# Operational Quantum Gravity: Canonical Scale Covariance and the Zero-Extension Boundary of Black-Hole Matter

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## Abstract

Operational Quantum Gravity (OQG) treats rods, clocks, detectors, and gravitating matter as physical Standard-Model systems whose measured scales may change with their state. This paper asks a narrow question about collapse: can a finite material system ever become an ordinary quantum state with exactly zero operational extension? A resolved internal material coordinate and its conjugate momentum are used to represent a scale-bearing sector. Every finite, reversible rescaling of that pair preserves the canonical quantum relation, the full uncertainty condition, state normalization, and the information required to undo the transformation. The exact zero-size endpoint is qualitatively different. It occurs only at an infinite scaling boundary, where the map ceases to have an inverse, and it has no normalizable representative in the regular canonical state space. If the accumulated scale response remains finite over every finite loading interval, finite mass-energy loading cannot reach that boundary. In OQG, a geodesic is not a fundamental path through a physical spacetime but a long-wavelength bookkeeping curve reconstructed from matter response. The terminal central geodesic of general relativity therefore has no corresponding material endpoint on the stated regular branch. If that branch remains available through every finite stage, matter continues through successively smaller positive scales; zero extension can be approached only asymptotically and is never attained. The result does not predict a universal minimum length, a bounce, a stable core, or a unique interior configuration, and it does not prevent a one-way horizon.

**Keywords:** operational quantum gravity; canonical dilation; uncertainty principle; normal quantum states; black-hole interiors; singularity boundary

**Mathematics Subject Classification (MSC 2020):** 81S05; 83C57

## 1. Introduction

Imagine following a small but finite piece of matter as a massive star collapses. Its measured size might become one half, one millionth, or an even smaller positive fraction of its original size. An exactly zero-size state is qualitatively different. It would not merely be very small; every resolved spatial width of the material system would have vanished. This paper asks whether that exact endpoint is an admissible quantum state. It does not ask how quickly collapse proceeds, whether a horizon forms, or whether the material eventually bounces. Keeping those questions separate is essential to understanding both the result and its limits.

Operational Quantum Gravity (OQG) begins from the fact that a rod, clock, atom, detector, or collapsing material element is itself a physical Standard-Model system. Its measured scale can depend on its physical state. In the OQG program, quantum matter is fundamental, its completed causal stress response carries the physics, and spacetime geometry is reconstructed as the long-wavelength covariant description used to compare material standards. The present paper adds one explicit bridge assumption rather than attributing it to the companion derivations: while a resolved scale-bearing sector remains on one regular quantum branch, a finite common rescaling of its internal position-like and momentum-like observables is represented by an ordinary canonical dilation. The theorem that follows is conditional on that representation.

A finite contraction can then be made as large as desired and still be represented by a reversible quantum scaling. Every such contraction retains some positive operational width. The exact value zero is not the last member of that family; it is a boundary obtained only after an unbounded amount of logarithmic scaling. Ordinary quantum mechanics permits states whose widths are arbitrarily close to zero, but it does not supply a normalizable state whose position-like width is exactly zero. Under an additional regularity condition stated later, a finite amount of accumulated physical loading also cannot drive the system all the way to that boundary.

The basic scale notation can be understood before any operator theory is introduced. Let $L_{\mathrm{op}}$ denote the operational extension: a root-mean-square measure formed from declared internal material coordinates relative to a stated material center. Let $L_0 > 0$ be its value in a declared reference state. Their dimensionless ratio is $b = L_{\mathrm{op}}/L_0$. Thus $b = 1$ means no relative change, $0 < b < 1$ means contraction, and $b > 1$ means expansion. It is useful to replace this multiplicative scale factor by the logarithmic variable $\chi = -\ln b$. For example, $b = 1/2$ corresponds to $\chi = \ln 2$. Successive contractions multiply their $b$ values but add their $\chi$ values. No finite $\chi$ gives $b = 0$; rather, $b$ approaches zero only when $\chi$ tends to positive infinity.

The quantum notation expresses the same distinction. A hat marks an observable operator. The operator $\hat{X}$ represents a resolved, center-relative, position-like collective coordinate, such as a Cartesian internal displacement of a finite material element, and $\hat{P}$ is its canonically conjugate momentum-like observable. They are not assumed to be a universal relativistic particle-position operator. A finite scale change takes the reciprocal form

$$\hat{X} \mapsto b\hat{X}, \qquad \hat{P} \mapsto b^{-1}\hat{P}, \qquad b > 0.$$

Contracting $X$ therefore expands the conjugate momentum scale. This reciprocal change preserves the canonical commutator $[\hat{X}, \hat{P}] = i\hbar\hat{I}$, where $[A, B] \equiv AB - BA$, $i$ is the imaginary unit, $\hbar$ is Planck's constant divided by $2\pi$, and $\hat{I}$ is the identity operator. A transformation that preserves this position-momentum structure is called *canonical* or *symplectic*. When it is represented by a unitary operator, it is reversible and preserves the normalization of quantum states. These terms are made precise in Sections 2 and 3.

This scaling does not create a universal minimum length. With the ordinary, undeformed commutator, normalized wave packets may be chosen with an arbitrarily small positive root-mean-square displacement from the declared center while their momentum width becomes arbitrarily large [9, 13, 14]. The important distinction is between an arbitrarily small positive number and the number zero itself. Exactly zero root-mean-square displacement would require an exact position eigenstate at the center. In the usual continuous position representation, that object is a Dirac delta distribution, not a square-integrable and therefore not a normalizable wavefunction. The paper extends this familiar pure-state statement to normal mixed states, meaning density operators that represent ordinary statistical quantum states with total probability one.

General relativity defines a gravitational singularity through *geodesic incompleteness*: at least one freely falling or lightlike curve cannot be continued to arbitrary values of its natural path parameter, even though it has not reached an ordinary edge of the geometric manifold. The singularity theorems establish such incomplete causal geodesics under specified geometric and energy assumptions [4, 6, 12]. That conclusion is mathematical; the further claim that physical evolution itself ends there comes from treating the geometric manifold and its geodesics as fundamental. OQG rejects that interpretation. Its starting objects are quantum matter states and their causal Standard-Model response. The metric and the geodesics drawn from it are reconstructed, long-wavelength bookkeeping devices for comparing matter with matter. If such a derived curve cannot be extended while the underlying matter states remain well defined, the bookkeeping description has reached its boundary; the matter has not reached a physical edge of existence. The physical singularity question is therefore whether the material state sequence terminates, not whether an emergent geometric curve does.

Two layers of the argument must therefore be distinguished. The first is a *state-space theorem*. It asks what states and transformations exist at all. A quantum state will be denoted by $\rho$ (the Greek letter rho). Within a regular canonical scale sector, finite transformations have $b > 0$, and no normal state $\rho$ has exactly zero operational extension. The second layer is a *dynamical corollary*. It asks whether a physical history can reach the state-space boundary. The dimensionless parameter $\lambda$ will label cumulative loading, and $\chi(\lambda)$ will describe the accumulated logarithmic contraction. If the absolute integral of the ordinary response rate $d\chi/d\lambda$ remains finite over every finite loading interval, then $\chi$ remains finite and $b = e^{-\chi}$ remains positive. If the regular branch remains available for every finite loading stage, this becomes an operational continuation statement: however small the material scale becomes, every finite stage is another positive-extension quantum state. Exact zero remains only the infinite-scale boundary. If the response instead diverges at finite loading, or if a phase change destroys the chosen material coordinate, new matter variables and dynamics are required; neither event turns zero extension into a regular state.

This separation prevents the uncertainty argument from being asked to determine an equation of state or a detailed collapse history. The paper does not claim a fixed minimum radius, a pressure-supported remnant, a bounce, a unique interior configuration, or the absence of a horizon. It establishes the more fundamental point needed first: covariance can continue through indefinitely smaller positive material scales, but the zero-extension boundary is not itself a covariant state. On the stated continuation conditions, the incomplete central geodesic of the emergent metric marks the limit of geometric bookkeeping rather than the termination of matter. No independent smoothing of a fundamental spacetime curvature is required, because spacetime curvature is not the underlying OQG object.

This is a companion theorem paper within the OQG program. The first related manuscript, *Operational Quantum Gravity from Standard-Model Stress Response: Static Matching, Causal Tensor Dynamics, and Einstein-Equivalent Gravity*, version 3.4, is under consideration by this journal as QSMF-D-26-00349. It develops the completed Standard-Model stress response, its causal tensor dynamics, and the Einstein-equivalent long-wavelength field equations. The second, *Operational Quantum Gravity in Practice: Spectral Stiffness, Matter Scaling, Equivalence, and Cosmology without Dark Energy*, version 0.3, was submitted to this journal as QSMF-S-26-00421. It develops operational scale comparisons of physical clocks and rulers, loading and unloading, equivalence, and cosmology. Because both manuscripts remain unpublished, they are identified here by title, version, and submission number rather than presented as published results. Their derivations, datasets, figures, and applications are not repeated. This paper uses only their declared OQG premises and adds a canonical state-space theorem, an endpoint analysis, and a conditional finite-loading result.

The distinction also prevents fragmentation of one calculation into minimally different reports. The first manuscript derives the response and field structure. The second develops practical and cosmological applications. The present paper asks a mathematically different question: which scale endpoints belong to the regular quantum state space? No Standard-Model loop calculation, cosmological fit, equation-of-state model, or collapse simulation is reused.

For reference, the recurring symbols are $L_{\mathrm{op}}$ for operational extension, $L_0$ for a positive reference extension, $b$ for their positive ratio, $\chi = -\ln b$ for logarithmic scale distance, $\rho$ for a quantum state, $\hat{X}$ and $\hat{P}$ for a collective canonical pair, $\hbar$ for the reduced Planck constant, and $\lambda$ for cumulative loading. Additional symbols are defined where they first appear.

Section 2 defines these quantities and the admissible state space. Section 3 constructs the canonical dilation and proves uncertainty preservation. Section 4 states and proves the no-zero-extension theorem. Section 5 gives the regular finite-loading corollary. Section 6 translates the result into the black-hole problem and compares it with other notions of singularity resolution. Section 7 states the limitations and the calculation that would turn the conditional corollary into a dynamical OQG result. A single compact appendix records the operator domain and endpoint-limit details.

## 2. Operational scale and the admissible state space

An operational length is a length produced by a specified material construction in a specified physical state. In other words, it is what a stated physical ruler or resolved material mode would actually report, not a coordinate distance introduced without a measurement prescription. The reference cannot be omitted. The Practice companion distinguishes epoch, source-established ambient, prepared, and causally unloaded states precisely so that two different baselines are not silently mixed. In this paper, $\rho$ denotes the state being examined and $\rho_0$ denotes whichever one of those physically appropriate reference states has been declared for that comparison. The symbol $L_{\mathrm{op}}[\rho]$ is the resulting operational extension, with $0 < L_{\mathrm{op}}[\rho_0] < \infty$. Define

$$b[\rho \mid \rho_0] \equiv \frac{L_{\mathrm{op}}[\rho]}{L_{\mathrm{op}}[\rho_0]} > 0, \qquad \chi[\rho \mid \rho_0] \equiv -\ln b[\rho \mid \rho_0]. \tag{1}$$

The vertical bar in $b[\rho \mid \rho_0]$ means "state $\rho$ compared with reference state $\rho_0$"; it is not a conditional probability. The positive, dimensionless number $b$ is the relative material scale. The natural logarithm ln converts multiplicative scale changes into the additive coordinate $\chi$: contraction gives $\chi > 0$, no relative change gives $\chi = 0$, and expansion gives $\chi < 0$.

The companion OQG papers reserve $A_{2,E}[\rho]$ for the static, Euclidean, transverse-traceless two-derivative Kubo stiffness extracted from the completed state-dependent stress response. On the homogeneous self-similar branch studied in QSMF-S-26-00421, and only when the dimensionless construction data are held fixed or treated separately, the Practice paper obtains the common cross-state ruler relation

$$b[\rho(\lambda) \mid \rho(\lambda_0)] = \frac{L_{\mathrm{op}}[\rho(\lambda)]}{L_{\mathrm{op}}[\rho(\lambda_0)]} = \left(\frac{A_{2,E}[\rho(\lambda)]}{A_{2,E}[\rho(\lambda_0)]}\right)^{1/2}.$$

Here $\lambda_0$ labels the declared reference point on that homogeneous state path. The equality is a constitutive result of the companion paper, not the definition of $b$, and it cannot be carried unchanged into a general anisotropic black-hole interior. Directional loading requires the full tensor response. The argument therefore uses only the kinematic ratio in (1) and does not assume a microscopic formula for $b$.

The variable $b$ is a transformation parameter, not a new position operator and not a postulated quantum of length. Restricting it to positive values describes scale changes that preserve spatial orientation rather than reflecting an axis. The question whether $b = 0$ can be added will be answered rather than assumed. One can formally write a map that sends a configuration coordinate to zero, but to count as a covariant canonical scale transformation it must preserve the position-momentum algebra, possess an inverse, and act on admissible quantum states.

To avoid disputed claims about a universal relativistic particle-position operator [11], the quantum coordinate used below is a *resolved collective scale-bearing mode*. "Collective" means that the coordinate describes an organized internal motion of many underlying degrees of freedom, not the absolute position of one fundamental particle. One example is a signed Cartesian displacement of a material part relative to a declared center of energy. More generally, a finite set of field observables can be averaged, or *smeared*, with smooth sampling functions over the region measured by an apparatus and then reduced to regular canonical pairs. Local quantum field theory is naturally formulated in terms of observables assigned to finite regions rather than undefined point measurements [5]. The result therefore concerns the regular finite-mode sector selected by an actual operational protocol, not a global particle-position operator for an interacting relativistic field theory.

Let $(\hat{X}, \hat{P})$ be one such canonical pair on a Hilbert space $\mathcal{H}$, the vector space in which normalized quantum state vectors are represented. The hats indicate operators rather than ordinary numerical variables. Self-adjointness of $\hat{X}$ and $\hat{P}$ is the mathematical condition that their measurement values are real. Because these operators can be unbounded, the rigorous starting point is the regular Weyl form of the canonical commutation relations. With $s$ and $t$ arbitrary real probe labels carrying the reciprocal units needed to make the exponents dimensionless,

$$e^{is\hat{X}} e^{it\hat{P}} = e^{-i\hbar st} e^{it\hat{P}} e^{is\hat{X}}. \tag{2}$$

Here $e^{is\hat{X}}$ and $e^{it\hat{P}}$ are bounded unitary operators generated by $\hat{X}$ and $\hat{P}$, $i$ is the imaginary unit, and $\hbar$ is the reduced Planck constant. Equation (2) is the exponentiated, domain-safe statement of the familiar relation $[\hat{X}, \hat{P}] = i\hbar\hat{I}$. *Regularity* means that an arbitrarily small change in $s$ or $t$ changes the corresponding unitary continuously when it acts on any state vector. Stone's theorem then supplies self-adjoint generators [15]. For finitely many canonical degrees of freedom, every irreducible regular representation is unitarily equivalent to the Schrödinger representation under the usual hypotheses [16, 17]. "Irreducible" means that the representation does not secretly split into independent invariant sectors. Working with (2) avoids treating the formal commutator of unbounded operators as if questions about their allowed domains did not exist.

An admissible state for the theorem is a *normal state* represented by a positive trace-class operator $\rho$. Positivity guarantees nonnegative probabilities, and $\mathrm{tr}\rho = 1$ says that total probability is one; tr denotes the trace, the operator analogue of summing diagonal probabilities. "Trace class" is the finiteness condition that makes this sum well defined. The state is also assumed to have finite second moments. Choose each internal coordinate relative to the declared material center and write

$$\ell_X(\rho)^2 \equiv \mathrm{tr}(\rho\hat{X}^2). \tag{3}$$

Thus $\ell_X(\rho) \geq 0$ is the root-mean-square distance carried by the selected center-relative material coordinate. Unlike a variance about that coordinate's own mean, it retains any nonzero mean displacement from the declared material center and therefore can represent extension rather than only statistical uncertainty. For several resolved coordinates define

$$L_{\mathrm{op}}(\rho)^2 \equiv \sum_{j=1}^{n} w_j \ \mathrm{tr}(\rho\hat{X}_j^2), \qquad w_j > 0. \tag{4}$$

Here $n$ is the finite number of resolved coordinates, the index $j$ labels them, and each fixed weight $w_j > 0$ states how strongly that coordinate contributes to the declared operational extension. The weights include any needed unit conversion or geometric normalization. Zero operational extension in this sector means that every included center-relative coordinate has zero root-mean-square magnitude. This definition is deliberately conservative. It does not identify a body's extension with the uncertainty of a fundamental particle position; it defines extension through the finite internal degrees of freedom that the measurement protocol actually resolves. It also makes the theorem conditional on the continued validity of that canonical sector. If a phase transition destroys the chosen collective coordinate, the appropriate conclusion is a change of physical branch and a need for new variables, not an extrapolation beyond the region where that coordinate is meaningful.

## 3. Uncertainty-preserving canonical scale covariance

Consider one finite change of operational scale. The coordinate operator $\hat{X}$ is multiplied by the positive scale factor $b$, while its conjugate $\hat{P}$ is multiplied by the reciprocal factor $b^{-1} = 1/b$:

$$\hat{X}_b = b\hat{X}, \qquad \hat{P}_b = b^{-1}\hat{P}, \qquad b > 0. \tag{5}$$

The reciprocal form is forced by the canonical algebra:

$$\left[\hat{X}_b, \hat{P}_b\right] = \left[b\hat{X}, b^{-1}\hat{P}\right] = i\hbar\hat{I}. \tag{6}$$

Here $\hat{I}$ is the identity operator. Thus contraction of a material coordinate is accompanied by expansion of its conjugate momentum scale, and the fundamental commutator remains unchanged. For $n$ pairs, collect all position-like and momentum-like operators into the column vector $\hat{\mathbf{R}} = \left(\hat{X}_1, \dots, \hat{X}_n, \hat{P}_1, \dots, \hat{P}_n\right)^T$ and let

$$S_b \equiv \begin{pmatrix} bI_n & 0 \\ 0 & b^{-1}I_n \end{pmatrix}, \qquad \Omega \equiv \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix}, \qquad S_b \Omega S_b^T = \Omega. \tag{7}$$

The superscript $T$ means matrix transpose. The symbol $I_n$ is the $n \times n$ identity matrix, each $0$ is an $n \times n$ zero block, $S_b$ is the matrix that performs the scale change, and $\Omega$ is the standard symplectic matrix encoding which $X_j$ is conjugate to which $P_j$. A real matrix is called *symplectic* when it preserves $\Omega$, as the last equality in (7) states. Also, $\det S_b = 1$, where $\det$ is the determinant. The transformation therefore preserves phase-space volume and the canonical pairing; it is not a dissipative projection that erases degrees of freedom.

Let $\Sigma$ be the statistical covariance matrix of the resolved operators,

$$\Sigma_{\alpha\beta} \equiv \frac{1}{2}\mathrm{tr}\rho\{\Delta\hat{R}_\alpha, \Delta\hat{R}_\beta\}, \qquad \Sigma_b = S_b \Sigma S_b^T. \tag{8}$$

The Greek indices $\alpha$ and $\beta$ label entries of $\hat{\mathbf{R}}$ and should not be confused with the scale factor $b$. The centered operator is $\Delta\hat{R}_\alpha \equiv \hat{R}_\alpha - \mathrm{tr}\left(\rho\hat{R}_\alpha\right)$, and $\{A, B\} \equiv AB + BA$ is the anticommutator. Accordingly, the diagonal entries of $\Sigma$ are variances and its off-diagonal entries describe correlations. The subscript on $\Sigma_b$ means "after scaling by $b$." The Robertson-Schrödinger uncertainty condition is the positive-semidefinite matrix inequality [13, 14]

$$\Sigma + \frac{i\hbar}{2}\Omega \succcurlyeq 0. \tag{9}$$

The notation $M \succcurlyeq 0$ means that the Hermitian matrix $M$ has no negative expectation value: $\mathbf{z}^\dagger M\mathbf{z} \geq 0$ for every complex vector $\mathbf{z}$. This compact statement contains all pairwise uncertainty bounds and correlations. Multiplying on the left and right by the real symplectic matrix gives

$$\Sigma_b + \frac{i\hbar}{2}\Omega = S_b\left(\Sigma + \frac{i\hbar}{2}\Omega\right)S_b^T \succcurlyeq 0. \tag{10}$$

Because a positive-semidefinite matrix remains positive-semidefinite under this congruence transformation, the full uncertainty condition is preserved. For one pair, (9) reduces to

$$(\Delta X)^2(\Delta P)^2 - \mathrm{Cov}(X,P)^2 \geq \frac{\hbar^2}{4}, \qquad \det\Sigma_b = \det\Sigma. \tag{11}$$

Here $\bar{X}_\rho \equiv \mathrm{tr}(\rho\hat{X})$ is the mean value, $\Delta X \equiv \sqrt{\mathrm{tr}\left[\rho\left(\hat{X} - \bar{X}_\rho\right)^2\right]}$ is the usual statistical uncertainty, and $\Delta P$ is defined in the same way for momentum. The quantity $\mathrm{Cov}(X,P)$ is the symmetrized $X$-$P$ covariance. The uncertainty $\Delta X$ should not be confused with the operational extension $L_{\mathrm{op}}$ defined from center-relative second moments in (4), although both scale by the same factor. The uncertainty content is preserved exactly at every finite scale. In particular,

$$\Delta X_b = b\,\Delta X, \qquad \Delta P_b = b^{-1}\Delta P. \tag{12}$$

The down-arrow notation $b \downarrow 0$ means that positive values of $b$ approach zero from above. Equations (11)-(12) are not a generalized uncertainty principle and do not yield a nonzero lower bound on $\Delta X$. They permit $\Delta X_b$ to become as small as desired as $b \downarrow 0$, provided the conjugate width $\Delta P_b$ becomes correspondingly large. The distinction between arbitrarily small and exactly zero is the center of the theorem.

The transformation also has a finite unitary implementation. A *unitary* operator preserves inner products, probabilities, and normalization and has an inverse equal to its adjoint. On a common invariant core - a dense set of sufficiently well-behaved states on which all displayed products are defined - introduce the symmetric dilation generator

$$\hat{D} \equiv \frac{1}{2}\left(\hat{X}\hat{P} + \hat{P}\hat{X}\right), \qquad \left[\hat{D},\hat{X}\right] = -i\hbar\hat{X}, \qquad \left[\hat{D},\hat{P}\right] = i\hbar\hat{P}. \tag{13}$$

The operator $\hat{D}$ is the generator of dilations, just as momentum generates translations. With $\chi = -\ln b$, the strongly continuous one-parameter group

$$U(\chi) \equiv \exp\left(\frac{i\chi\hat{D}}{\hbar}\right), \qquad U(\chi)^\dagger\hat{X}U(\chi) = e^{-\chi}\hat{X}, \qquad U(\chi)^\dagger\hat{P}U(\chi) = e^{\chi}\hat{P} \tag{14}$$

implements (5). The dagger $\dagger$ denotes the adjoint, so $U(\chi)^\dagger = U(\chi)^{-1}$. The exponential is an operator exponential, and $U(\chi)$ is unitary for every finite real value of $\chi$. It represents the scale comparison itself; it is not being asserted to be the physical time-evolution operator of a collapsing star. Finite dilations compose as

$$U(\chi_2)U(\chi_1) = U(\chi_1 + \chi_2), \qquad b(\chi_1 + \chi_2) = b(\chi_1)b(\chi_2). \tag{15}$$

Equation (15) says that performing two scale changes in succession adds their logarithmic parameters and multiplies their ordinary scale factors. Equivalently, define the transformed Schrödinger-picture state by $\rho_b \equiv U(\chi)\rho U(\chi)^\dagger$; moments of $\left(\hat{X},\hat{P}\right)$ in $\rho_b$ equal the corresponding moments of $\left(\hat{X}_b,\hat{P}_b\right)$ in $\rho$. "Schrödinger picture" here simply means that the state is transformed while the named observables are held fixed.

The identity transformation is $\chi = 0$ (equivalently $b = 1$), and the inverse of $U(\chi)$ is $U(-\chi)$. The word *group* means that finite transformations can be composed, have an identity, and have inverses. This group has the same composition structure as the real numbers under addition, written $(\mathbb{R}, +)$, or, through $b = e^{-\chi}$, as the positive real numbers under multiplication, written $(\mathbb{R}_+,\times)$. It contains every finite contraction and expansion. It contains no element $b = 0$; in boundary-limit notation,

$$b \downarrow 0 \quad \Leftrightarrow \quad \chi \to +\infty, \qquad b = 0 \notin \mathbb{R}_+. \tag{16}$$

At the attempted $b = 0$ boundary, the coordinate map sends every value of $X$ to the same value and therefore loses the information required for an inverse. At the same time, $b^{-1}\hat{P}$ is undefined. No finite value of the self-adjoint generator's parameter $\chi$ produces that map. These are algebraic facts; the independent state-normalizability argument follows next.

This is the central logical hinge, not a hidden assumption that every conceivable endpoint must be invertible. A canonical scale covariance is, by definition, an *automorphism*: a reversible, structure-preserving map of the canonical observable algebra. It preserves the symplectic form, the rule for composing transformations, and the information needed to recover the prior canonical variables. Equation (6) shows that setting only $\hat{X} \mapsto 0$ cannot meet those requirements, while the reciprocal assignment for $\hat{P}$ ceases to exist. The physical claim is therefore conditional but noncircular: if the OQG matter-scale sector continues to be represented by regular canonical covariance, its transformations have $b > 0$. A proposed noncanonical endpoint, a nonregular representation, or the destruction of the collective mode would constitute new physics outside that branch and must be supplied with its own observables and dynamics.

## 4. No-zero-extension theorem

*Theorem 1 (no zero extension in the regular canonical scale sector).* Let $(\hat{X}_j, \hat{P}_j)$, $j = 1, \dots, n$, be regular canonical pairs representing a finite set of resolved internal material coordinates. Each $\hat{X}_j$ measures a signed displacement from the declared material center, and $\hat{P}_j$ is its conjugate momentum. Assume that these coordinates have the ordinary continuous behavior of position near the center: normalized wave packets can be concentrated as narrowly as desired around $X_j = 0$, but no normalizable state has the exact value $X_j = 0$ with certainty. In operator language, the projector onto an exact zero eigenstate vanishes, $E_j(\{0\}) = 0$, equivalently $\ker\hat{X}_j = \{0\}$. Let $\rho$ be an ordinary normalized quantum state, pure or mixed, for which the mean-square displacements are finite, and let $L_{\mathrm{op}}(\rho)$ be defined by (4). Then $L_{\mathrm{op}}(\rho) > 0$. Every state obtained from $\rho$ by a finite canonical dilation also has positive operational extension. Exact zero extension is not a state in this regular canonical state space.

*Proof.* Consider one coordinate $\hat{X}_j$. A measurement of that coordinate in the state $\rho$ has an ordinary probability distribution, denoted $\mu_{\rho,j}$. More precisely, $E_j(B)$ is the spectral projector selecting measurement results in a set $B$ of coordinate values, and $\mu_{\rho,j}(B) = \mathrm{tr}\big[\rho E_j(B)\big]$ is the probability of obtaining a value in that set. The mean-square displacement is therefore

$$\ell_{X_j}(\rho)^2 = \mathrm{tr}\big(\rho\hat{X}_j^2\big) = \int_{\mathbb{R}} x^2 \, d\mu_{\rho,j}(x). \tag{17}$$

Every contribution to the integral is nonnegative because $x^2 \geq 0$. The integral could equal zero only if the probability of every nonzero value vanished, placing total probability one at the single value $x = 0$. That would require $\mu_{\rho,j}(\{0\}) = 1$. The theorem's coordinate assumption gives the opposite result, $\mu_{\rho,j}(\{0\}) =$

$\mathrm{tr}\big[\rho E_j(\{0\})\big] = 0$. The assumed zero mean-square extension would therefore require the same probability to be both one and zero, which is impossible. Hence $\ell_{X_j}(\rho) > 0$. Equation (4) is a sum of these positive mean-square extensions with positive weights, so $L_{\mathrm{op}}(\rho) > 0$. Under a finite dilation, every coordinate is multiplied by $b > 0$, every term in (4) is multiplied by $b^2$, and $L_{\mathrm{op}}(\rho_b) = bL_{\mathrm{op}}(\rho) > 0$. QED.

In plain language, zero operational extension would require every resolved internal coordinate to be found exactly at the declared material center with one hundred percent probability. A continuous position-like coordinate has no normalizable state of that kind. It has ordinary wave packets that can be made narrower and narrower, but the supposed perfectly sharp endpoint is a Dirac distribution, an idealized point spike rather than a square-integrable wavefunction in $L^2(\mathbb{R})$. Equation (17) covers pure wave packets and statistical mixtures in the same way: averaging several positive-width states cannot manufacture a zero-width state.

The theorem asserts *nonattainment*, not a positive minimum. The symbol inf below denotes the infimum, or greatest lower bound, over all admissible normal states:

$$\inf_{\rho\ \mathrm{normal}} \ell_X(\rho) = 0, \qquad \ell_X(\rho) \neq 0 \text{ for every admissible } \rho. \tag{18}$$

Equation (18) is analogous to the open interval $0 < x < 1$: positive numbers can be selected as close to zero as desired, yet zero is not a member of the interval. Likewise, normalized states can have root-mean-square extensions closer and closer to zero, but no member has zero extension. Along the unitary scale orbit the same distinction appears as finite $\chi$ versus the boundary notation $\chi = \infty$. The limiting wave-packet sequence has no *strong* Hilbert-space limit, meaning it does not converge in vector norm. For standard packets it converges *weakly* to zero, meaning its overlap with every fixed test vector tends to zero, even while its coordinate probability measures concentrate toward a Dirac point measure. One may enlarge the mathematical framework to generalized vectors or nonregular states, but that enlargement changes the state space and loses the finite regular canonical representation assumed here. Appendix A records the short domain and limiting argument.

Theorem 1 is stronger than the statement that the map at $b = 0$ is noninvertible, because it independently excludes an exactly zero-extension normal state even if one tries to append such a state by hand. By itself it classifies the regular state space rather than selecting a collapse history. Section 5 adds the finite-loading condition needed to prevent a physical trajectory from reaching the boundary at a finite loading stage, and Section 6 states the corresponding OQG continuation result. If the regular canonical branch changes, the matter description must change with it; the missing variables cannot be supplied by declaring the geometric bookkeeping singular.

> *Corollary 1 (finite scale distance).* A finite sequence of canonical OQG scale transformations cannot produce zero extension. Let $N$ be the number of finite scale steps, let $\delta\chi_k$ be the logarithmic scale change at step $k$, and let $\Delta\chi_N \equiv \sum_{k=1}^{N} \delta\,\chi_k$ be their accumulated change. Then

$$b_N = b_0 e^{-\Delta\chi_N} = b_0 \exp\!\left(-\textstyle\sum_{k=1}^{N} \delta\,\chi_k\right) > 0 \qquad (N < \infty). \tag{19}$$

Here $b_0 > 0$ is the initial scale ratio and $b_N$ is the ratio after $N$ steps. This finite-step statement is unconditional but deliberately modest. Infinitely many ever-smaller layers can accumulate a divergent sum, and a continuous trajectory can have a response rate whose total diverges. The conditions under which finite physical loading excludes those possibilities are stated next.

## 5. Regular finite-loading corollary

Theorem 1 classifies admissible states; it does not by itself describe motion from one state to another. To state a limited dynamical result, let $\lambda$ be a nondecreasing, dimensionless measure of cumulative mass-energy loading of the selected material sector, with any units removed by one fixed positive reference energy. "Nondecreasing" means that $\lambda$ serves as an ordered progress variable along the chosen history. The initial state has $b(0) > 0$ and therefore finite $\chi(0)$. A finite loading interval is written $0 \leq \lambda \leq \lambda_f < \infty$, where the subscript $f$ marks the finite endpoint being examined. This bookkeeping variable does not assume that all supplied energy becomes compression work; radiation, transport, binding, phase change, and other channels remain part of the complete OQG energy ledger.

Assume that the logarithmic scale coordinate $\chi(\lambda)$ is *absolutely continuous* on every finite loading interval. Operationally, this means that its total change is obtained by integrating an ordinary response rate and that no unaccounted finite jump is hidden at a point:

$$\chi(\lambda) = \chi(0) + \int_0^{\lambda} \frac{d\chi}{du}\, du. \tag{20}$$

Here $u$ is only a dummy integration variable. The derivative $d\chi/du$ is the logarithmic scale change per unit loading; a positive value describes increasing contraction. The regular finite-loading hypothesis is

$$\int_0^{\lambda_f} \left|\frac{d\chi}{du}\right| du < \infty \quad \text{for every finite } \lambda_f. \tag{21}$$

The absolute value prevents divergent contraction and expansion from being hidden by cancellation. Equation (21) therefore says that the total magnitude of the logarithmic scale response is finite whenever the accumulated loading is finite.

> *Theorem 2 (regular finite loading).* Under (20)-(21), finite accumulated loading cannot reach the zero-extension boundary.

*Proof.* Absolute integrability gives

$$|\chi(\lambda_f)| \leq |\chi(0)| + \int_0^{\lambda_f} \left|\frac{d\chi}{du}\right| du < \infty, \qquad b(\lambda_f) = e^{-\chi(\lambda_f)} > 0. \tag{22}$$

The first inequality is the triangle inequality for the integral. It bounds the final logarithmic scale distance by its finite initial value plus a finite accumulated response. Therefore $\chi(\lambda_f)$ remains finite, its exponential $b(\lambda_f) = e^{-\chi(\lambda_f)}$ remains strictly positive, and the endpoint belongs to the positive dilation orbit. QED.

The discrete form is equally transparent. A countable layer decomposition labels its layers by $k$ and assigns each layer an increment $\delta\chi_k$. If $\sum_k |\delta\chi_k| < \infty$ for every finite total loading, then the product $b = b_0 \prod_k e^{-\delta\chi_k}$ converges to a positive number. The symbol $\prod$ denotes repeated multiplication, just as $\sum$ denotes repeated addition. The continuum condition (21) is the corresponding absolute-summability statement in the declared cumulative-loading parameter; it is not a claim that every microscopic OQG response automatically satisfies it.

The contrapositive - the logically equivalent statement obtained by reversing and negating the implication - is physically useful. If an OQG trajectory has $b(\lambda) \to 0$ at some finite $\lambda_f$, then at least one assumption used in Theorem 2 fails:

$$b(\lambda) \to 0 \text{ as } \lambda \uparrow \lambda_f < \infty \;\Rightarrow\; \begin{cases} \int_0^{\lambda_f} \left|\frac{d\chi}{du}\right| du = \infty, \text{ or} \\ \chi \text{ is not absolutely continuous, or} \\ \text{the regular canonical scale branch terminates before } \lambda_f. \end{cases} \tag{23}$$

The notation $\lambda \uparrow \lambda_f$ means that $\lambda$ approaches the finite endpoint $\lambda_f$ from below. Equation (23) turns a vague appeal to "finite mass" into a testable regularity requirement. Finite total mass-energy alone does not bound $\chi$ unless the scale response per unit loading is integrable. A pole or critical divergence in $d\chi/d\lambda$, an impulsive jump not represented by the integral, or a phase change that destroys the collective coordinate can invalidate the finite-loading corollary. Such behavior would not make $b = 0$ a regular state; it would mean that the evolution reaches or exposes the boundary through a breakdown of the assumptions. This is exactly why Theorems 1 and 2 must not be conflated.

The missing dynamical calculation is now narrow. The full state-dependent Standard-Model response must determine $d\chi/d\lambda$ along controlled high-density histories and establish, or fail to establish, (21). In physical terms, one must calculate how much additional logarithmic contraction is produced by each additional unit of loading and whether the accumulated magnitude stays finite. No arbitrary pressure floor, fixed minimum length, or prescribed bounce may substitute for that calculation.

## 6. Black-hole interpretation and comparison with other approaches

Apply the scale sector directly to a finite material configuration inside a collapsing body. The physical object is the evolving matter state $\rho(\lambda)$ and its causal response, not a curve embedded in a fundamental spacetime. At long wavelength the same history may be drawn geometrically as a center-of-energy *worldtube*, but in OQG that worldtube is a record of the matter evolution rather than the object that evolves. Let $L_0 > 0$ be the declared reference extension of the selected matter state, let $\lambda$ label its cumulative loading as in Section 5, and let

$$L_{\mathrm{op}}(\lambda) = b(\lambda)L_0 = e^{-\chi(\lambda)}L_0. \tag{24}$$

Equation (24) says that the physical extension at loading $\lambda$ is the reference extension multiplied by the relative scale factor. A realized zero-extension center would require a normal state at $L_{\mathrm{op}} = 0$. Theorem 1 excludes that state within the regular canonical OQG scale sector. If, in addition, the physical loading history obeys (20)-(21), Theorem 2 excludes reaching the boundary under finite accumulated loading. Matter may pass through successively smaller covariant states without any member being the zero-extension endpoint.

The scale map does not discard the mass-energy carried into the interior. A canonical dilation redistributes the resolved state between a contracted coordinate width and an expanded conjugate momentum width while preserving normalization and the canonical information content. It is not, by itself, a time-evolution operator, so unitarity of the dilation should not be misquoted as conservation of the expectation value of an arbitrary fixed Hamiltonian. Stress-energy conservation belongs to the complete OQG dynamical equations and their matter, field, binding, transport, and radiative channels. At the level proved here, the important point is narrower: neither (5) nor the limit $b \downarrow 0$ contains an operation that deletes matter or stress-energy. A physical interior solution must keep that energy in occupied matter, field, and stress degrees of freedom, and must calculate how the increasing conjugate fluctuations enter the energy ledger. The theorem classifies the zero-extension endpoint without pretending to supply that partition.

Nothing in this reasoning prevents formation of a horizon. Operationally, an OQG horizon is a one-way causal boundary produced by the completed retarded Standard-Model response: after matter crosses it, no outward chain of physical signals reaches distant exterior matter. General relativity represents the same causal fact with light cones, null geodesics, and trapped surfaces, but those geometric objects are the long-wavelength record of the causal response, not its source. Once the one-way structure forms, every locally forward causal route available to infalling matter remains directed deeper into the loaded interior. There is no route back out and no reason in this theorem for the inward scaling to stop. The conclusion is instead that inward evolution cannot terminate by turning the matter into an exact zero-extension state. Under the continuation conditions stated below, a body can be hidden behind a horizon while passing through an unending succession of smaller positive material scales.

General relativity calls the central endpoint singular because a causal geodesic in its metric description ends after a finite value of its affine or proper-time parameter [4, 6, 12]. That definition is internally consistent, but its physical interpretation assumes that the geometric curve is fundamental. In OQG the order is reversed. Matter and its causal response are fundamental; the metric is reconstructed bookkeeping, and a geodesic is a derived line in that bookkeeping. An incomplete derived line cannot terminate the matter that generated the description. It shows that the geometric extrapolation has reached the end of its valid range.

*Corollary 2 (continued positive-scale evolution).* Suppose the regular canonical matter branch remains available for every finite loading value $\lambda \geq 0$, and suppose (20)-(21) hold on each finite interval. Then the material extension is positive at every finite stage. If continued contraction makes the extension tend toward zero, this can occur only as both the loading parameter and logarithmic scale distance become unbounded:

$$L_{\mathrm{op}}(\lambda) > 0 \quad \text{for every } \lambda < \infty, \qquad L_{\mathrm{op}}(\lambda) \to 0 \;\Rightarrow\; \lambda \to \infty, \quad \chi(\lambda) \to +\infty. \tag{25}$$

*Proof.* Theorem 2 gives $L_{\mathrm{op}}(\lambda) > 0$ at every finite $\lambda$. Suppose $L_{\mathrm{op}}(\lambda_n) \to 0$ along a sequence of loading values $\{\lambda_n\}$ that is bounded above. Because $\lambda_n \geq 0$, the sequence has a subsequence $\lambda_{n_k} \to \lambda_f$ for some finite $\lambda_f$. Absolute continuity makes $\chi$ continuous and finite at $\lambda_f$. Equation (24) therefore gives $L_{\mathrm{op}}(\lambda_{n_k}) \to L_{\mathrm{op}}(\lambda_f) = e^{-\chi(\lambda_f)} L_0 > 0$, contradicting the assumed convergence to zero. The loading values must therefore become unbounded. Since $\lambda$ is nondecreasing along the history, this means $\lambda \to \infty$; equation (24) then shows that $L_{\mathrm{op}} \to 0$ only when $\chi \to +\infty$. QED.

Equation (25) is the precise meaning of "it keeps scaling." There is no final infinite contraction performed in one step and no last state at zero size. For every finite stage there is a normal, positive-extension matter state, and any further finite change leads to another one. The limit may become arbitrarily small, but the limit is not a member of the sequence. Within the assumptions of the corollary, the central geodesic incompleteness of general relativity is therefore removed as a physical singularity: what ends is the range of the emergent geometric bookkeeping, not the OQG matter evolution. OQG does not need to complete a fundamental geodesic because no fundamental geodesic exists. It must keep the quantum matter description well defined, which is exactly what the stated regular-branch and finite-interval conditions provide.

This conclusion remains conditional in a scientifically useful way. The full Standard-Model response must decide whether the same canonical variables survive every finite stage. A phase change may require a new set of matter observables, and a nonintegrable response at finite loading would signal that the present branch has lost validity. Either outcome demands a new physical description of matter; neither licenses a zero-extension state or turns a failed geometric extrapolation into the destruction of matter.

This conclusion is distinct from several established research programs. Generalized uncertainty-principle models modify the usual position-momentum algebra and can produce a nonzero minimum position uncertainty or a black-hole remnant [1, 8]. Here the commutator remains $[\hat{X}, \hat{P}] = i\hbar\hat{I}$ and no universal minimum length is postulated. Loop quantum cosmology quantizes a symmetry-reduced cosmological geometry and can replace a classical big-bang singularity by a bounce [2]. Wheeler-DeWitt approaches instead impose a quantum wave equation on gravitational configurations; recent wave-packet calculations study whether those dynamics delay or avoid the Schwarzschild interior singularity [3]. The present result requires neither a bounce nor a smallest radius. It removes the zero-extension endpoint from the material state space and identifies the geometric singularity as a boundary of the emergent description.

Horowitz and Marolf proposed a different criterion: a classically singular static spacetime may be called quantum mechanically nonsingular for a test particle when its evolution is uniquely determined without an extra boundary condition at the singularity [7]. Mathematically, that criterion concerns *essential self-adjointness*: whether the probe Hamiltonian has one unique self-adjoint completion and therefore one unique unitary evolution. It is evaluated on a fixed background spacetime. OQG does not grant that background fundamental status; its criterion is continuation of the normal matter state under regular canonical scaling. Affine quantization also uses dilation-type kinematics for classical variables restricted to be positive [10]. In the present construction, however, $b$ is the parameter of an ordinary unitary canonical transformation, while the collective quadratures retain the undeformed Weyl algebra. Theorem 1 does not import an affine gravitational metric degree of freedom.

The comparison shows where the novelty lies. The unitary dilation, the Weyl relations, and the uncertainty inequalities are standard mathematics. The new claim is their bounded OQG assembly: on the explicit canonical-branch assumption, the scale of physical matter standards is carried into the black-hole interior; finite common-scale changes are canonical and uncertainty-preserving; the exactly zero-extension endpoint is outside that regular state space; finite loading excludes the endpoint when the OQG response is integrable; and the geodesic endpoint is reclassified as the failure boundary of an emergent geometric record rather than the end of matter. This is a *state-space obstruction* with a declared dynamical gate and an operational continuation criterion. It is not a new uncertainty relation.

## 7. Scope, falsifiability, and the next calculation

The paper proves four statements within explicit assumptions. First, reciprocal scaling of a regular canonical pair is symplectic and preserves the full Robertson-Schrödinger uncertainty condition. Second, a center-relative position-like coordinate with no normalizable zero eigenstate has no normal state of exactly zero root-mean-square magnitude, so $b = 0$ cannot be represented as a normal zero-extension state. Third, an absolutely continuous OQG scale history with integrable logarithmic response on finite loading intervals remains at $b > 0$. Fourth, if that regular branch is available through every finite loading stage, an extension tending toward zero can do so only along an unbounded sequence of stages; there is no terminal zero-extension matter state.

It does not prove a nonzero universal lower bound $L_{\min}$, where $L_{\min}$ would mean one fixed smallest allowed length. It does not calculate a turning radius, equilibrium core, equation of state relating quantities such as pressure and energy density, pair-production wall, or phase transition. It does not determine whether contraction proceeds monotonically, pauses, reverses, or changes material phase; those are questions for the full Standard-Model response. Nor does it attempt to smooth a fundamental spacetime curvature or complete a fundamental geodesic, because OQG treats neither as fundamental. Its result is instead operational: under the stated continuation conditions, failure of those derived geometric quantities does not terminate the matter-state evolution. The exterior long-wavelength field equation established in the first companion manuscript is unchanged.

The assumptions can be challenged in specific ways. A controlled derivation could show that the strong-field OQG scale sector is not represented by a regular canonical pair, that the selected collective coordinate loses validity, or that the relevant commutator is deformed. Any of those findings would limit Theorem 1's physical application while leaving its mathematics intact. A calculated loading response with a nonintegrable divergence at finite cumulative energy would defeat Theorem 2 on that branch. A branch-changing transition could require a new canonical chart - a new set of valid position-like and momentum-like variables - and a new extension observable. Conversely, a full Standard-Model response satisfying (21) on every finite physical loading interval, together with a valid canonical description through those intervals, would establish the operational continuation criterion throughout the strong-field domain.

The decisive next object is therefore not another collapse survey. It is the logarithmic scale susceptibility along a declared high-density state path. Schematically, the calculation must determine

$$\frac{d\chi}{d\lambda} = -\frac{d}{d\lambda}\ln\left[\frac{L_{\mathrm{op}}(\lambda)}{L_0}\right]. \tag{26}$$

Equation (26) is simply the precise version of "contraction produced per unit loading." This derivative must be obtained from the same completed Standard-Model state response used by OQG. If the state admits the static limit and local gradient expansion required by the companion papers, $A_{2,E}[\rho]$ may supply the applicable stiffness input. A driven or nonstationary strong-field history instead requires the full retarded transverse-traceless response kernel, conventionally written $\Pi^{R}_{\mathrm{TT}}(\omega, \mathbf{k}; \rho)$, where $\omega$ is frequency and $\mathbf{k}$ is spatial wave vector. The calculation must include thresholds at which new processes begin, binding energy, material composition, energy transport, dissipation, and matching across phases without counting the same contribution twice. It must also identify the state domain in which the collective scale coordinate and its canonical conjugate remain valid. The pass condition is the integrability bound (21) with controlled errors. A resolved nonintegrable divergence would show that the present regular branch loses validity at finite loading and that a new matter description is required; it would not establish a physical zero-extension state. An unresolved coefficient remains inconclusive.

There is also an immediate mathematical audit. The field-theoretic reduction should be written with smooth smearing functions - sampling profiles that average a field over finite regions - and a finite measurement algebra, then checked for regularity of the induced Weyl representation. Infinite-dimensional quantum field theory can describe physically inequivalent sectors that cannot all be connected by unitary transformations. For that reason, the finite-mode Stone-von Neumann uniqueness statement cannot simply be applied to the full field algebra. The theorem has been formulated to avoid that overreach: it applies to the resolved regular scale sector, and Appendix A displays the corresponding smeared-pair normalization.

## 8. Conclusion

Finite OQG scale covariance is an invertible quantum transformation. For every $b > 0$, reciprocal scaling of collective position-like and momentum-like quadratures preserves the commutator, the symplectic form, and the uncertainty matrix, and is represented by a finite unitary dilation. Exact zero extension is not the endpoint member of that family. It lies at $\chi = \infty$, makes the inverse momentum scaling undefined, and has no normalizable zero-extension representative in the regular canonical state space.

Under an additional and explicit regularity condition, finite loading also cannot reach that boundary: integrability of $d\chi/d\lambda$ on finite loading intervals keeps $\chi$ finite and $b$ positive. If the regular canonical branch remains available through every finite stage, matter can continue through arbitrarily small positive extensions without ever becoming a zero-extension state. In OQG this removes the central geodesic endpoint as a physical singularity. The incomplete object is the emergent geometric bookkeeping curve, not the quantum matter evolution that the curve was constructed to represent. Horizons and branch changes remain possible, but neither requires a point endpoint. The remaining calculation is sharply defined: determine the full Standard-Model scale susceptibility and test whether the regular finite-loading and branch-continuation conditions survive throughout the strong-field regime.

## Appendix A. Dilation domain, endpoint limit, and smeared modes

In the one-mode Schrödinger representation, a state is a square-integrable wavefunction $\psi(x)$ in $L^2(\mathbb{R}, dx)$, where $x$ is the numerical value of the internal coordinate and $dx$ denotes integration over the real line. Take $\hat{X}\psi(x) = x\psi(x)$ and $\hat{P}\psi(x) = -i\hbar\, d\psi/dx$ on the Schwartz space, the dense set of smooth wavefunctions that, together with all their derivatives, decrease faster than any inverse power of $|x|$. The symmetric generator (13) acts as $\widehat{D} = -i\hbar(x\, d/dx + 1/2)$ and is essentially self-adjoint on a standard invariant core, meaning that this initial domain determines a unique self-adjoint generator. Its unitary group acts by

$$[U(\chi)\psi](x) = e^{\chi/2}\psi(e^{\chi}x) = b^{-1/2}\psi(x/b), \qquad b = e^{-\chi} > 0. \tag{A1}$$

The prefactor $b^{-1/2}$ is exactly what preserves the norm $\int |\psi(x)|^2 dx$, and a direct change of variables proves (12). The formula also fixes the sign convention: increasing $\chi$ contracts the $X$ distribution and expands the $P$ distribution.

For a wavefunction $\psi \in L^1 \cap L^2$ - both absolutely integrable and square-integrable - and a bounded test wavefunction $\varphi$ that vanishes outside a finite region,

$$\langle \varphi, U(\chi)\psi \rangle = b^{1/2} \int_{\mathbb{R}} \varphi\,(by)^* \psi(y)\, dy \to 0 \qquad (b \downarrow 0). \tag{A2}$$

Here $y$ is a dummy integration variable. The brackets denote the Hilbert-space inner product, and the star denotes complex conjugation. Density extends this weak convergence to all test vectors in $L^2$. Since $\| U(\chi)\psi \| = \| \psi \| \neq 0$, the family has no strong, or norm, limit. Its coordinate probability distributions can nevertheless concentrate toward a Dirac point measure. That distributional limit is not a normalized Hilbert-space vector, which is the endpoint distinction used in the main text.

For a field-theoretic measurement sector, let $\hat{\phi}(\mathbf{x})$ be a field operator and $\hat{\pi}_\phi(\mathbf{x})$ its equal-time conjugate momentum field at spatial point $\mathbf{x}$. These symbols are deliberately distinct from the OQG response kernel $\Pi^R_{\mathrm{TT}}$. Let $f(\mathbf{x})$ and $g(\mathbf{x})$ be smooth sampling functions that vanish outside finite regions. Smearing means averaging the fields against these functions rather than attempting an infinitely sharp point measurement. Their resolved pair obeys

$$\hat{X}_f \equiv \int d^3x\, f(\mathbf{x})\hat{\phi}(\mathbf{x}), \qquad \hat{P}_g \equiv \int d^3x\, g(\mathbf{x})\hat{\pi}_\phi(\mathbf{x}), \qquad \left[\hat{X}_f, \hat{P}_g\right] = i\hbar \int d^3x\, f(\mathbf{x})g(\mathbf{x}). \tag{A3}$$

The volume element $d^3x$ indicates integration over three-dimensional space. With the overlap normalized by $\int f(\mathbf{x})g(\mathbf{x})\, d^3x = 1$, equation (A3) gives one canonical quadrature pair. Theorem 1 applies only to an induced finite, normal, regular measurement sector whose selected $X$ operator has no normalizable zero-eigenvalue vector; it makes no uniqueness claim for the full interacting field algebra.

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