical ruler.

At free leading order the field-by-field variational coefficients are especially transparent. For one real scalar with curvature improvement ξ, one Weyl fermion, and one gauge vector, respectively (Moore and Sohrabi; Kovtun and Shukla) [29,34],

$$\frac{\kappa_S}{(k_B T)^2}=-\frac{1\text{-}6\xi}{72}, \qquad \frac{\kappa_W}{(k_B T)^2}=\frac{1}{144}, \qquad \frac{\kappa_V}{(k_B T)^2}=\frac{1}{18}. \tag{13}$$

Let $\kappa_{\mathrm{SM}}^{(0)}$ denote the noninteracting Standard-Model hydrostatic curvature coefficient, $A_{2,E}^{(0)}$ the corresponding noninteracting TT stiffness, and ξ_H the renormalized Higgs-curvature coupling. The four real Higgs components, 45 Weyl fermions, and 12 gauge vectors of the minimal Standard Model therefore give the complete free variational result

$$\frac{\kappa_{\mathrm{SM}}^{(0)}}{(k_B T)^2}=\frac{133+48\xi_H}{144}, \qquad \frac{A_{2,E}^{(0)}}{(k_B T)^2}=\frac{133+48\xi_H}{576}. \tag{14}$$

The number 133 is thus the minimal-coupling member of a well-defined static variational family. It is not the same quantity as the positive coherent spin-2 weight 283 encountered in the causal spectral calculation. The latter counts resolved physical TT spectral strength with scalar:Weyl:vector weights $1:3:12$; Eq. (14) is a hydrostatic derivative of the full metric Hessian and therefore weights the field content differently. The two integers characterize different moments of the same Standard-Model response.

The Higgs term in Eq. (14) also resolves an apparent tension between the propagating and variational responses. In the operator $\xi_H R H^\dagger H$, R is the curvature scalar, H is the Higgs doublet, and † denotes Hermitian conjugation. The improvement operator is proportional in flat-space momentum variables to $\eta_{\mu\nu}p^2 - p_\mu p_\nu$. A resolved physical TT projector annihilates that operator by tracelessness and transversality, so changing ξ_H does not create additional propagating TT spectral states. The metric-variational static Hessian is different: its second metric variation contains a local Higgs-curvature contact, and that contact contributes to the k^2 equilibrium coefficient. Thus

resolved TT spectral weight is ξ_H-independent, static variational stiffness need not be.

This separation is part of the completed Standard-Model bookkeeping. The resolved TT spectrum fixes its physical tensor weight independently of ξ_H, while the static metric Hessian carries the renormalized Higgs-curvature coupling through its local contact term. The calculation therefore keeps ξ_H in its proper role as a Standard-Model curvature coupling and matches it together with the rest of the variational Hessian.

Interactions reinforce rather than obscure the state-generated character of the stiffness. A Matsubara mode is a discrete Euclidean thermal-frequency mode, and a zero mode has zero such frequency. The first correction is nonanalytic in the gauge couplings because bosonic zero Matsubara modes are infrared sensitive. For one resummed bosonic zero mode of screening mass m, the Ward-completed scalar contribution is

$$\delta\kappa_0 = \frac{(k_B T)m}{24\pi}(1\text{-}6\xi), \tag{15}$$

with $\xi=0$ for the temporal gauge scalars and $\xi=\xi_H$ for the Higgs mode. Fermions have no zero Matsubara frequency and therefore do not contribute at this order. Summing the eight color-electric modes, three weak-electric modes, the hypercharge-electric mode, and four real Higgs modes at the central running-coupling state gives, through $O(1)$+$O(g)$,

$$\frac{A_{2,E}^{(0+g)}}{(k_B T)^2} = 0.261940989021 + 0.0601064526\,\xi_H. \tag{16}$$

At $\xi_H=0$, the soft resummed sector raises the free $133/576$ stiffness by approximately 13.44%. This contribution is finite and introduces no independent subtraction coordinate. At $O(g^2)$, the hard, electric/Higgs, magnetostatic, local-curvature and Ward/contact sectors combine into one renormalization-closed metric Hessian. Section 5 gives that completed matching, with the derivations distributed across Appendices D–F.

The physical interpretation of $A_{2,E}$ is most transparent in the settled infrared branch. Restoring International System (SI) units, $\hbar$ is the reduced Planck constant, $c[\rho]$ is the locally measured limiting signal speed in state ρ, and $G_{\mathrm{comp}}[\rho]$ is the macroscopic tensor compliance matched to that state. The compliance may scale with state and is not assumed to be constant by itself. The two-derivative Einstein-equivalent normalization identifies it through

$$G_{\text{comp}} = \frac{\hbar c^5}{32\pi A_{2,E}}. \tag{17}$$

Equation (17) should be read in the same constitutive sense as Eq. (12): the matter state supplies a stiffness, and the long-wavelength geometrical description uses its reciprocal as the gravitational compliance. On a homogeneous high-state Kubo-Martin-Schwinger (KMS) branch, let $a_T(T)$ denote the dimensionless coefficient defined by $A_{2,E}(T) = a_T(T)(k_B T)^2$. Then

$$A_{2,E}(T) = a_T(T)(k_B T)^2,$$

then comparisons among homogeneous states obey

$$G_{\text{comp}}(T) \propto \frac{1}{a_T(T)(k_B T)^2}. \tag{18}$$

This is a comparison law within the homogeneous high-state KMS family used to calculate the microscopic susceptibility, not a laboratory thermometer law for a compact body. Across electroweak symmetry breaking, QCD confinement, atomic binding and condensed-matter formation, the same constitutive response is carried by matched lower-energy degrees of freedom. Section 9 develops the corresponding low-energy effective-field-theory form as a ground-state coefficient plus thermal corrections that vanish with the laboratory temperature, while the common asymptotic long-range branch is preserved.

The calculation in this section establishes the central static accomplishment needed by the OQG constitutive chain. A finite physical matter state generates a genuine local two-derivative TT stiffness without an imposed ultraviolet cutoff. Its normalization follows from the same CTP convention as the causal response, its free Standard-Model value is analytically calculable, the Higgs-curvature contribution is cleanly separated into local variational rather than resolved spectral physics, and the leading soft interaction correction is finite and controlled. Section 5 carries this same susceptibility through the full hard/electric/local $O(g^2)$ matching and reduces its magnetostatic sector to two universal pure-Yang-Mills inputs with a defined lattice extraction.

5. Complete Standard-Model $O(g^2)$ static matching

This section completes the perturbative Standard-Model calculation of the static TT stiffness through $O(g^2)$. The central difficulty is not one especially large diagram; it is that several physically different momentum ranges contribute at the same order and must be matched without double counting. “Hard” modes carry momenta of order $\pi k_B T$; the screened electric and Higgs zero modes live at the softer $g k_B T$ scale; and the spatial non-Abelian zero modes form a confining magnetostatic sector at still longer distance. Matsubara modes are the discrete thermal frequencies that appear in the Euclidean finite-temperature formalism.

The accomplishment is that these regions can be assembled as parts of one Standard-Model metric Hessian. Their artificial factorization-scale dependence cancels, no independent gravitational counterterm is introduced, and no external physical cutoff is required. The final unresolved numbers belong only to the genuinely nonperturbative magnetostatic Yang-Mills sectors, for which Supplementary Appendix F gives a lattice extraction.

The useful organization separates the free contribution $A_{2,E}^{(0)}$, the leading resummed soft contribution $A_{2,E}^{(g)}$, the strict-hard two-loop term $A_{\mathrm{hard}}^{(g^2)}$, the electric/Higgs term $A_{\mathrm{E/H}}^{(g^2)}$, the matched local-curvature term $A_{R,\mathrm{match}}^{(g^2)}$, and the remaining magnetostatic contribution A_M^{NP}, where the superscript NP denotes nonperturbative:

$$A_{2,E}=A_{2,E}^{(0)}+A_{2,E}^{(g)}+A_{\mathrm{hard}}^{(g^2)}+A_{\mathrm{E/H}}^{(g^2)}+A_{R,\mathrm{match}}^{(g^2)}+A_M^{\mathrm{NP}}. \tag{19}$$

Here NLO means next-to-leading order. The four-dimensional modified minimal subtraction ($\overline{\mathrm{MS}}$, or MS-bar) scale is $\bar{\mu}$; the three-dimensional renormalization scale introduced below is μ_3. The gauge couplings g_3, g_2, and g_Y denote the strong, weak-isospin, and hypercharge couplings, respectively; y_t, y_b, and y_τ are the top-quark, bottom-quark, and tau-lepton Yukawa couplings; and λ is the Higgs quartic coupling. The Yukawa invariant entering the curvature running is $Y_2 = 3y_t^2 + y_b^2 + y_\tau^2$ at the retained accuracy. The scale separation follows the Standard-Model dimensional-reduction program of Kajantie, Laine, Rummukainen and Shaposhnikov [27]. Supplementary Appendix D contains the strict-hard reduction, Supplementary Appendix E the electric/Higgs and local-R matching, and Supplementary Appendix F the magnetostatic Yang-Mills and lattice construction.

The terms in Eq. (19) are therefore bookkeeping pieces, not separate phenomenological forces. Dimensional reduction means that short-distance thermal modes are integrated out and their effects are encoded in a lower-dimensional effective theory for the slower zero modes. Each momentum range is calculated with the degrees of freedom appropriate to it, and the arbitrary matching scales must disappear from the completed result. This organization also prevents double counting: an NLO screening mass cannot be inserted into the $O(g)$ ring term and then counted again through the two-loop three-dimensional graphs.

At the hard scale, $p \sim \pi k_B T$, the complete general-momentum Standard-Model TT operator algebra of Ghiglieri, Jackson, Laine and Zhu [24] contains five gauge-independent interaction classes: Higgs self-coupling, gauge-gauge, Yukawa, scalar-gauge, and fermion-gauge. In the following displayed slopes, the subscripts λ, gg, y, sg, and fg label those five classes, respectively. Their static k_z^2 master slopes can all be reduced analytically in dimensional regularization. At the central running point $\bar{\mu} = 2\pi k_B T$, the finite contributions to the OQG stiffness are

$$\frac{\delta A_{\lambda,\mathrm{hard}}}{(k_BT)^2}=-0.000015100213339,$$
$$\frac{\delta A_{gg,\mathrm{hard}}}{(k_BT)^2}=-0.001055183207970,$$
$$\frac{\delta A_{y,\mathrm{hard}}}{(k_BT)^2}=-0.000001464199449,$$
$$\frac{\delta A_{sg,\mathrm{hard}}}{(k_BT)^2}=+0.000029717504960,$$
$$\frac{\delta A_{fg,\mathrm{hard}}}{(k_BT)^2}=-0.000933360454277.$$

The subscript op denotes the operational, source-normalized sum of these strict-hard sectors. Thus

$$\frac{\delta A^{(2)}_{\mathrm{hard,op}}}{(k_BT)^2}=-0.001975390570075. \tag{20}$$

Let H_{fg} denote the dimensionless reduced fermion-gauge slope. This sector provides a useful analytic check on the reduction. Ninety-seven printed master structures collapse, after static power counting and mixed Bose/Fermi integration-by-parts identities, to nineteen products of one-loop thermal tadpoles. All double and simple ultraviolet poles cancel within that sector, and the exact reduced slope is

$$H_{fg}=-\frac{7}{432\pi^2}.$$

Equation (20) is the complete strict-hard connected-operator contribution. The metric-variational Hessian then adds the Higgs curvature contact derived in Sec. 4 and Supplementary Appendix C. Its hard interaction term and the hard pole/logarithmic data are precisely the matching information carried into the three-dimensional theory, where the Ward-completed variational response closes.

The renormalization check is especially restrictive. The beta function β_{ξ_H} describes the renormalization-scale running of ξ_H. The curved-space Standard-Model result derived by Markkanen, Nurmi, Rajantie and Stopyra gives [32]

$$16\pi^2\beta_{\xi_H}=\left(\xi_H-\frac{1}{6}\right)\left[12\lambda+2Y_2-\frac{3}{2}g_Y^2-\frac{9}{2}g_2^2\right],$$

so the complete explicit $O(g^2)$ scale dependence of the state-dependent stiffness must satisfy

$$\frac{\partial}{\partial\ln\bar{\mu}}\left[\frac{A^{(2)}_{2,E}}{(k_BT)^2}\right]_{\mathrm{explicit}}=-\frac{\beta_{\xi_H}}{12}. \tag{21}$$

At the central running state the right-hand side is

$$-\frac{\beta_{\xi_H}}{12}=-0.000080194657458+0.000481167944750\,\xi_H.$$

The full hard-plus-soft calculation reproduces this coefficient. In particular, the hard Higgs contact pole is canceled by the matched three-dimensional Higgs contact, and the surviving ξ_H-dependent logarithm is exactly the one required by Eq. (21). The scalar curvature coupling is therefore carried as an ordinary renormalized Standard-Model input; no special gravitational value is selected by the matching.

Below the hard scale, dimensional reduction leaves the Higgs and temporal gauge scalars as the electrostatic degrees of freedom. The Higgs-condensate matching follows Laine and Meyer, while the three-dimensional spin-2/local-curvature structure is anchored by the stress-tensor analysis of Corianò, Delle Rose and Skenderis [20,31]. Let $\delta A^{(2)}_{3D,\mathrm{pot}}$ denote the order-g^2 three-dimensional scalar-potential contribution. The finite nonderivative scalar-potential graphs give

$$\frac{\delta A^{(2)}_{\mathrm{3D,pot}}}{(k_B T)^2}=-0.000249653005245+0.001159183483123\,\xi_H.$$

The derivative/gauge sector is more subtle because spatial gauge exchange and its covariant-kinetic seagull must be kept together. Here q is the external three-momentum taken to zero in the massive scalar-gauge threshold calculation; it is distinct from the earlier tensor source $q_{\mu\nu}$. Evaluating the curvature coefficient directly at $q \to 0$ gives the finite threshold constant

$$c_{sg}^{(m)}=\frac{5}{18}-\ln 2=-0.415369402782168. \tag{22}$$

The corresponding complete massive electric derivative/gauge loop is

$$\frac{\delta A^{(2)}_{E,\mathrm{op}}}{(k_B T)^2}=+0.005669486183825.$$

A temporary magnetic infrared regulator mass m_M makes the Ward structure visible: the scalar-current exchange graph and the $A^2\phi^\dagger\phi$ seagull contain equal and opposite $1/m_M$ terms. Their cancellation shows that the electric calculation does not import an additional $O(g)$ magnetic contribution and leaves the pure magnetostatic sector cleanly separated.

The same calculation exposes two independent renormalization coordinates. The four-dimensional hard theory uses $\bar{\mu}$, whereas the three-dimensional effective theory has its own scale μ_3. Let D_{soft} and $D_{R,\mathrm{diag}}$ denote the coefficients of the soft-scale and diagonal curvature logarithms, respectively, in the three-dimensional matching. The local three-dimensional curvature coefficient is

$$C_R^{3D}(\bar{\mu},\mu_3)=D_{\mathrm{soft}}\ln\frac{\bar{\mu}}{\mu_3}+D_{R,\mathrm{diag}}\ln\frac{\bar{\mu}}{2\pi k_B T}, \tag{23}$$

with

$$D_{\text{soft}}\text{=-}0.000520047975283, \qquad D_{R,\text{diag}}\text{=-}0.000613202644155.$$

The first coefficient cancels the intrinsic electric-plus-magnetic three-dimensional running, while the second is the residual derivative observed when $\bar{\mu}$ and μ_3 are moved together. At the central choice

$$\bar{\mu}\text{=}2\pi k_B T, \qquad \mu_3\text{=}g_2^2 k_B T,$$

Eq. (23) gives

$$C_R^{3D}\text{=-}0.001671619097302. \tag{24}$$

A complete second-metric-variation audit leaves no additional state-dependent finite k^2 boundary. The scalar sector's only two-derivative contact is the already included $\xi_H R H^\dagger H$ term; the Dirac and Yang-Mills sectors contain no analogous two-metric-derivative contact; and a state-independent four-dimensional Einstein term cancels from the state-subtracted thermal Hessian. Thus the perturbative hard, electric/Higgs, Ward/contact, and local-R matching problem closes without an arbitrary finite gravitational constant.

Combining the pieces through this point gives

$$\frac{A_{2,E}}{(k_B T)^2}\text{=}0.263713812532203\text{+}0.066623718018839\,\xi_H\text{+}\frac{A_M^{\text{NP}}(\mu_3)}{(k_B T)^2}. \tag{25}$$

The final term in Eq. (25) is the pure non-Abelian magnetostatic contribution and is naturally expressed in the confining three-dimensional Yang-Mills theories. After the hard and electric scales are integrated out, the remaining non-Abelian magnetic theories are pure three-dimensional $SU(3)$ and $SU(2)$ Yang-Mills. Writing $g_{M,3}^2$ and $g_{M,2}^2$ for the three-dimensional magnetostatic gauge couplings, their intrinsic scales are

$$\lambda_3\text{=}3g_{M,3}^2, \qquad \lambda_2\text{=}2g_{M,2}^2,$$

The known ultraviolet running reduces the finite magnetostatic information to one pure number for each gauge group. For group G, λ_G is its intrinsic three-dimensional scale, c_G^{NP} is its finite nonperturbative constant, and $D_{M,G}$ is its known magnetostatic logarithmic coefficient:

$$\frac{A_{M,G}^{\text{NP}}}{(k_B T)^2}\text{=}D_{M,G}\left[\ln\frac{\mu_3}{\lambda_G}\text{+}c_G^{\text{NP}}\right].$$

At the central matching scale,

$$D_{M,SU(3)}\text{=}0.002999279573124, \qquad D_{M,SU(2)}\text{=}0.000799379975567,$$

and therefore the complete static result is

$$\frac{A_{2,E}}{(k_B T)^2} = 0.260056643133723 + 0.066623718018839\,\xi_H$$
$$+0.002999279573124\,c_3^{\text{NP}} + 0.000799379975567\,c_2^{\text{NP}}. \tag{26}$$

The remaining constants in Eq. (26) are not unspecified OQG couplings. They are finite pure-Yang-Mills numbers in a fixed state-subtracted $\overline{\mathrm{MS}}$ convention. Existing glueball masses and stress-tensor residues do not determine them because the published superconvergent observable deliberately removes the local polynomial k^2 information that the present susceptibility requires.

That apparent obstacle can nevertheless be converted into a sharply defined lattice observable. For one magnetostatic group, let g_{ij} denote the three-dimensional spatial metric and $h(z)$ a weak off-diagonal shear source varying along z. Impose the generating shear

$$g_{ij}(z)=\begin{pmatrix} 1 & h(z) & 0 \\ h(z) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Let F^a_{ij} denote the Yang-Mills field strength, g_M the magnetostatic gauge coupling, S the Euclidean action, and $\mathcal{L}_{\mathrm{YM}}$ its Lagrangian density. The first and second source derivatives are exactly

$$T_{12}=\frac{1}{g_M^2}F^a_{13}F^a_{23}, \qquad \left.\frac{\delta^2 S}{\delta h^2}\right|_{h=0}=\mathcal{L}_{\mathrm{YM}},$$

Let R denote the three-dimensional Ricci scalar constructed from g_{ij}. The local curvature term obeys

$$\int d^3x\,\sqrt{g}\,R=-\frac{1}{2}\int d^3x\,(\partial_z h)^2+O(h^3).$$

Thus the coefficient of k^2 in the full shear Hessian is exactly the renormalized three-dimensional curvature coefficient. More importantly, an absolute lattice Einstein counterterm need not be calculated. Let a denote the lattice spacing, $\mathcal{G}^L_G(k)$ the full lattice shear Hessian for group G, and $\mathcal{S}^L_G(k_a,k_b)$ its finite-difference k^2 slope between momenta k_a and k_b. Define the secant

$$\mathcal{S}^L_G(k_a,k_b)=\frac{\mathcal{G}^L_G(k_a)-\mathcal{G}^L_G(k_b)}{\hat{k}_a^2-\hat{k}_b^2}, \qquad \hat{k}=\frac{2}{a}\sin\frac{ak}{2},$$

Subtract an ultraviolet (UV) secant from an infrared (IR) secant on the same lattice. The momentum pair k_{i1},k_{i2} is taken in the infrared window and k_{u1},k_{u2} in the ultraviolet matching window. Every additive local-R contribution cancels. Let $C_G^{\overline{\mathrm{MS}}}(\mu_3)$ denote the continuum curvature coefficient and $\mathcal{S}^{\overline{\mathrm{MS}}}_{G,\mathrm{pert}}$ the perturbative ultraviolet secant that restores the absolute continuum scheme. With the three-loop pure-Yang-Mills ultraviolet anchor of Caron-Huot, Pokraka and Zahraee, the matching is [16]

$$\begin{aligned} C_G^{\overline{\mathrm{MS}}}(\mu_3)=&\mathcal{S}^{\overline{\mathrm{MS}}}_{G,\mathrm{pert}}(k_{u1},k_{u2};\mu_3)\\ &+\lim[\mathcal{S}^L_G(k_{i1},k_{i2})-\mathcal{S}^L_G(k_{u1},k_{u2})]. \end{aligned} \tag{27}$$

The continuum limit is taken first, the infinite-volume limit second, and the infrared $k \to 0$ limit last. The background shear is only a generating device, so the measurement can be performed on ordinary flat $SU(3)$ and $SU(2)$ ensembles; no curved-metric ensemble is required.

Let $C_3^{\overline{\mathrm{MS}}}$ and $C_2^{\overline{\mathrm{MS}}}$ denote the continuum-matched $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ infrared slopes, respectively. Once they are known, the magnetostatic stiffness can be inserted directly without introducing the intermediate c_G^{NP} notation:

$$\frac{A_{2,E}}{(k_B T)^2}=0.263713812532203+0.066623718018839\,\xi_H+\frac{C_3^{\overline{\mathrm{MS}}}+C_2^{\overline{\mathrm{MS}}}}{2k_B T}. \tag{28}$$

Equation (28) marks the natural boundary of the analytic static program. The complete perturbative Standard-Model matching is fixed, the Ward and two-scale renormalization ledgers close, the finite massive electric threshold is known, and the remaining magnetic information has been reduced to two well-defined pure-Yang-Mills observables with an explicit lattice-to-$\overline{\mathrm{MS}}$ extraction. No finite coefficient has been chosen to force the result. The next section turns to the independent causal side of the same completed Standard-Model response and shows that its physical finite-(ω, k) structure is a massless long-range branch together with an ordinary underdamped matter resonance rather than a forced weak-coupling critical merger.

6. Finite-temperature causal TT kernel and the natural underdamped branch

Sections 4 and 5 determined the *static* strength of the TT response. This section asks the complementary dynamical question: what happens when the same Standard-Model state is driven at finite frequency and finite wave number? The answer requires the language of complex response functions. A **pole** represents a discrete resonance, a **branch cut** represents a continuum of allowed excitations, and a different **Riemann sheet** is the analytic continuation reached by crossing such a cut.

With those terms fixed, the physical picture is simple. In the controlled weak-coupling regime the TT response contains a massless long-range branch, two continua (a Landau/scattering continuum and a two-particle-production continuum), and one ordinary underdamped resonance on the two-particle-production sheet. The resonance describes microscopic coherence and damping of matter; it is not needed to generate the long-range interaction and it does not naturally approach critical damping in the ordinary state.

The long-range part is already visible in the two-derivative inverse response derived in Sec. 3. Let $D_{TT,E}(0,k)$ denote the static Euclidean transverse-traceless compliance at spatial wave-number magnitude k. In the static Euclidean projection,

$$D_{TT,E}(0,k)=\frac{1}{A_{2,E}k^2+O(k^4)} \qquad (k \to 0). \tag{29}$$

Thus the compliance contains the massless $1/k^2$ branch required for a long-range interaction before any finite-frequency resonance is discussed. The role of the causal calculation is instead to determine the additional microscopic dispersive and absorptive structure carried by the same stress Hessian.

A thermal equilibrium state selects a rest frame, so temporal frequency and spatial wave number become independent response coordinates. Let z be the complex retarded frequency, k the spatial wave-number magnitude, T the equilibrium temperature, k_B Boltzmann's constant, c the locally measured limiting signal speed, and $\hbar$ the reduced Planck constant. It is useful to write

$$u=\frac{\hbar z}{k_B T}, \qquad y=\frac{\hbar c k}{k_B T}, \tag{30}$$

The real-time free-field structures for scalar, Weyl and gauge sectors follow the thermal spinning-field response of Ota, Zhu and Zhu and the finite-temperature graviton self-energy decomposition of Brandt and Frenkel [11,36]. Let ω denote a real positive angular frequency. The collisionless tensor response contains two physically distinct continua. The scattering or Landau cut occupies $0 < \omega < ck$, while the two-particle-production cut begins at $\omega > ck$ for massless constituents. These two cuts are not artifacts of a particular representation; they are the real-time manifestation of the distinct kinematic processes available to the thermal matter state.

This is also why the dynamical and static limits must be kept separate. The Landau continuum extends all the way to the origin along the real-frequency direction, so the response remembers the path by which (ω, k) approaches $(0,0)$. A static shear experiment and a collisionless low-frequency experiment therefore probe different limits of the same kernel. In general,

$$\lim_{\omega\to 0}\lim_{k\to 0}\Pi^R_{TT}(\omega,k) \neq \lim_{k\to 0}\lim_{\omega\to 0}\Pi^R_{TT}(\omega,k). \tag{31}$$

The Euclidean static stiffness of Sec. 4 is therefore a legitimate equilibrium projection, but it should not be confused with the collisionless retarded limit taken along another path. Both belong to the same matter response, but they answer different physical questions.

The general-kinematics thermal master functions of Jackson and the complete Standard-Model TT algebra of Ghiglieri, Jackson, Laine and Zhu make this construction explicit [24,26]. The finite-ω, k kernel can be constructed without relying on unstable near-cut two-dimensional quadrature. After subtracting the local polynomial terms already assigned to the static response, let $R(z,k)$ denote the resulting dimensionless nonlocal thermal TT response factor and $R^I(z,k)$ its principal-sheet reconstruction. The notation Im means imaginary part, and $+i0$ means that the real axis is approached from the retarded upper side. For fixed real k, the principal-sheet response is obtained from the exact real-axis discontinuity by the subtracted spectral relation

$$R_{\mathrm{I}}(z,k)=\frac{2}{\pi}\int_0^\infty d\,\omega\;\frac{\omega\,\mathrm{Im}\,R(\omega+i0,k)}{\omega^2\text{-}z^2}. \tag{32}$$

The two-particle-production and Landau/scattering intervals are integrated separately. The scalar, Weyl and gauge-vector contributions combine with the same physical TT weights established in Supplementary Appendix B. Numerically, this representation reproduces direct imaginary-axis evaluations of the full collisionless Standard-Model thermal kernel to better than one part in 10^9 at the benchmark wave number, which makes it suitable for distinguishing true sheet poles from discretized representations of a continuum. Supplementary Appendix G gives the full spectral reconstruction, sheet continuation, pole-convergence, Landau-cut, and interaction-topology derivations summarized in this section.

A decaying resonance is found after the retarded boundary value is continued onto the adjacent sheet of the appropriate cut. Write $\mathrm{Disc}_P R$ for the discontinuity across the positive-frequency two-particle-production cut, use the label II, P for the adjacent two-particle-production sheet, and let s denote the local argument of the logarithmic factor whose branch cut is crossed. The label P denotes the two-particle-production cut throughout, and Log on sheet I is the principal complex logarithm. Then

$$R_{\mathrm{II},P}(z,k)=R_{\mathrm{I}}(z,k)+\mathrm{Disc}_P R(z,k), \qquad \mathrm{Log}_{\mathrm{II},P} s=\mathrm{Log} s\text{-}2\pi i, \tag{33}$$

where the logarithmic continuation is applied only to the cut actually crossed. The Landau continuation uses its own thermal discontinuity and does not inherit this vacuum-log shift. This sheet-by-sheet construction is essential: a global replacement of every logarithm by Log-$2\pi i$ does not represent the retarded analytic structure correctly.

Let $F_{\mathrm{II},P}(u,y)$ be the dimensionless zero-forming inverse-response eigenvalue on that positive-frequency two-particle-production sheet, so a pole satisfies $F_{\mathrm{II},P}(u_P,y)=0$. At

$$y=0.758224684,$$

the physical positive-frequency two-particle-production sheet contains the stable pole

$$u_P=1.353089335\text{-}0.093439111\,i, \qquad \left|\partial_u F_{\mathrm{II},P}(u_P)\right|=5.39112. \tag{34}$$

The derivative is decisively nonzero. The pole is therefore simple. Here $\mathrm{Re}u_P$ and $\mathrm{Im}u_P$ denote the real and imaginary parts of the pole coordinate. Its damping-to-oscillation ratio is

$$\frac{\text{-}\mathrm{Im}\,u_P}{\mathrm{Re}\,u_P}=0.0691.$$

It is an ordinary underdamped resonance rather than an exceptional point. The distinction is not semantic: at an exceptional point both the inverse response and its first derivative vanish, whereas Eq. (34) shows that the known causal resonance branch on the two-particle-production sheet is far from that condition.

The same pole can be tracked continuously over real wave number. Representative collisionless values are

y	$u_P(y)$
0.4	0.9600-0.0629 i
0.7582	1.35309-0.09344 i
1.0	1.58615-0.10564 i
2.0	2.48635-0.11775 i
3.0	3.38920-0.10557 i
4.0	4.31703-0.08972 i
5.0	5.26477-0.07568 i

and the magnitude of $\partial_u F$ grows rather than tends toward zero across the upper part of this range. Extending the branch much farther in y preserves the same qualitative behavior: the ordinary resonance remains underdamped, and the damping fraction decreases at sufficiently large momentum.

The Landau sector behaves differently. Finite-quadrature representations generate many apparent zeros immediately below the Landau interval, but their imaginary parts shrink in inverse proportion to the spectral resolution. They therefore collapse onto the branch cut rather than converge to isolated physical poles. The causal matter response contains a Landau continuum, not a hidden forest of stable long-lived modes. This numerical convergence test is important because near-cut discretization can otherwise mimic precisely the sort of extra pole one might incorrectly interpret as new gravitational physics.

Interactions do not change this topology easily. The first controlled Standard-Model interaction analysis separates three effects: thermal dispersion shifts, ordinary quasiparticle widths, and the finite Ward-completed vertex/self-energy deformation. The need to keep self-energy and gravitational-vertex resummation together follows the Ward-complete thermal polarization analysis of Nachbagauer, Rebhan and Schwarz; the kinetic-theory domain and hard damping estimates are consistent with Arnold, Moore and Yaffe and with Burgess and Marini [6,12,35]. The first two effects preserve root multiplicity when treated as translations of the existing analytic structure. Let F_0 denote the unbroadened inverse response and u_0 one of its simple pole coordinates. If Γ is a physical decay rate, define the dimensionless width $\hat{\Gamma} \equiv \hbar\Gamma/(k_B T)$. A common hard width then produces

$$F_\Gamma(u,y)=F_0\big(u+i\hat{\Gamma},y\big) \quad \Rightarrow \quad u_\Gamma=u_0-i\hat{\Gamma}, \qquad F_\Gamma{}'(u_\Gamma)=F_0{}'(u_0). \tag{35}$$

A width moves the pole deeper into the lower half-plane but cannot make a simple root double. The same statement holds for a pure invariant thermal-mass translation as long as the shifted pole remains away from the origin.

A two-particle-production threshold also does not become an exceptional point merely because a pole reaches it. Near a two-particle branch point $u_{\rm th}$, introduce the uniformizing coordinate

$$w=\sqrt{u-u_{\rm th}}. \tag{36}$$

Let $F(w)$ denote the inverse response re-expressed in the uniformizing coordinate w. It is locally analytic in w. A simple pole passing through $w=0$ changes sheets while remaining a simple zero. An exceptional point requires the additional independent condition

$$F(0)=0, \qquad \partial_w F(0)=0. \tag{37}$$

This distinction is central to the interacting topology. Threshold crossing is generically a sheet transition, not a critical pole merger.

The quasiparticle-pole interpretation used in this high-momentum discussion follows Weldon, while the non-Abelian asymptotic-mass organization and its nonperturbative context are discussed by Moore and Schlusser [33,43]. The leading Standard-Model thermal masses make the nearest gauge thresholds explicit. Let $m_{\infty,a}$ denote the leading asymptotic thermal quasiparticle mass for field class a, and define its frequency scale by $\Omega_{\infty,a}=m_{\infty,a}c^2/\hbar$. Then

$$\omega_{\mathrm{th},a}=\sqrt{c^2k^2+4\Omega_{\infty,a}^2}. \tag{38}$$

At the benchmark wave number, the leading dressed thresholds are approximately $1.233752\,k_BT/\hbar$ for gluons, $1.225000\,k_BT/\hbar$ for the weak gauge bosons, and $1.187933\,k_BT/\hbar$ for hypercharge, while the real part of the causal resonance is $1.353089\,k_BT/\hbar$. Including the corresponding leading widths moves the nearest branch points into the lower half-plane. Their complex separations from the pole are approximately $0.120\,k_BT/\hbar$ for the gluon two-particle branch point and $0.134\,k_BT/\hbar$ for the weak two-particle branch point. The pole is close enough that the exact finite interaction correction matters for precision, but it is not coincident with either threshold.

The size of the deformation required to change the local topology can be estimated directly from the known simple root u_0. Let δF be a perturbation of the inverse response about that root and δu the induced pole shift. To first order,

$$\delta u=-\frac{\delta F(u_0,y)}{F_0{}'(u_0,y)}+O(\delta F^2). \tag{39}$$

Dragging the benchmark pole all the way to the nearest dressed gluon branch point requires a normalized change of the inverse response of approximately 0.65, and reaching the weak branch point requires approximately 0.72. Those are order-one deformations of the zero-forming function, not ordinary percent-level shifts. The complete two-loop Standard-Model TT algebra is gauge independent in its assembled sectors, and the benchmark kinematics lies in the hard timelike regime addressed by the general-kinematics thermal master functions, well outside the external soft/light-cone region where fixed-order thermal perturbation theory loses its ordinary power counting. No known soft, collinear, threshold, polarization-mixing or kinetic mechanism supplies the anomalous enhancement needed to change the local root count.

The helicity structure supplies one further stability check. In an isotropic parity-invariant state, rotations about the propagation axis keep helicities +2 and -2 distinct while parity keeps their eigenvalues equal. The physical TT block therefore remains proportional to the identity in the two-helicity subspace. This degeneracy is semisimple: the two polarization eigenvectors remain independent, so ordinary Standard-Model interactions cannot turn polarization degeneracy itself into a non-Hermitian exceptional point.

The resulting causal picture is consequently simple and physically interpretable:

$$
\begin{aligned}
&\text{massless long-range branch} \\
&\quad + \text{one underdamped causal TT resonance} \\
&\qquad \text{on the two-particle-production sheet} \\
&\quad + \text{Landau/scattering and two-particle-production continua.}
\end{aligned}
\tag{40}
$$

The controlled interaction deformation adds ordinary dispersion and damping while leaving that topology intact. The nonzero derivative at the pole and the interaction bounds fix the branch classification: higher-order corrections refine its coordinate, reactive dispersion scale and damping rate without changing the established simple underdamped topology in the absence of a new singular mechanism.

This result also restores a useful physical hierarchy. Long-range attraction is carried by the massless inverse-response branch of Eq. (29). The finite-frequency pole characterizes the microscopic causal response of the matter state and may set a finite correlation or dispersion scale. It does not have to become critically damped in order for gravity to exist. Ordinary weakly coupled Standard-Model matter instead realizes an underdamped response, while the critical-damping idea belongs, if realized at all, to a separate strong-field boundary. Section 7 then places the principal-sheet analytic double-zero benchmark, finite spatial response and strong-field boundary into their proper relation while preserving the ordinary causal pole derived here as its own physical branch.

7. Finite wave number, analytic benchmark, and the strong-field boundary

This section separates three response structures that must not be confused: the ordinary retarded resonance of weakly coupled matter, an exact double zero of a simplified analytic kernel, and the proposed critical self-loading boundary of an extreme strong-field state. The first is a physical causal pole, the second is a mathematical benchmark, and the third is a constitutive boundary condition. Keeping those roles separate prevents an ordinary matter resonance from being forced to satisfy a horizon-scale condition.

For readers following the complex analysis, the **principal sheet** is the default branch of the logarithm before any physical cut is crossed; the **retarded sheet** relevant to a decaying resonance is reached by analytic continuation through the appropriate cut. Supplementary Appendix G gives the detailed sheet construction, convergence tests, interaction-topology analysis, and strong-field comparison. The general-momentum Standard-Model tensor organization and thermal spectral masters follow Ghiglieri, Jackson, Laine and Zhu and Jackson, while the Ward-complete interaction consistency check follows Nachbagauer, Rebhan and Schwarz [24,26,35].

Finite spatial momentum is the first place where this distinction becomes unavoidable. A Kubo-Martin-Schwinger (KMS) equilibrium state selects a rest frame, so the temporal and spatial response coordinates are no longer related only by a Lorentz-invariant combination. Let z denote the complex frequency, let $k \equiv |\mathbf{k}|$ denote the real spatial wave-number magnitude, let T be the physical KMS temperature, let k_B be Boltzmann's constant, and let c be the locally measured limiting signal speed. Define the dimensionless variables $u \equiv \hbar z/(k_B T)$ and $y \equiv \hbar c k/(k_B T)$. Writing the pole on the two-particle-production sheet in terms of a real reactive coordinate $\mathcal{E}_P(y)$ and a positive dimensionless damping coordinate $\gamma_P(y)$,

$$u_P(y)=\mathcal{E}_P(y)-i\,\gamma_P(y), \tag{41}$$

the real part $\mathcal{E}_P$ describes reactive dispersion and $\gamma_P > 0$ describes decay. This decomposition is preferable, once interactions are present, to treating the entire quantity u_P^2-y^2 as one fixed complex "mass." Thermal forward scattering shifts the reactive dispersion while collisions supply an additional width, and those two effects need not share the same large-y asymptote.

The collisionless calculation nevertheless provides a useful guide to the finite-wave-number structure. At small real wave number the tracked resonance branch is gapless and behaves approximately as

$$u_P^2(y) \simeq 2.28\,y \qquad (y \ll 1), \tag{42}$$

whereas at large real wave number the collisionless branch approaches

$$u_P^2(y)\text{-}y^2 \to 2.931\text{-}0.480\,i. \tag{43}$$

Equation (43) is not promoted to a permanent complex particle mass. It is the asymptotic form of the collisionless thermal branch. In the interacting theory the cleaner observables are the reactive dispersion scale obtained from $\mathcal{E}_P^2$-y^2 and the damping rate γ_P separately. Ordinary hard Standard-Model collisions dominate the collisionless $1/y$ decay width beyond several thermal momenta while the fractional damping continues to decrease, so the large-y continuation remains naturally underdamped rather than evolving toward a critical pole merger.

The static finite-wave-number response provides an independent projection. The scalar, Weyl and vector expressions can all be evaluated at zero Matsubara frequency while retaining nonzero spatial momentum, and their small-y slopes independently reproduce the Kubo coefficients of Sec. 4. This is a strong normalization check because the finite-y functions are not obtained by fitting the zero-momentum answer. Let $F_{\text{stat}}(y)$ denote the dimensionless static transverse-traceless inverse-response factor in this projection. When the leading resummed soft contribution is combined with the free thermal functions, F_{stat} develops two positive roots,

$$F_{\text{stat}}(y_i)=0, \qquad {F_{\text{stat}}}'(y_i) \neq 0, \qquad i=1,2. \tag{44}$$

Neither root is a spatial critical double zero. More importantly, when the same free thermal model is recomputed without the soft ring contribution, the zeros disappear and are replaced by a shallow positive minimum. The finite-y roots are therefore useful diagnostics of thermal spatial dispersion rather than fundamental correlation lengths. They show directly that a thermal rest frame breaks the simple temporal/spatial coincidence of the vacuum skeleton; they do not authorize the introduction of a new spatial scale by hand.

The leading one-log model provides a useful analytic benchmark because it can be solved exactly. Here Log$x$ means the principal complex logarithm, and the Lambert-W function used below is defined as the inverse of $w \mapsto we^{w}$. Let x be the dimensionless invariant coordinate of this one-log representation and let $\mathcal{A} > 0$ denote its dimensionless matter-response coefficient. The zero-forming factor is

$$\mathcal{F}_{\mathrm{an}}(x;\mathcal{A})=1+\frac{x\mathrm{Log}x}{\mathcal{A}}. \tag{45}$$

Its nonzero roots satisfy

$$x\mathrm{Log}x=-\mathcal{A},$$

and are generated by the two real Lambert-W branches W_0 and W_{-1}, defined by $W(q)e^{W(q)}=q$,

$$x_{\pm}=\exp\left[W_{0,-1}(-\mathcal{A})\right]. \tag{46}$$

At

$$\mathcal{A}_c=\frac{1}{e}$$

the two principal-sheet roots merge at

$$x_c=\frac{1}{e},\qquad \mathcal{F}_{\mathrm{an}}(x_c)=0,\qquad \mathcal{F}_{\mathrm{an}}{}'(x_c)=0,\qquad \mathcal{F}_{\mathrm{an}}{}''(x_c)=e^2\neq 0. \tag{47}$$

Equation (47) is an exact nondegenerate double zero of the leading analytic representation. The assembled leading interaction correction alters the coefficient that maps the Standard-Model matter content into $\mathcal{A}$ but does not destroy this Lambert-W branch topology, and the leading thermal statistical memory likewise leaves a well-defined principal-sheet benchmark. At the central thermal state the corresponding dimensionless root magnitude is 3.507691515625.

The causal meaning is nevertheless different. A decaying physical resonance is obtained by continuing the retarded response through the appropriate cut into the lower half-plane. As Sec. 6 emphasized, that continuation must be performed cut by cut. The derivative condition satisfied by Eq. (47) on the principal logarithmic sheet is changed by the second-sheet discontinuity. The leading retarded-sheet calculation therefore does not identify the Lambert merger with the physical resonance on the two-particle-production sheet of Eq. (34). The principal-sheet double zero remains valuable as an analytic benchmark against which interaction and memory corrections can be organized; it is not relabeled as a critical-damping pole merely because both calculations contain a double-root condition.

This separation also clarifies the role of complex spatial continuation. Let $m_\infty^2 \equiv 2.931 - 0.480\,i$ denote the dimensionless collisionless asymptotic constant in Eq. (43), and let m_∞ denote a chosen square root of m_∞^2, with the branch selected consistently for the reduced dispersion. If one approximates the high-y collisionless branch by

$$u_\pm(y) \simeq \pm\sqrt{y^2+\mathfrak{m}_\infty^2},$$

the reduced dispersion has square-root branch points at formally complex wave number y=$\pm i\mathfrak{m}_\infty$. That observation is useful as a guide to a spatial coherence problem, but it is not an established exceptional point of the complete thermal kernel. Continuing the exact thermal integrals to complex y rotates internal integration contours through the poles of the Bose and Fermi distributions, so the correct continuation requires contour deformation and the associated thermal residues. Spatial screening, temporal pole position and causal attenuation are therefore distinct response observables in a thermal state.

The strong-field critical boundary belongs to a third level of the construction. The earlier damping formulation used the conventional damping ratio ζ, with an underdamped parameter range $0 \le \zeta < 1$ and the ordinary real-frequency branch terminating at ζ=1. In the present language, however, the strong-field boundary should be defined directly from the complete retarded tensor response rather than from a constant-coefficient Markov oscillator. Let $\mathcal{K}_{TT}^R(z,k;\mathcal{S})$ denote the relevant eigenvalue of the retarded inverse transverse-traceless response in physical state $\mathcal{S}$. For a putative critical state $\mathcal{S}_\star$, let $z_\star$ and $k_\star$ denote its complex frequency and spatial wave number. The non-Markovian critical condition is

$$\mathcal{K}_{TT}^R(z_\star,k_\star;\mathcal{S}_\star)=0, \qquad \partial_z\mathcal{K}_{TT}^R(z_\star,k_\star;\mathcal{S}_\star)=0, \qquad \partial_z^2\mathcal{K}_{TT}^R(z_\star,k_\star;\mathcal{S}_\star) \neq 0. \tag{48}$$

Equation (48) is the proper field-theoretic analogue of critical damping. The weakly coupled KMS resonance calculated in Sec. 6 does not satisfy it and is not expected to. If such a state is realized at all, it belongs to a separate extreme self-loaded branch rather than to ordinary matter.

The older operational model associated this limiting condition with the Schwarzschild horizon. The defensible statement is correspondingly asymptotic. Let $\Omega_\star$ denote the critical state's angular frequency and define its pole energy by

$$E_\star \equiv \hbar\Omega_\star$$

and microscopic correlation length

$$\xi_\star=\frac{\hbar c}{E_\star}.$$

For that same state, let the state-specific macroscopic compliance be $G_{\mathrm{comp}}[S_\star] \equiv \hbar c^5/\big(32\pi A_{2,E}[S_\star]\big)$. Its gravitational radius is defined from the asymptotic Einstein-equivalent infrared field,

$$r_g(E_\star;\mathcal{S}_\star)=\frac{2G_{\mathrm{comp}}(\mathcal{S}_\star)E_\star}{c^4},$$

rather than by assuming that the local Schwarzschild metric remains the microscopic field equation at r=$\xi_\star$. The OQG strong-field self-loading identification is then

$$r_g(E_\star;\mathcal{S}_\star)=\xi_\star.$$

Using this state-specific infrared stiffness/compliance relation gives

$$\frac{A_{2,E}(\mathcal{S}_\star)}{E_\star^2}=\frac{1}{16\pi}. \tag{49}$$

Equation (49) is therefore a constitutive boundary condition on a *critical strong-field state*. It is not the normalization of the ordinary KMS resonance, it is not used to define an auxiliary ultraviolet energy, and it is not imposed on the static Standard-Model coefficient calculated in Sec. 5. The state entering Eq. (49) would have to be the same state whose full causal kernel actually satisfies Eq. (48).

This removes the apparent conflict between the natural underdamped branch and the older critical-damping intuition. Ordinary weakly coupled matter realizes Eq. (40): a massless long-range response together with an underdamped finite-frequency resonance and the two-particle-production and Landau/scattering continua. The principal-sheet Lambert merger of Eq. (47) is an exact analytic feature of a leading representation. The horizon-scale condition of Eq. (49) is a separate proposed strong-field boundary. None has to be substituted for another.

The distinction is physically useful. The long-range gravitational interaction does not depend on the ordinary resonance becoming critical; it is already contained in the massless $1/k^2$ compliance. The ordinary resonance pole on the two-particle-production sheet describes microscopic coherence, dispersion and damping of the matter state. The analytic double zero supplies a controlled benchmark for the response function. Critical self-loading, if Nature realizes it, characterizes an extreme branch beyond the weakly coupled state treated here. The next section can therefore construct the Einstein-equivalent infrared description directly from the massless two-helicity sector without asking either the ordinary resonance or the principal-sheet benchmark to carry the burden of generating gravity.

8. Minimal Weyl-transverse-diffeomorphism (WTDiff) infrared completion and vacuum-shift blindness

This section performs two infrared tasks. First, it translates the matter-generated long-range TT response into the minimal local geometrical language appropriate at macroscopic distance. Second, it shows what that fixed-measure representation does—and does not do—to a homogeneous vacuum-stress offset.

The Standard-Model calculation supplies the response coefficient; it does not by itself choose the geometrical redundancy. Fixed-measure Weyl-transverse diffeomorphism (WTDiff) supplies the selected massless two-helicity infrared representation. At the minimal local two-derivative level, conservation and the contracted Bianchi identity then lead to the Einstein-equivalent macroscopic equation with a cosmological integration datum. The same fixed-measure construction makes an arbitrary homogeneous metric-proportional stress *zero point* invisible to the local connected source, while leaving physical vacuum response intact.

This completion uses the fixed-measure operational metric $\hat{g}_{\mu\nu}$ introduced in Eq. (8). Its determinant is not an independently varied local degree of freedom, and the associated Weyl-transverse redundancy preserves the selected massless two-helicity representation of the Standard-Model TT response [4,5,7,15]. The microscopic Standard Model itself is not being declared Weyl invariant. Masses, spontaneous symmetry breaking, running couplings and the trace anomaly remain in the matter theory and in its completed Hessian. WTDiff is the symmetry of the *infrared geometrical representative* of the long-range response.

At the lowest local derivative order, let $R_{\mu\nu}$ and R denote the Ricci tensor and Ricci scalar of $g_{\mu\nu}$; let $T_{\mu\nu}$ be the macroscopic matter stress with trace $T \equiv g^{\mu\nu}T_{\mu\nu}$; and let $c[\rho]$ be the locally measured limiting signal speed in state ρ. The quantity $G_{\text{comp}}[\rho]$ is the macroscopic compliance matched to that branch's state through $A_{2,E}[\rho]$ in Eq. (17), not a universal microscopic constant. Trace-free variation gives

$$R_{\mu\nu}-\frac{1}{4}\hat{g}_{\mu\nu}R=\frac{8\pi G_{\text{comp}}}{c^4}\left(T_{\mu\nu}-\frac{1}{4}\hat{g}_{\mu\nu}T\right). \tag{50}$$

Equation (50) is the nonlinear completion of the same infrared tensor sector whose quadratic kernel begins with Eq. (10). No measured value of Newton's constant G is inserted into the microscopic calculation to obtain the tensor stiffness; the geometrical equation simply expresses the reciprocal matter compliance in its conventional macroscopic form.

The significance of Eq. (50) is easiest to see by following the integrability condition. Let ∇_μ denote the Levi-Civita covariant derivative compatible with $g_{\mu\nu}$. Within the local two-derivative equation for a connected settled branch, the common OQG scaling makes $G_{\text{comp}}[\rho]/c[\rho]^4$ covariantly invariant. This—not constancy of G_{comp} by itself—is the condition used when taking the divergence. The complete matter stress obeys covariant conservation on that branch,

$$\nabla^\mu T_{\mu\nu}=0,$$

while the contracted Bianchi identity gives

$$\nabla^\mu\left(R_{\mu\nu}-\frac{1}{2}\hat{g}_{\mu\nu}R\right)=0.$$

Taking the divergence of Eq. (50) therefore yields

$$\nabla_\nu\left[R+\frac{8\pi G_{\text{comp}}}{c^4}T\right]=0. \tag{51}$$

On each connected settled branch, Eq. (51) implies the existence of a branchwise integration datum Λ of the macroscopic geometrical representation, defined by

$$R+\frac{8\pi G_{\mathrm{comp}}}{c^4}T=4\Lambda. \tag{52}$$

Define the Einstein tensor by $G_{\mu\nu} \equiv R_{\mu\nu}\text{-}\,1/2\,\hat{g}_{\mu\nu}R$. Substituting Eq. (52) back into the trace-free equation gives

$$G_{\mu\nu}+\Lambda\,\hat{g}_{\mu\nu}=\frac{8\pi G_{\mathrm{comp}}}{c^4}T_{\mu\nu}. \tag{53}$$

Thus the minimal local two-derivative WTDiff completion is Einstein-equivalent in the infrared. The result follows from the matter-generated TT stiffness, the fixed-measure tensor representation, conservation and the Bianchi identity. General covariance is a consistency property of the macroscopic description; it is not being used to manufacture the microscopic coefficient that controls the response.

This ordering also fixes the status of the nonlinear statement. The Standard-Model two-point calculation determines the quadratic long-range sector and its stiffness. Equation (53) is the minimal local, two-derivative nonlinear completion that preserves the WTDiff gauge structure of that sector. It does not assert that the connected three-stress and four-stress kernels (conventionally denoted TTT and $TTTT$), or the higher connected stress kernels, have already been evaluated and shown to generate every nonlinear vertex uniquely. The result established here is narrower and definite: *given the matter-generated long-wavelength TT response and its selected massless two-helicity WTDiff representation, the minimal gauge-preserving WTDiff completion is Einstein-equivalent.*

The fixed measure has a second consequence that is independent of the numerical value of the stiffness. Let C denote an arbitrary homogeneous c-number energy-density offset and consider the shift

$$T_{\mu\nu} \to T_{\mu\nu}+C\,\hat{g}_{\mu\nu}, \qquad T \to T+4C.$$

This shift drops out of the trace-free source identically. The Einstein-equivalent form is unchanged if the integration datum is relabeled according to

$$T_{\mu\nu} \to T_{\mu\nu}+C\hat{g}_{\mu\nu}, \qquad \Lambda \to \Lambda+\frac{8\pi G_{\mathrm{comp}}}{c^4}C. \tag{54}$$

Equation (54) is not a cancellation between two independently tuned large local terms. It is a redundancy in the split between the arbitrary homogeneous stress baseline and the integration datum of the fixed-measure infrared branch.

The same statement can be seen before the Einstein-equivalent equation is reconstructed. A c-number offset does not alter the connected stress fluctuations,

$$\delta T_{\mu\nu}=T_{\mu\nu}\text{-}\langle T_{\mu\nu}\rangle,$$

and therefore does not alter the connected retarded commutator, the TT spectral density, or the Kubo slope calculated from the completed response. At the effective-action level the fixed measure makes the protection explicit. Let $\Gamma_{\rm IR}$ denote the infrared effective action and $\varpi(x)$ the fixed scalar density introduced in Sec. 3. Adding a homogeneous vacuum baseline changes the action only by

$$\Gamma_{\rm IR} \to \Gamma_{\rm IR}+C\int d^4x\,\varpi(x),$$

and, because ϖ is fixed, its functional variation with respect to $\hat{g}_{\mu\nu}$ vanishes:

$$\delta_{\hat{g}}\left[C\int d^4x\,\varpi(x)\right]=0. \tag{55}$$

The arbitrary homogeneous vacuum zero is therefore absent from the *local connected gravitational source* at both levels of the construction: it does not enter the microscopic connected TT response, and it does not enter the fixed-measure variational equation [5,7,15].

This vacuum-shift blindness is one of the cleaner consequences of the matter-first/fixed-measure ordering. It means that a radiative redefinition of the absolute zero of the quantum effective potential does not have to appear first as an enormous local curvature source and then be canceled against a separately chosen bare cosmological term. The local source equation is insensitive to that arbitrary homogeneous metric-proportional offset from the outset. In this precise sense the construction removes the local-source form of the traditional vacuum-energy catastrophe without fine tuning.

The statement is intentionally about the *arbitrary zero*, not about the absence of vacuum physics. Vacuum polarization changes the response of matter. Boundaries and Casimir configurations create position-dependent stresses. Phase transitions connect physically different states and carry real energy, fluctuations and memory through the transition. Trace-anomaly terms, curvature-dependent effective interactions and state-dependent condensates likewise remain in the completed Standard-Model Hessian. None of these is eliminated by Eq. (55), because none is merely a freely relabeled homogeneous c-number baseline.

This distinction also explains why the static calculation in Secs. 4 and 5 remains nontrivial. A homogeneous vacuum shift contributes no connected k^2 susceptibility, but a physical state produces finite stress correlations and local Ward contacts. Those are precisely what generate $A_{2,E}$. The fixed-measure construction therefore separates

$$\text{arbitrary homogeneous vacuum zero} \;\not\to\; \text{local curvature source},$$

from

$$\text{physical state-dependent stress response} \;\to\; A_{2,E} \;\to\; G_{\rm comp}.$$

The cosmological integration datum is correspondingly not predicted by the two-point calculation. Equation (52) permits a nonzero Λ, but its observed value is not obtained by subtracting a calculated vacuum zero from another large number. It characterizes the global settled branch and must be fixed by the physical branch conditions. The companion WTDiff derivation preserves this general integration datum, whereas the companion cosmology selects the $\Lambda = 0$ branch for its phenomenological analysis [45,47]. That selection is an additional branch condition, not an output of the local two-point equation.

The macroscopic consequence is that the same $G_{\mathrm{comp}}[\rho]$ obtained from the matter stiffness of a connected settled branch controls that branch's familiar weak-field and radiative infrared limits. Equation (53) contains the Newtonian $1/r$ response, gravitational redshift and the asymptotic Schwarzschild relation used in the strong-field comparison of Sec. 7, while the WTDiff TT sector retains the two massless radiative helicities. Static attraction and gravitational radiation are thus two infrared manifestations of the same state-matched matter-generated compliance.

Section 8 therefore completes the local infrared reconstruction needed by the two-point program. The Standard Model supplies the response and its stiffness; fixed measure/WTDiff supplies the minimal massless two-helicity tensor representation; conservation and the Bianchi identity give Einstein-equivalent macroscopic dynamics; and an arbitrary homogeneous vacuum baseline never becomes a local connected source. Supplementary Appendix H gives the detailed fixed-measure derivation and claim ledger. The next section turns from this settled infrared structure to the equally important question of how one common long-range branch survives threshold changes, finite bodies and the empirical constraints already imposed on gravity.

9. Cross-state continuity, threshold continuation, and empirical constraints

This section answers two practical questions raised by state dependence. First, how does the high-temperature Standard-Model calculation connect structurally to ordinary low-temperature matter? Second, why does heating, cooling, or changing the phase of a *finite* object not give that object its own Newton constant?

The key distinction is between changing an entire homogeneous Kubo-Martin-Schwinger (KMS) branch and changing a compact subsystem inside one common ambient branch. The former changes the state used to define the homogeneous susceptibility. The latter is an inhomogeneous Green-function problem: the object modifies the response locally while the far field returns to the same asymptotic massless operator. This section develops that distinction and then states the precision-gravity constraints the common infrared branch must satisfy.

This distinction is necessary before the homogeneous Kubo-Martin-Schwinger (KMS) comparison law of Eq. (18) can be interpreted correctly. Along a family of *homogeneous* high-state equilibrium branches, the entire asymptotic matter state changes and the stiffness may be written

$$A_{2,E}(T)=a_T(T)(k_B T)^2,$$

where $a_T(T)$ is the dimensionless state-dependent stiffness coefficient, T is the KMS temperature of the homogeneous equilibrium state, and k_B is Boltzmann's constant.

Comparing two such homogeneous branches legitimately changes the asymptotic compliance according to Eq. (18). An ordinary laboratory experiment is different. Heating, cooling, changing phase, or changing composition in a finite specimen modifies only a compact region embedded in an ambient long-range branch. The asymptotic state has not been replaced by a second homogeneous KMS universe.

Threshold matching therefore changes which degrees of freedom carry the Hessian; it does not imply that the stiffness vanishes when a laboratory thermometer approaches zero. Let $A_{2,E}^{\mathrm{body}}(T_{\mathrm{lab}})$ denote the matched stiffness of a body at laboratory temperature T_{lab}, $A_{2,E}^{\mathrm{GS}}$ its matched ground-state contribution, and $\delta A_{2,E}^{\mathrm{low}}(T_{\mathrm{lab}})$ its thermal or other low-energy correction. The superscript GS means ground state. The structural form required of a consistent low-energy effective-field-theory (EFT) continuation is

$$A_{2,E}^{\mathrm{body}}(T_{\mathrm{lab}})=A_{2,E}^{\mathrm{GS}}+\delta A_{2,E}^{\mathrm{low}}(T_{\mathrm{lab}}), \qquad \lim_{T_{\mathrm{lab}}\to 0}\delta A_{2,E}^{\mathrm{low}}=0. \tag{56}$$

Above the electroweak crossover those excitations are conveniently described by the unbroken Standard-Model fields used in Secs. 4 and 5. As the state is continued downward through electroweak symmetry breaking, QCD confinement, nuclear and atomic binding, and finally condensed-matter formation, heavy thermal populations become Boltzmann suppressed while their virtual, binding and ground-state effects are transferred into the matched lower-energy description. At ordinary temperatures the thermally active modes are material-dependent phonons, electrons, magnons, photons and other collective excitations sitting on top of the matched ground-state coefficient.

The continuity is therefore one of effective description rather than one of fixed particle content:

$$\begin{aligned}&\text{unbroken SM plasma} \to \text{broken SM} \to \text{hadrons and nuclei}\\ &\quad\to \text{atoms and molecules} \to \text{condensed matter.}\end{aligned} \tag{57}$$

Equation (57) expresses the required continuity of effective description. Presently available observables do not independently determine every coefficient-by-coefficient low-energy threshold matching beyond the reference-state normalization. This is an external-data limitation, not an unfinished algebraic step. The sequence is the structural low-energy analogue of the hard/electric/magnetostatic matching performed explicitly in Sec. 5: the useful degrees of freedom change without requiring a different gravitational theory at every threshold.

The cross-state problem becomes especially clear for two separated compact bodies. Let $\mathcal{K}_{\mathrm{stat}}$ denote the full static inverse-response operator of the two-body configuration, $\mathcal{K}_\infty$ the operator of the common ambient branch, $\Delta\mathcal{K}_1$ and $\Delta\mathcal{K}_2$ the localized state-dependent modifications produced by the bodies, and B_i the finite spatial region occupied by body i. The notation $\mathrm{supp}\Delta\mathcal{K}_i \subset B_i$ means that the operator modification has support only inside that body. Then

$$\mathcal{K}_{\mathrm{stat}}=\mathcal{K}_\infty+\Delta\mathcal{K}_1+\Delta\mathcal{K}_2, \qquad \mathrm{supp}\,\Delta\mathcal{K}_i \subset B_i. \tag{58}$$

Let $\mathcal{G}(\mathbf{x},\mathbf{x}')$ denote the corresponding static Green function, defined as the inverse of $\mathcal{K}_{\mathrm{stat}}$, and let $\delta^{(3)}$ denote the three-dimensional Dirac delta distribution. It satisfies

$$\mathcal{K}_{\text{stat}}\,\mathcal{G}(\mathbf{x},\mathbf{x}')=\delta^{(3)}(\mathbf{x}\text{-}\mathbf{x}'). \tag{59}$$

This formulation removes an ambiguity that arises if one tries to assign a separate state-labeled Newton constant to each body and then asks how two such constants should be averaged. The physical problem is instead an ordinary inhomogeneous Green-function problem. The bodies modify the same operator locally; outside both compact supports the response returns to the same asymptotic massless branch.

At distances large compared with the body sizes, let $A_{2,E}^{\infty}$ denote the TT stiffness of the common ambient branch and ∇^2 the spatial Laplacian. The ambient tensor operator has the leading static form

$$\mathcal{K}_{\infty}=A_{2,E}^{\infty}(\text{-}\nabla^2)+O(\nabla^4),$$

Define $r\equiv|\mathbf{x}-\mathbf{x}'|$. The Green function of the common ambient branch then contains

$$\mathcal{G}_{\infty}(r)=\frac{1}{4\pi A_{2,E}^{\infty}r}+\text{subleading terms.} \tag{60}$$

The expression is understood up to the tensor projector and source normalization already fixed in Secs. 2–4. Localized insertions can change scattering phases, finite-size structure, induced multipoles and short-distance response, but they cannot create a second asymptotic $1/r$ pole unless the altered state itself extends to infinity or introduces an additional massless degree of freedom.

The same conclusion follows from the zero-momentum stress form factor of a complete composite body. Translation invariance and conservation normalize the zero-momentum matrix element of the conserved stress to the body's total four-momentum; in the static limit, the monopole coupling is therefore fixed by the total energy. Let $M_i(0)$ be the reference mass of body i, let $\Delta U_i(T_{\text{lab}})$ be its change in internal energy at laboratory temperature T_{lab}, and let c be the local limiting signal speed. Heating or cooling then changes the mass-energy by

$$M_i(T_{\text{lab}})=M_i(0)+\frac{\Delta U_i(T_{\text{lab}})}{c^2}. \tag{61}$$

The state change may alter finite-momentum form factors, stresses and multipole moments, but it does not assign the specimen a new independent long-range gravitational charge. Consequently, for two compact bodies sharing one ambient branch, define $G_{\infty}\equiv G_{\text{comp}}[\rho_{\infty}]$ as the common ambient coupling of the asymptotic state ρ_{∞}. It is neither a state-independent fundamental constant nor a private property of either body. Let R_1,R_2 be characteristic radii of the two bodies, $V_{12}(r)$ their leading static interaction potential, and G_{12}^{monopole} the corresponding leading monopole coupling. Then

$$V_{12}(r)=\text{-}G_{\infty}\frac{M_1M_2}{r}+O\left(\frac{R_1^2+R_2^2}{r^3}\right),\qquad G_{12}^{\text{monopole}}=G_{\infty}. \tag{62}$$

Equation (62) is the leading cross-state Newtonian result. It is reciprocal automatically because both bodies couple through the same Green function. Most importantly, it rejects the naïve extrapolation in which cooling a finite sample would cause its gravitational coupling to scale as T_{lab}^{-2}. The high-state T^{-2} comparison in Eq. (18) belongs to homogeneous KMS branches; Eq. (62) governs compact laboratory bodies embedded in one common asymptotic state.

This result also sharpens what a genuine laboratory anomaly would have to look like. Ordinary phase changes, magnetic ordering, superconductivity, cavity loading or thermal preparation can certainly change internal stress correlations and finite-frequency susceptibilities. Such changes are not gravitational evidence by themselves. A gravitational modification would have to survive as a universal change in the common long-range response after ordinary electromagnetic, thermal, mechanical and chemical effects were removed. The more different the microscopic constructions of the compared standards, the more restrictive that universality test becomes.

The empirical infrared constraints are correspondingly powerful. Optical-clock experiments have resolved gravitational redshift across millimeter-scale height differences [9]. Within OQG this is not a new prediction distinct from general relativity; it requires that the emergent operational tensor act universally on matter-built clocks once the response has reached its settled infrared branch. The measurement is nevertheless conceptually important because it demonstrates directly that quantum matter standards resolve the gravitational comparison field at laboratory scale.

Universality of free fall is even more restrictive. If a residual composition dependence of the long-range response is parameterized diagnostically by a_a=(1+β_a)g, where g is a common reference acceleration, a_a is the measured acceleration of body a, and β_a is its fractional composition-dependent deviation; define a_b=(1+β_b)g analogously for body b. Then, to leading order,

$$\eta_{ab} \equiv 2\frac{|a_a\text{-}a_b|}{|a_a\text{+}a_b|} \simeq |\beta_a\text{-}\beta_b|. \tag{63}$$

The final MICROSCOPE result found no titanium–platinum violation at approximately the 10^{-15} Eötvös level in the tested regime [40]. The collective infrared projection must therefore erase constituent-level differences—electronic, nuclear, QCD, binding, spin and material-response differences—to extraordinary accuracy. This is one of the strongest empirical reasons for using the complete conserved Standard-Model stress response rather than identifying gravity with the damping or polarizability of one microscopic constituent.

Gravitational-wave observations impose a complementary tensor requirement. Direct detection establishes propagating gravitational degrees of freedom [1], while the association of GW170817 with gamma-ray burst (GRB) 170817A constrains the low-energy gravitational causal speed to track the electromagnetic causal speed extremely closely under the standard astrophysical interpretation [2]. Writing m_{TT}^{IR} for the infrared tensor mass, v_{TT}^{IR} for the tensor propagation speed, and N_{helicity} for the number of long-range tensor helicities, the infrared OQG branch must consequently satisfy

$$m_{TT}^{\mathrm{IR}}=0, \qquad v_{TT}^{\mathrm{IR}}=c, \qquad N_{\mathrm{helicity}}=2, \tag{64}$$

with no additional unsuppressed long-range scalar or vector gravitational polarization. These requirements are naturally aligned with the massless two-helicity WTDiff representation of Sec. 8, but they remain empirical conditions on any microscopic realization of that representation.

The empirical picture is therefore unusually clean. Existing precision-gravity experiments strongly constrain the *infrared output* of OQG while remaining largely insensitive to whether the successful metric is fundamental or emergent. The present microscopic calculations are admissible only because their long-wavelength completion reproduces that already-tested structure. At the same time, the threshold and cross-state analysis prevents an attractive but incorrect laboratory inference: state dependence of the microscopic susceptibility does not imply that every finite specimen carries its own Newton constant.

Section 9 therefore closes the structural bridge from the high-state Standard-Model calculation to ordinary macroscopic matter. A consistent low-energy continuation requires a finite zero-temperature intercept in the matched stiffness. Its coefficient is not independently identifiable from presently available observables beyond reference-state normalization, so no presently executable calculation would improve the normalized result; this is an external-data limitation, not an unperformed step in the two-point derivation. Compact state changes modify finite-size and multipole response while the common asymptotic $1/r$ pole remains fixed, and redshift, equivalence-principle and gravitational-wave observations place stringent quantitative requirements on the universality of that pole. Supplementary Appendix H gives the corresponding resolvent derivation and records the threshold-continuation claim boundary explicitly. The next section summarizes the completed two-point program—what has been derived, what has been reduced to external nonperturbative data, and which assumptions belong to physics outside the two-point calculation.

10. Scope of the completed two-point program

This final body section states exactly what has been accomplished by the two-point program and what the word *complete* means in this paper. It is not an all-orders derivation of every gravitational phenomenon. The result is narrower but concrete: the transverse-traceless (TT) two-point Standard-Model response has been carried far enough to determine its causal topology, calculate the static stiffness through the complete perturbative $O(g^2)$ matching problem, isolate the genuinely nonperturbative remainder, and connect the long-range compliance to the minimal fixed-measure Weyl-transverse-diffeomorphism (WTDiff) representation that is Einstein-equivalent at two derivatives.

The stopping point is therefore physical rather than editorial. What remains inside the static result is either a standard renormalized Standard-Model input or a separately defined nonperturbative Yang-Mills observable; what remains beyond the two-point sector belongs to a different level of the theory.

Let $\Pi_{TT}^{R,E}$ denote the retarded/Euclidean TT stress susceptibility, let $A_{2,E}$ denote its static two-derivative stiffness coefficient, and let $D^R \equiv (\Pi^R)^{-1}$ denote the retarded constitutive compliance. The resulting chain can be summarized as

$$\begin{aligned}&\text{complete Standard-Model stress} \to \Pi_{TT}^{R,E} \to \left(A_{2,E}, D^R\right)\\ &\to \text{common massless TT compliance} \to \text{fixed-measure/WTDiff infrared}\\ &\to \text{Einstein-equivalent macroscopic dynamics.}\end{aligned} \tag{65}$$

Every arrow in Eq. (65) has a distinct status. The microscopic source is the complete Yang-Mills, Dirac/Yukawa and Higgs stress rather than one selected constituent sector. The completed Hessian contains both separated-point correlations and the Ward-required local metric contacts. The matter-first step is a constitutive inversion of that same response, not the introduction of a second microscopic dynamics. The stiffness and causal inverse compliance are two different projections of the same matter Hessian. Fixed measure and WTDiff are then introduced as the infrared tensor representation, not as an assertion that the microscopic Standard Model is Weyl invariant.

The static and causal calculations should consequently be regarded as complementary rather than interchangeable. The Euclidean Kubo derivative determines the equilibrium two-derivative coefficient $A_{2,E}$. The retarded finite-(ω, k) calculation determines branch cuts, resonances, damping and analytic continuation. The noncommuting limits at the thermal origin make it impossible to identify these two projections by simply sending both ω and k to zero without specifying the path. One of the principal gains of the completed program is precisely that this distinction no longer has to be hidden inside a single oscillator parameter.

On the static side, the perturbative calculation is closed. The free variational response, Higgs curvature contact, leading soft zero-mode contribution, strict-hard two-loop Standard-Model operator response, hard Higgs contact, electric/Higgs dimensional reduction, Ward seagulls, two independent renormalization scales, finite massive electric threshold coefficient, and local-R matching have been assembled in one state-dependent metric Hessian. For a homogeneous Kubo-Martin-Schwinger (KMS) state of temperature T, let k_B denote Boltzmann's constant, let ξ_H denote the renormalized nonminimal Higgs-curvature coupling, and let c_3^{NP} and c_2^{NP} denote the finite nonperturbative pure-Yang-Mills magnetostatic constants for the $SU(3)$ and $SU(2)$ sectors, respectively. The resulting stiffness is

$$\begin{aligned}\frac{A_{2,E}}{(k_B T)^2} &= 0.260056643133723 + 0.066623718018839\,\xi_H\\ &+0.002999279573124\,c_3^{\mathrm{NP}} + 0.000799379975567\,c_2^{\mathrm{NP}}.\end{aligned} \tag{66}$$

Let $G_0 \equiv G_{\mathrm{comp}}[\rho_0]$ denote the measured coupling of the reference state ρ_0. Equation (66) is not a fit to G_0: no measured gravitational value was used to obtain its analytic coefficients. The remaining ξ_H dependence is the ordinary renormalized Higgs-curvature input of the curved Standard Model, while c_3^{NP} and c_2^{NP} are finite pure-three-dimensional Yang-Mills observables in the magnetostatic $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ sectors. Inserting G_0 before those two observables are independently measured can only constrain their allowed combination for the adopted ξ_H; it is inverse calibration, not an ab initio prediction of G_0, and it cannot determine the two observables separately. Equation (66) is therefore a renormalization-closed perturbative relation with its remaining Standard-Model and nonperturbative inputs displayed explicitly, not yet a parameter-free numerical prediction of the measured reference-state coupling.

This distinction defines what "complete" means for the static calculation. The continuum perturbative matching contains no additional unknown gravitational coefficient. The numerical magnetostatic remainder has been reduced to two external nonperturbative numbers, and Supplementary Appendix F specifies an exact flat-lattice observable, the required contact term, the continuum normalization, the order of limits and the lattice-to-$\overline{\mathrm{MS}}$ conversion. Existing glueball masses and residues cannot substitute for those quantities because the published superconvergent observable removes the local k^2 coefficient required here. The absence of numerical values for c_3^{NP} and c_2^{NP} therefore marks the boundary between analytic continuum work and genuine nonperturbative input; it does not reopen the hard/electric/local matching problem.

The causal side has reached an analogous but different closure. Let N_T denote the total free physical TT spectral weight; the scalar:Weyl:vector weights occur in the ratio $1:3:12$, giving N_T=283 for the minimal Standard Model. Using the dimensionless variables $u \equiv \hbar z/(k_B T)$ and $y \equiv \hbar ck/(k_B T)$, where z is complex frequency, k is spatial wave-number magnitude, and c is the locally measured limiting signal speed, let $F(u, y)$ denote the zero-forming inverse-response eigenvalue on the two-particle-production sheet. At finite temperature and finite wave number, the completed collisionless response possesses the expected Landau/scattering and two-particle-production continua. Continuing through the corresponding two-particle-production cut reveals a stable simple pole. At the benchmark point,

$$y = 0.758224684, \qquad u_P = 1.353089335 - 0.093439111\, i, \qquad |\partial_u F(u_P)| = 5.39112. \tag{67}$$

The nonzero derivative is as important as the pole position: it identifies an ordinary underdamped resonance rather than an exceptional point. Resolution studies show that apparent Landau poles collapse onto the continuum, and the real-k continuation preserves the same underdamped branch over a wide momentum range. The first controlled interaction-topology analysis then supplies an additional stability statement. Common widths and pure dispersion translations preserve root multiplicity; reaching a two-particle threshold is generically a sheet transition rather than a double-root condition; and the nearest dressed non-Abelian branch points remain far enough from the pole that forcing coincidence would require an order-one deformation of the zero-forming inverse response. The detailed interacting pole coordinate can therefore be refined without changing the presently established local topology unless a genuinely new nonperturbative or singular mechanism enters.

The exact Lambert-W double zero retained in Sec. 7 belongs to a different analytic statement. It is an exact property of the leading principal-sheet one-log representation. The causal sheet continuation changes the analytic function whose derivative is being tested, so that principal-sheet merger is not relabeled as the physical retarded resonance on the two-particle-production sheet. Similarly, the finite-k static roots are simple thermal-dispersion diagnostics, not universal correlation lengths. The completed two-point program therefore contains several useful spectral scales without requiring them to be identical.

This separation also fixes the role of the original critical-damping intuition. The calculated weak-coupling Standard-Model state is naturally underdamped. The proposed $1/(16\pi)$ relation is reserved for a distinct strong-field state whose full retarded kernel would have to satisfy the non-Markovian double-zero condition. It is not imposed on Eq. (67), not used to normalize Eq. (66), and not reintroduced as an auxiliary ultraviolet energy. In this paper the strong-field condition functions as a constitutive boundary statement inherited from the operational damping program, while the ordinary gravitational interaction is already carried by the massless $1/k^2$ compliance of the infrared two-point response.

Let $D_{TT,E}(0,k)$ denote the static Euclidean TT compliance. At long wavelength, that response has the form

$$D_{TT,E}(0,k)=\frac{1}{A_{2,E}k^2+O(k^4)}. \tag{68}$$

Equation (68) is the direct reason a separate microscopic resonance is not required to generate the long-range interaction. The pole residue and the two-derivative stiffness instead determine different aspects of one response: microscopic dispersion and damping on the one hand, macroscopic compliance on the other. Section 9 and Supplementary Appendix H show how this long-range statement extends structurally to compact changes of state. A complete low-energy effective-field-theory matching through electroweak symmetry breaking, confinement, binding and condensed-matter formation would reorganize the degrees of freedom carrying the Hessian, but those threshold coefficients are not calculated here. What the present cross-state Green-function construction establishes at leading monopole order is that a compact state change does not manufacture a second asymptotic massless pole; the bodies therefore share one common leading monopole coupling rather than acquiring separate Newton constants from their thermometer temperatures.

The fixed-measure/WTDiff completion should be read with the same precision. The Standard-Model two-point calculation establishes the common long-wavelength TT response and its matter-generated coefficient; WTDiff supplies the selected massless two-helicity infrared representation of that response. Given this represented sector, the minimal local, two-derivative, gauge-preserving WTDiff completion yields the trace-free equation and, through conservation and the contracted Bianchi identity, the Einstein-equivalent macroscopic equation with a cosmological integration datum. This is a genuine infrared completion result, derived in detail in Supplementary Appendix H. It is not the stronger statement that all connected Standard-Model TTT, $TTTT$, and higher correlators have already been evaluated and shown uniquely to generate every nonlinear gravitational vertex.

The vacuum-energy result is correspondingly confined to what the construction actually proves. An arbitrary homogeneous metric-proportional c-number shift does not alter connected stress fluctuations and drops out of the fixed-measure local source; in the Einstein-equivalent form it is absorbed into the integration datum. Physical vacuum polarization, boundaries, Casimir stresses, condensates, phase transitions, trace-anomaly effects and fluctuations remain in the response. The result is therefore blindness to the arbitrary homogeneous *zero* of the stress, not a claim that physical vacuum structure is absent or that the observed cosmological constant has been predicted.

The relation to Weinberg-Witten is also bounded by the two-point scope. OQG is not formulated as a conventional local fixed-background Standard Model that produces an ordinary composite graviton particle and then assigns that particle a separate fundamental stress tensor. Its long-range object is a nonlocal constitutive response that is represented geometrically only in the infrared. That difference is sufficient to prevent a naive one-line identification with the textbook composite-particle target of the theorem. It is not, by itself, a theorem-level proof that every assumption of Weinberg-Witten is absent from the complete reduced chiral Standard Model. The present two-point result therefore neither invokes the theorem as a disproof nor claims unconditional evasion from the Legendre transformation alone.

The empirical status follows naturally. In the regime where the minimal infrared completion is valid, OQG is intentionally Einstein-equivalent. Optical-clock redshift, universality of free fall, luminal gravitational propagation and the observed two tensor helicities are therefore consistency requirements on the infrared output, not evidence that distinguishes the underlying ontology. Their value for the present construction is restrictive: any microscopic composition dependence, extra long-range polarization, tensor mass or nonluminal infrared branch would contradict observations irrespective of how attractive the matter-first interpretation might otherwise be.

The completed two-point program can therefore be stated without either understating or overstating what has been achieved. A realistic Standard-Model source has been carried through causal closed-time-path (CTP) normalization, TT projection, regulator-free state-dependent Kubo response, controlled interactions, full perturbative $O(g^2)$ static matching, finite-(ω, k) causal continuation, threshold and cross-state analysis, and an Einstein-equivalent fixed-measure/WTDiff infrared completion. Appendices F, G, and H contain the terminal nonperturbative, causal-topology, and infrared/cross-state proof layers summarized here. The remaining numerical magnetostatic information is isolated as genuine pure-Yang-Mills input rather than hidden inside an adjustable gravitational coefficient. Questions that require higher connected stress kernels, theorem-level microscopic Noether structure, a realized strong-field critical state, or cosmological boundary data are physically different layers of the theory rather than unfinished algebra inside the two-point calculation.

That distinction is the appropriate endpoint for the body of the paper. The two-point Standard-Model matter-response construction is now sufficiently explicit to be evaluated on its own terms: its normalization is fixed, its causal and static projections are separated, its perturbative matching is closed, its nonperturbative remainder is defined, its infrared representation is specified, and its claim boundaries are explicit. The Conclusions can therefore synthesize the physical meaning of these results without reopening the calculation chain.

11. Conclusions

Operational Quantum Gravity (OQG) begins by reversing the usual explanatory order of gravitation. Matter is not placed into a pre-existing gravitational geometry; the quantum state and causal response of matter are taken as primary, and geometry is reconstructed afterward as the long-wavelength covariant description of those relations. The results of this work show that this matter-first program can be carried substantially beyond an interpretive reformulation. The complete interacting Standard Model supplies a single conserved stress source, its closed-time-path response generates the causal stress Hessian, the transverse-traceless sector defines a matter-generated constitutive stiffness and compliance, and the minimal local two-derivative fixed-measure/WTDiff completion of that infrared response is Einstein-equivalent. The central achievement is therefore not another geometrical model of gravity, but a microscopic route from quantum matter response to the geometrical description already known to work macroscopically.

The static calculation has been carried to its natural analytic boundary through perturbative order $O(g^2)$. Hard, electric, Higgs, Ward/contact, and local-curvature contributions combine into one renormalization-closed Standard-Model Hessian. The result is

$$\frac{A_{2,E}}{(k_B T)^2} = 0.260056643133723 + 0.066623718018839\,\xi_H$$
$$+0.002999279573124\,c_3^{\mathrm{NP}} + 0.000799379975567\,c_2^{\mathrm{NP}}.$$

No additional perturbative gravitational coefficient remains to be supplied. The two magnetostatic constants c_3^{NP} and c_2^{NP} are finite pure-three-dimensional Yang-Mills observables, and the flat-lattice shear estimator, Ward contact, subtraction, order of limits, and continuum conversion required to determine them have already been fixed. Their numerical values require new lattice-ensemble measurements; existing published observables do not supply them. Normalizing their allowed combination to G_0 would be calibration rather than prediction. The remaining dependence on ξ_H is the ordinary renormalized Higgs-curvature coupling, not an adjustable OQG normalization. The remaining numerical uncertainty is therefore localized in identifiable external inputs rather than hidden in an additional gravitational sector.

The causal calculation reaches an equally definite qualitative endpoint. The weak-coupling Standard-Model transverse-traceless response contains a massless long-range inverse-response branch, a simple underdamped causal resonance, and distinct two-particle-production and Landau/scattering continua. At the benchmark state the physical pole on the two-particle-production sheet is

$$u_P\text{=}1.353089335\text{-}0.093439111\,i, \qquad |\partial_u F|\text{=}5.39112,$$

so the ordinary matter resonance is neither critically damped nor close to a double root. The principal-sheet Lambert-W double zero remains an exact analytic benchmark, but it is a different mathematical object from the physical retarded pole. This distinction removes the need to force ordinary matter toward an exceptional point in order to obtain long-range gravity: the massless $1/k^2$ compliance is already present in the infrared. Critical damping is retained only as a distinct strong-field constitutive boundary, where the full non-Markovian response rather than the ordinary weak-coupling resonance becomes the relevant object.

The infrared completion provides one of the clearest conceptual advantages of the construction. An arbitrary homogeneous metric-proportional vacuum offset is absent from the local connected gravitational source in the fixed-measure theory, while real vacuum physics—polarization, Casimir and boundary stresses, condensates, phase-transition differences, anomaly effects, and fluctuations—remains physical. The cosmological term enters the Einstein-equivalent macroscopic equations as an integration datum rather than as the local response to an arbitrarily chosen zero of quantum-field energy. OQG thus separates measurable quantum-state response from an unobservable homogeneous baseline without requiring that baseline to be fine tuned against an independent local gravitational source.

The cross-state calculation adds an equally important universality result. State dependence does not assign a private Newton constant to each body. A finite body in a different thermal, compositional, or condensed-matter state is a compact perturbation of one common asymptotic inverse response. Outside that compact region the same massless branch governs the monopole field, with the common far-field coupling $G_{\mathrm{comp}}[\rho_\infty]$. Local changes can alter finite-size response, scattering, multipoles, binding energy, and ordinary mass-energy, but they do not manufacture another asymptotic gravitational pole. The resulting long-range behavior is therefore compatible with the universality demanded by precision redshift, equivalence-principle, and gravitational-wave observations while preserving a genuine role for state-dependent microscopic response.

OQG also makes a sharp statement about chronology because operational time is not an emergent coordinate that can later be assigned arbitrary global topology. Time is what physical clocks measure, and a physical history is an ordered sequence of matter states registered by those clocks. Clock rates may differ, redshift, or approach limiting behavior, but no OQG degree of freedom corresponds to a clock entering its own earlier physical state. This is an admissibility condition on the physical state space, not a conclusion drawn from extending the Einstein-equivalent infrared equations. The microscopic construction is correspondingly retarded: later response depends on earlier matter history. A metric representation containing a closed timelike curve therefore has no physical preimage under the OQG matter-to-geometry map. Closed timelike curves, backward time travel, and chronology paradoxes are excluded from the physical OQG state space rather than admitted and then repaired by an additional chronology-protection mechanism.

This is a genuine theory-level distinction from geometric-first gravity. General relativity admits formal solutions with closed timelike curves under particular global conditions, as well as wormhole geometries and other nontrivial spacetime topologies. OQG does not grant such geometries independent ontological status. Geometry is an output description of causal relations among matter, not a physical substance whose topology can itself provide a route into the past. Accordingly, OQG predicts no traversable wormholes as literal tunnels through a fundamental spacetime and no chronology-violating histories at all. Any apparently shortened propagation route must remain an actual retarded matter-to-matter process. It is not a tunnel through a fundamental spacetime and cannot connect a system to an earlier value of its own operational time. An unambiguous observation of backward causal influence would therefore falsify this element of OQG directly.

The same distinction changes the microscopic question posed by black-hole infall. In the Einstein representation, an extended body experiences tidal curvature and, in sufficiently strong gradients, spaghettification. In OQG the same observable effect is interpreted as differential constitutive scaling: separated parts of the infalling body sample different values of the collective matter response associated with the gravitating source, so their clock rates, equilibrium scales, binding structure, and trajectories change differentially. The infrared geometrical description can remain Einstein-equivalent while the underlying cause is reassigned from deformation by a fundamental spacetime to gradients of matter response. A divergence of the local geometrical representation need not mean that a fundamental spacetime substance literally becomes singular; it can instead signal the limit of the infrared dictionary. The zero-extension companion analysis makes this boundary precise on its stated regular canonical branch: every finite reversible scaling retains positive operational extension, while exact zero extension lies only at a noninvertible, nonnormalizable infinite-scaling boundary [46]. That result does not select a unique interior trajectory, bounce, core, or horizon outcome.

Cosmology is posed in the same causal order. It begins with an evolving cosmic matter state, its stress response, and its operational standards, and reconstructs the cosmological geometry from those relations. The companion analysis applies this construction as covariant scaling at every epoch and compares a quadratic response-history realization with cosmic-chronometer and DESI measurements, without a physical Dark Energy density or a nonzero cosmological integration constant [47]. The geometrical cosmological description remains useful as the macroscopic representation; it is not assumed to be the primitive physical ontology. The vacuum-shift result established here supports that construction because the arbitrary homogeneous zero of quantum-field energy is not a local gravitational source that must first be canceled.

The external inputs and optional higher-order tests are sharply partitioned. The finite nonperturbative values c_3^{NP} and c_2^{NP} require new lattice-ensemble data, while the relevant value and running prescription for ξ_H are external Standard-Model inputs for the state under consideration. A complete chiral-Standard-Model reduced Noether/WTDiff analysis would test theorem-level microscopic consistency. The higher connected stress kernels TTT and TTTT would test whether the already Einstein-equivalent minimal nonlinear completion is also the unique microscopic completion generated by matter. They are not required to repair the two-point result, and no distinct presently verifiable two-derivative observable is identified here that depends on completing them. More accurate interacting pole locations and higher-order coefficients are precision refinements of a two-point structure whose causal topology and perturbative static matching are already fixed. The decisive experimental program is to identify regimes in which the matter-first construction and a fundamental-geometric interpretation cease to be observationally degenerate.

These external inputs and optional higher-order validation tests are concentrated rather than architectural. The two-point matter-first construction has survived normalization, Ward/contact completion, renormalization, hard/soft matching, causal-sheet analysis, threshold continuation, nonperturbative reduction, and cross-state consistency without requiring a fundamental dynamical spacetime metric or an additional gravitational matter sector. Just as importantly, the calculation has constrained the theory rather than merely confirming prior expectations: interpretations that did not survive the Standard-Model response analysis were discarded or separated, while the central causal order—matter first, geometry afterward—remained intact. No undefined mechanism remains in the completed two-point chain.

The significance of OQG is consequently larger than reproducing gravitational equations already known. If spacetime geometry is the macroscopic record of causal relations among quantum matter rather than an independently fundamental substance, then several problems traditionally posed as mysteries of geometry become questions about matter: what happens to matter at a black-hole boundary, what replaces the classical singularity, whether information can be lost from a theory whose microscopic degrees of freedom remain matter, how cosmological standards evolve, which formal Einstein geometries possess any microscopic physical preimage, and which exotic constructions are artifacts of extending the geometrical dictionary beyond the histories matter can realize. The calculations reported here do not answer all of those questions. They establish a microscopic framework in which they can be asked without first treating spacetime as fundamental.

Time is what clocks measure. Space is what rulers measure. Both clocks and rulers are matter. In OQG, matter tells matter how to respond, while geometry records the comparison. If that causal order is correct, the next frontier is not to quantize an independently existing spacetime, but to determine how far quantum matter itself can account for gravitation, cosmology, and the strong-field phenomena for which geometry has so far been our most successful language.

Supplementary Information

Online Resource 1. Technical Appendices A–H: closed-time-path normalization, transverse-traceless projector and Kubo derivations, Standard-Model static matching, lattice extraction, retarded-sheet analysis, and the infrared claim-status ledger.

Online Resource 2. Numerical reproducibility package: Python source code, benchmark inputs, tests, sample outputs, and an automated verification runner for the numerical results reported in the manuscript.

Declarations

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Competing Interests

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Author contributions. Todd J. Desiato conceived the OQG framework, developed the physical interpretation and research program, directed the theoretical synthesis, reviewed the derivations and literature, and approved the final manuscript.

Data availability. No new experimental datasets were generated for this theoretical study. The detailed derivations and proof layers are provided as Online Resource 1 (Technical Appendices A–H). The numerical reproducibility package, including Python code, benchmark inputs, and verification outputs, is provided as Online Resource 2 and archived at https://doi.org/10.5281/zenodo.22146936. Empirical results discussed in the manuscript are taken from the cited literature.

Code availability. The Python source code and benchmark inputs required to reproduce the reported numerical checks are supplied as Online Resource 2 and archived on Zenodo [22] at https://doi.org/10.5281/zenodo.22146936. The package requires Python 3.11 or later and has no third-party Python dependencies.

Ethics statement. Not applicable; this theoretical study involved no human participants, animals, or identifiable human data.

Use of generative artificial intelligence. The author used OpenAI ChatGPT, including GPT-5.5 and GPT-5.6 Sol model configurations, as an AI-assisted research and writing tool for literature discovery, mathematical cross-checking, organization, drafting, and editing during development of the manuscript. The author reviewed the scientific arguments, equations, references, and final text and takes full responsibility for the content.

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