Static Transverse-Traceless Stress Susceptibility of the High-Temperature Standard Model through Second Order in the Couplings

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Abstract

We calculate the static transverse-traceless stress susceptibility of the symmetric high-temperature Standard Model through second order in the interaction couplings, using the stated top-Yukawa truncation. The observable is the thermal-minus-vacuum second metric variation of the equilibrium generating functional, so connected stress fluctuations and the local contacts required by Ward identities are varied and renormalized together. The calculation combines the free response, screened bosonic zero modes, five hard two-loop sectors, Higgs-curvature matching, the electrostatic effective theory, and local-curvature renormalization. The Higgs-contact and operator poles close against their specified counterterms, the three-dimensional factorization-scale dependence cancels, and the remaining four-dimensional scale dependence agrees with the running of the Higgs-curvature coupling. We obtain the massive scalar-gauge threshold analytically. The confining magnetostatic remainder reduces to two finite response coefficients of pure three-dimensional Yang-Mills theory. A flat-lattice infrared-minus-ultraviolet construction determines these coefficients while canceling the additive curvature counterterm. The resulting expression contains only the renormalized Higgs-curvature coupling and the two defined Yang-Mills responses. It therefore closes the perturbative and effective-theory matching problem through second order and establishes a finite local metric stiffness derived from matter stress response.

Keywords: Standard Model; finite-temperature field theory; stress tensor; transverse-traceless response; dimensional reduction; three-dimensional Yang-Mills

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1. Introduction

Equilibrium stress-tensor correlators describe the response of a quantum state to weak geometry. In the transverse-traceless (TT) shear channel, the coefficient of the term quadratic in spatial wave number is a hydrostatic curvature susceptibility. It is local in a derivative expansion, but it cannot be extracted from a separated-point correlator alone. A metric variation differentiates both the statistical weight and the metric dependence of the action; connected stress fluctuations and local second-variation contacts are therefore parts of one completed Hessian [11,15]. This distinction is the central organizing principle of the calculation. Omitting the contact term changes the quantity being computed and prevents the hard and reduced theories from matching consistently.

The high-temperature Standard Model provides a stringent example. Here and below, g denotes a generic Standard-Model gauge, Yukawa, or scalar coupling when no species label is attached. Nonzero Matsubara modes carry momenta of order $\pi k_B T$; screened temporal gauge fields and the Higgs zero mode live at order $g k_B T$; and spatial non-Abelian zero modes form confining three-dimensional Yang-Mills theories at order $g^2 k_B T$ [2,9]. Contributions nominally of order g^2 consequently arise in more than one effective theory. Intermediate terms contain ultraviolet poles and logarithms of independent matching scales. A valid result must close those ledgers after the variational contacts are included; numerical smallness by itself is not an adequate check.

The necessary ingredients exist separately. Hydrostatic Kubo relations are given in Refs. [11,15]. The general-momentum two-loop Standard-Model shear correlator was organized into five gauge-independent sectors in Ref. [7], using the thermal master technology of Ref. [8]. Standard-Model dimensional reduction and Higgs-condensate matching are available in Refs. [2,9,13], while the curved-space running of the nonminimal Higgs coupling is known [14]. Three-dimensional stress form factors and ultraviolet Yang-Mills behavior are reported in Refs. [5,6], and curved multiloop heat-kernel methods supply the method used here to evaluate the massive scalar-gauge threshold [3,4]. The missing step is to normalize these pieces as one static metric Hessian and demonstrate their joint pole, contact, and scale closure.

The objective is to determine whether a local two-derivative metric stiffness can be defined from a renormalized matter response, with its contact terms and effective-theory matching exposed. This response-theoretic question is related to proposals for effective gravitational equations in matter [12], but the observable studied here is the equilibrium static Hessian itself. Establishing that Hessian as a finite, scheme-defined field-theory quantity is a necessary step before any causal or infrared gravitational interpretation can be formulated.

The calculation provides the static reduction of all five hard interaction classes, their variational completion by the Higgs contact, an analytic massive scalar-gauge threshold, a two-scale matching ledger that distinguishes derived closures from matching conditions, and a counterterm-free lattice construction for the two remaining pure-Yang-Mills constants. The main text contains the definitions and reductions needed to identify the observable, follow the matching across scales, and determine the remaining nonperturbative coefficients.

Throughout, $\Theta \equiv k_B T$ is the thermal energy, with k_B Boltzmann's constant and T the temperature. The symbols g_Y, g_2, and g_3 denote the hypercharge, weak, and strong gauge couplings; y_t is the top Yukawa coupling; and λ is the Higgs quartic coupling. We use the conventional top-Yukawa truncation consistently, neglecting the bottom and tau Yukawa couplings y_b and y_τ at this order and defining $Y_2 \equiv 3y_t^2$. This is a stated approximation, not a claim that those couplings vanish. At the benchmark couplings, restoring y_b and y_τ changes Y_2 by 0.038% and the associated ξ_H running target by 0.041%; these figures quantify the truncation but are not folded into the reported coefficients. The leading color-electric, weak-electric, hypercharge-electric, and Higgs screening masses are denoted by m_{D3}, m_{D2}, m_{DY}, and m_H, respectively. The scale $\bar{\mu}$ is the four-dimensional modified-minimal-subtraction renormalization scale. The coupling values in Table 1 together with the analytic screening-mass relations below define a dimensionless benchmark state at $\bar{\mu} = 2\pi\Theta$; no unique physical temperature is inferred from them. Extra digits are retained only to audit arithmetic and cancellations, not as uncertainty estimates; the corresponding checks are supplied in Online Resource 1.

Table 1. Benchmark inputs and leading screening masses.

Running coupling	**Value**	**Screening mass**	**Value divided by Θ**
g_3	0.486635	m_{D3}	0.688205817
g_2	0.502460	m_{D2}	0.680334056
g_Y	0.477571	m_{DY}	0.646634191
y_t	0.401965	m_H	0.291877592
λ	-0.0335869		
$Y_2 = 3y_t^2$	0.484727584		

The mass ratios in Table 1 follow the leading symmetric-phase relations

$$\frac{m_{D3}^2}{\Theta^2} = 2g_3^2, \qquad \frac{m_{D2}^2}{\Theta^2} = \frac{11}{6}g_2^2, \qquad \frac{m_{DY}^2}{\Theta^2} = \frac{11}{6}g_Y^2, \qquad \frac{m_H^2}{\Theta^2} = \frac{3g_2^2 + g_Y^2}{16} + \frac{\lambda}{2} + \frac{y_t^2}{4}.$$

The mass ratios are displayed to enough digits to reproduce the numerical rows below; the coupling inputs and analytic mass relations, rather than rounded mass ratios alone, define the benchmark.

2. Completed static susceptibility

Let z be one Cartesian spatial coordinate and let k be a real wave number directed along it. Introduce a static off-diagonal metric source in the xy shear channel,

$$h_{xy}(z) = h_k e^{ikz} + h_{-k} e^{-ikz}. \tag{1}$$

where h_k and $h_{-k} = h_k^*$ are the two Fourier amplitudes of the real source and the asterisk denotes complex conjugation. Because the only nonzero component is h_{xy} and the momentum points along z, the perturbation is transverse and traceless. Let $Z_E[h]$ be the Euclidean partition function in this background, let $W_E[h] \equiv -\ln Z_E[h]$ be its connected generating functional, and let V_4 be the Euclidean spacetime volume. Let T_{xy} be the xy component of the complete Standard-Model stress tensor, let angle brackets denote thermal Euclidean expectation values, let the subscript c select their connected part, and let $G_{E,\mathrm{contact}}(k)$ be the local second metric variation of the microscopic action. Every response below denotes the renormalized thermal value minus its zero-temperature value at the same renormalized parameters. With this convention, the completed response is the negative second derivative of W_E per unit volume. In schematic operator notation,

$$G_E(k) \equiv -\frac{1}{V_4}\frac{\delta^2 W_E}{\delta h_k \delta h_{-k}}\bigg|_{h=0} = \int d^4x\, e^{ikz}\langle T_{xy}(x)T_{xy}(0)\rangle_c - G_{E,\mathrm{contact}}(k). \tag{2}$$

The same source action therefore fixes both terms and their relative sign; the contact is not an optional correction added after the correlator is computed.

The superscript xy, xy used below only makes this shear channel explicit: $G_E^{xy,xy}(0,k)$ is the same completed response as $G_E(k)$ in Eq. (2), evaluated at zero Euclidean frequency.

The symbol ∂_k^2 denotes the ordinary second derivative with respect to the scalar wave number while every thermodynamic and renormalized parameter is held fixed. The conventional hydrostatic coefficient κ and the source-normalized susceptibility $A_{2,E}$ are defined by

$$\kappa = \partial_k^2 G_E^{xy,xy}(0,k)\big|_{k=0}, \qquad A_{2,E} = \frac{\kappa}{4}. \tag{3}$$

Equivalently, $G_E(0,k) = G_E(0,0) + (\kappa/2)k^2 + \mathcal{O}(k^4)$ near the origin. The factor $1/4$ follows from the symmetric tensor-source convention $\Pi_{TT,E} \equiv -G_E^{xy,xy}/2$ adopted here; it is only a normalization convention. The notation $\mathcal{O}(k^4)$ collects terms whose leading small-k behavior is fourth order or higher. The susceptibility has mass dimension two, so the dimensionless quantity quoted below is $A_{2,E}/\Theta^2$.

The calculation uses the thermal state-dependent Hessian. A state-independent local Einstein term cancels in the thermal-minus-vacuum subtraction. The remaining finite convention is modified minimal subtraction, denoted $\overline{\mathrm{MS}}$. The four-dimensional scale $\bar{\mu}$ and the three-dimensional scale μ_3 are held independent until matching is complete. Their roles are different: the μ_3 dependence must cancel among reduced-theory sectors, whereas the explicit $\bar{\mu}$ derivative must reproduce the running of ξ_H. A finite local matching coefficient fixed by that renormalization condition is not counted later as an independent prediction.

The operator $\xi_H R H^\dagger H$ is essential, where H is the Higgs doublet, the dagger denotes Hermitian conjugation, R is the Ricci scalar, and ξ_H is the renormalized Higgs-curvature coupling. Its first flat-space improvement vanishes in a resolved on-shell TT matrix element, but its second metric variation supplies a local two-derivative contact proportional to the thermal condensate $\langle H^\dagger H\rangle$. The remaining renormalizable Standard-Model action depends algebraically on the background metric: the Yang–Mills field strength contains no Levi-Civita connection, the Dirac kinetic term has no independent two-metric-derivative contact in this channel, and the quartic and Yukawa interactions generate only momentum-independent local seagulls. Those seagulls disappear from the k^2 derivative; the Higgs-curvature contact does not. It is therefore possible for ξ_H to affect a static susceptibility without adding a propagating TT degree of freedom.

We use six sector labels. The superscripts (0), (g), and (g^2) indicate the free, ring-resummed, and second-order terms. The labels hard, E/H, and R denote, respectively, nonzero Matsubara modes, the screened electrostatic/Higgs theory, and local-curvature matching. Finally, A_M^{NP} denotes the nonperturbative magnetostatic remainder. With these definitions, the factorized calculation is organized as

$$A_{2,E} = A_{2,E}^{(0)} + A_{2,E}^{(g)} + A_{\mathrm{hard}}^{(g^2)} + A_{E/H}^{(g^2)} + A_R^{(g^2)} + A_M^{\mathrm{NP}}. \tag{4}$$

These labels are factorization sectors, not separately observable forces. Only their completed sum has the claimed scale and contact independence. The organization also prevents double counting: bosonic zero modes removed from the strict-hard diagrams reappear once, with screening, in the reduced theory.

3. Calculation through second order

3.1 Free and ring-resummed terms

The free result is best understood species by species because that exposes the role of the scalar contact. For one real massless scalar with curvature coupling ξ, the separated operator bubble gives $\kappa_s^{\mathrm{op}}/\Theta^2 = -1/72$. The superscript op identifies the operator-correlator part before variational completion. The scalar's curvature contact adds $\xi/12$, so the completed coefficient is $\kappa_s/\Theta^2 = -(1-6\xi)/72$. One two-component Weyl fermion gives $1/144$, and one massless gauge vector, including its gauge-fixing and ghost completion, gives $1/18$.

The symmetric-phase Standard Model contains four real Higgs components with common coupling ξ_H, 45 Weyl fermions, and 12 gauge vectors. Summing those completed species contributions and applying the normalization in Eq. (3) gives

$$\frac{A_{2,E}^{(0)}}{\Theta^2} = \frac{133 + 48\xi_H}{576}. \tag{5}$$

At minimal Higgs curvature coupling, $\xi_H = 0$, Eq. (5) gives $133/576 = 0.230902777778$. The integer 133 is a count appropriate to the static completed Hessian. It is not a count of resolved graviton-like spectral polarizations, which would be a different response moment.

The bosonic zero mode is nonanalytic before resummation, so it cannot be handled by an ordinary Taylor expansion around zero mass. Giving one three-dimensional scalar mode a screening mass m changes the completed slope by $\delta\kappa_{s,0} = \Theta m(1-6\xi)/(24\pi)$, where the subscript zero labels the Matsubara zero mode and δ denotes the screened result minus its strict massless expansion. Temporal gauge scalars have $\xi = 0$, and fermions have no zero Matsubara mode. Let $\delta A_{2,E}^{(g)}$ denote the total first-order screened correction. Summing the eight color-electric, three weak-electric, one hypercharge-electric, and four real Higgs modes gives

$$\frac{\delta A_{2,E}^{(g)}}{\Theta^2} = \frac{8m_{D3} + 3m_{D2} + m_{DY} + 4m_H(1-6\xi_H)}{96\pi\Theta}. \tag{6}$$

This term is formally first order in a generic coupling because every screening mass is proportional to $g\Theta$ at leading order. Define $A_{2,E}^{(0+g)} \equiv A_{2,E}^{(0)} + \delta A_{2,E}^{(g)}$. At the benchmark point, the free and screened contributions combine to

$$\frac{A_{2,E}^{(0+g)}}{\Theta^2} = 0.261940621924 + 0.060106452558\,\xi_H. \tag{7}$$

For $\xi_H = 0$, the ring term raises the free value by 13.44%. Because Table 1 specifies a dimensionless benchmark rather than a complete running trajectory, no scale-variation band is inferred from this single snapshot.

3.2 Strict-hard two-loop sectors

The strict-hard term contains loop momenta of order $\pi\Theta$. Bosonic zero modes are excluded from this region and reserved for the three-dimensional theory, so hard describes a matching region rather than a new physical contribution. We start from the complete two-loop 12,12 operator correlator of Ref. [7]. Its five gauge-independent classes are Higgs self-coupling, gauge-gauge, Yukawa, scalar-gauge, and fermion-gauge.

For each class $x \in \{\lambda, gg, y, sg, fg\}$, let H_x denote the dimensionless second derivative of the corresponding hard master combination with respect to k, divided by Θ^2, and let C_x denote its Standard-Model coupling and multiplicity weight. The source-normalized contribution is then $\delta A_x/\Theta^2 = C_x H_x/4$. Dimensional regularization is performed in $D = 4 - 2\epsilon$ spacetime dimensions and $d = 3 - 2\epsilon$ spatial dimensions, where ϵ is the regulator taken to zero after renormalization.

Let $P = (P_0, \mathbf{p})$ and $Q = (Q_0, \mathbf{q})$ be Euclidean thermal four-momenta, with bosonic or fermionic Matsubara frequencies as assigned by the sector, and let $K = (0,0,0,k)$ be the external static four-momentum. Each thermal sum-integral contains a Matsubara sum multiplied by Θ and a spatial integral with measure $d^d\mathbf{p}/(2\pi)^d$; the double summation sign below contains one such factor for P and one for Q. The integers a, b, c, r, s are propagator powers, and f, g, h are frequency powers. Before the master is used, define the homogeneity index $y \equiv a + b + c + r + s - f - g - h - 2$. With this notation, the static expansion reduces masters of the form

$$I_{abcrs}^{fgh} = \sum_{P,Q} \frac{P_0^f Q_0^g (Q-P)_0^h (K^2)^y}{(P^2)^a (Q^2)^b [(Q-P)^2]^c [(K-P)^2]^r [(K-Q)^2]^s}. \tag{8}$$

Terms with $y \geq 2$ start at k^4; a $y = 1$ term is evaluated at $K = 0$ after the explicit factor of k^2 is extracted; and a $y = 0$ term survives only if an external propagator remains to be differentiated. Tensor numerators are reduced by rotational averaging in exact d dimensions before the Laurent expansion in ϵ. Thermal integration-by-parts identities and frequency relabelings then reduce the surviving terms to products of one-loop tadpoles. This order of operations is part of the calculation, because an evanescent factor proportional to ϵ can multiply a pole and leave a finite remainder.

For the gauge-gauge class, define the bosonic tadpole $\mathcal{I}_n \equiv \sum_P (P^2)^{-n}$ and let $\mathcal{C}_{gg}(d)$ be the rational coefficient left by tensor reduction. Integration by parts reduces the complete slope to $\mathcal{C}_{gg}(d)\mathcal{I}_1\mathcal{I}_2$. The two factors needed to expose the finite evanescent remainder are

$$\begin{aligned} \mathcal{C}_{gg}(d) &= \frac{(d-8)(d-3)(d-2)(d-1)^3(d+1)}{12d(d-5)}, \\ \mathcal{C}_{gg}(3-2\epsilon) &= -\frac{40}{9}\epsilon + O(\epsilon^2), \\ \mathcal{I}_1 &= \frac{\Theta^2}{12} + O(\epsilon), \qquad \mathcal{I}_2 = \frac{1}{16\pi^2\epsilon} + O(1). \end{aligned} \tag{9}$$

These factors combine so that the evanescent coefficient leaves $H_{gg} = -5/(108\pi^2)$. Setting $d = 3$ prematurely would erase this term. The result is a useful audit of the reduction because it arises entirely from the order in which tensor averaging and pole expansion are performed.

The fermion-gauge class is the largest independent check. Its 97 printed structures first reduce by static power counting and frequency relabeling; thermal integration by parts then collapses the survivors to 19 products of one-loop tadpoles. Both its double and simple poles cancel within the class. The finite slope is

$$H_{fg} = -\frac{7}{432\pi^2} = -0.001641778438649. \tag{10}$$

The remaining three classes provide complementary checks. The Higgs self-coupling and scalar-gauge classes reduce to bosonic tadpole products but retain simple poles that must be matched. The Yukawa class mixes bosonic and fermionic Matsubara assignments; after frequency mapping and integration by parts, its candidate structures also reduce to products of one-loop sums. Let γ_E be the Euler–Mascheroni constant, and let $\zeta'(-1)$ denote the derivative of the Riemann zeta function at -1. Before minimal subtraction, the three Laurent endpoints are

$$\begin{aligned}
H_\lambda &= \frac{1}{12\pi^2\epsilon} + \frac{-48\zeta'(-1) + 8\ln[\bar{\mu}/(4\pi\Theta)] - 3 + 4\gamma_E}{24\pi^2} + O(\epsilon),\\
H_y &= \frac{1}{72\pi^2\epsilon} + \frac{4\ln[\bar{\mu}/(2\pi\Theta)] - 24\zeta'(-1) - 6\ln 2 - 1 + 2\gamma_E}{72\pi^2} + O(\epsilon),\\
H_{sg} &= \frac{11}{288\pi^2\epsilon} + \frac{-528\zeta'(-1) + 88\ln[\bar{\mu}/(4\pi\Theta)] - 49 + 44\gamma_E}{576\pi^2} + O(\epsilon).
\end{aligned}$$

The Standard-Model weights are not independent fit parameters. In the stated top-Yukawa truncation they are $C_\lambda = \lambda/4$, $C_{gg} = (3g_2^2 + 12g_3^2)/4$, $C_y = 3y_t^2/4$, $C_{sg} = (g_Y^2 + 3g_2^2)/4$, and $C_{fg} = (5g_Y^2 + 9g_2^2 + 24g_3^2)/4$. At the benchmark these evaluate to -0.008396725, 0.899790408, 0.121181896, 0.246368054, and 2.274022931, respectively. Table 2 reports each minimally subtracted slope, its weight, and the resulting source-normalized contribution. The analytic reductions establish the structure; the decimal values are audit targets rather than substitutes for the derivation.

Table 2. Strict-hard operator contributions at $\bar{\mu} = 2\pi\Theta$.

Sector x	$H_x^{\overline{\mathrm{MS}}}$	C_x	$\delta A_x/\Theta^2 = C_x H_x/4$
Higgs self-coupling	+0.007193382340	-0.008396725	-0.000015100213339
Gauge-gauge	-0.004690795539	+0.899790408	-0.001055183207970
Yukawa	-0.000048330633	+0.121181896	-0.000001464199449
Scalar-gauge	+0.000482489584	+0.246368054	+0.000029717504960
Fermion-gauge	-0.001641778439	+2.274022931	-0.000933360454277
Strict-hard subtotal			**-0.001975390570075**

Let $A_{\mathrm{hard,op}}^{(g^2)}$ denote the sum of the five strict-hard operator terms at second order. The gauge-gauge and fermion-gauge classes are finite. The other three deliberately retain the pole and explicit scale data needed for matching:

$$\left.\frac{A_{\mathrm{hard,op}}^{(g^2)}}{\Theta^2}\right|_{1/\epsilon} = \frac{0.000263263990495}{\epsilon}. \tag{11}$$

Define the explicit hard logarithmic derivative by $D_{\mathrm{hard,op}} \equiv \partial\left(A_{\mathrm{hard,op}}^{(g^2)}/\Theta^2\right) / \partial\ln\bar{\mu}$ with all running couplings held fixed during the partial derivative. Its value is $D_{\mathrm{hard,op}} = 0.001053055961979$. The identity $D_{\mathrm{hard,op}} = 4P_{\mathrm{hard,op}}$, where $P_{\mathrm{hard,op}}$ is the coefficient of $1/\epsilon$ in Eq. (11), checks the normalization of the logarithms. The residual pole and derivative are retained as matching data.

3.3 Higgs contact, electric theory, and scale closure

The strict-hard operator correlator is not yet the completed metric Hessian. Its pole and scale derivative must be combined with the second variation of the nonminimal Higgs operator and with the screened zero modes. The Higgs variation contributes

$$A_{H,\text{contact}} = \frac{\xi_H}{2}\langle H^\dagger H\rangle. \tag{12}$$

At the benchmark point, define the dimensionless hard contact $\delta a_H \equiv \delta A^{(g^2)}_{H,\text{contact}}/\Theta^2$, its pole coefficient $p_H \equiv 0.000780071962924$, and its finite coefficient $f_H \equiv 0.002835981753442$. The hard condensate correction [13] gives

$$\delta a_H = -\xi_H\left(\frac{p_H}{\epsilon} + f_H\right). \tag{13}$$

To state the required four-dimensional running, define $\beta_{\xi_H} \equiv d\xi_H/d\ln\bar{\mu}$ and use the top-only invariant $Y_2 = 3y_t^2$ fixed above. The curved-space Standard-Model result [14] is

$$16\pi^2\beta_{\xi_H} = \left(\xi_H - \frac{1}{6}\right)\left(12\lambda + 2Y_2 - \frac{3}{2}g_Y^2 - \frac{9}{2}g_2^2\right), \tag{14}$$

Define D_{required} as the explicit derivative of the completed second-order coefficient with respect to $\ln\bar{\mu}$, holding the running parameters fixed inside that partial derivative. Renormalization-group invariance then requires

$$D_{\text{required}} = -\frac{\beta_{\xi_H}}{12}. \tag{15}$$

Numerically, $D_{\text{required}} = -0.000080194657458 + 0.000481167944750\,\xi_H$. This is an independent target, not a coefficient fitted to make the subsequent ledger close.

The electrostatic effective theory contains the Higgs zero mode, the temporal weak adjoint A_0^a with $a \in \{1,2,3\}$, and the temporal hypercharge scalar B_0. Its potential block consists of the Higgs quartic graph and the Higgs-electric figure-eight graphs. At leading order the three-dimensional couplings are $\lambda_3 = \lambda\Theta$, $h_3 = g_2^2\Theta/4$, and $h'_3 = g_Y^2\Theta/4$. For N_X real electric scalars X_A of mass m_X coupled through $h_X H^\dagger H X_A X_A$, the two finite contributions are

$$\frac{\delta A_{\lambda_3}}{\Theta^2} = -\frac{\lambda}{32\pi^2}(1 - 6\xi_H), \qquad \frac{\delta A_{HX}}{\Theta^2} = -\frac{(h_X/\Theta)N_X}{32\pi^2}\left[\left(\frac{1}{6} - \xi_H\right)\frac{m_X}{m_H} + \frac{1}{6}\frac{m_H}{m_X}\right].$$

The hypercharge block uses $(X, N_X, h_X, m_X) = (B_0, 1, h'_3, m_{DY})$, while the weak block uses $(A_0, 3, h_3, m_{D2})$. Their constant and ξ_H coefficients are, respectively, $(+0.000106345764, -0.000638074587)$, $(-0.000080242946, +0.000399966933)$, and $(-0.000275777025, +0.001397449055)$. Define the dimensionless sum $\delta a_{3D,\text{pot}} \equiv \delta A^{(g^2)}_{3D,\text{pot}}/\Theta^2$, its curvature-independent coefficient $q_0 \equiv -0.000249674206$, and its coefficient linear in ξ_H, $q_\xi \equiv 0.001159341402$. The summed potential contribution is

$$\delta a_{3D,\text{pot}} = q_0 + q_\xi\xi_H. \tag{16}$$

The same expression follows by inserting the soft one-loop mass shifts into the ring result. This equivalence is the double-counting check: next-to-leading screening masses and the explicit three-dimensional potential graphs represent the same physics and must not both be retained. In the present organization the ring term uses leading screening masses, and the second-order potential graphs are kept explicitly.

The pole ledger has two different parts. The reduced Higgs condensate supplies $+p_H\xi_H/\epsilon$, which cancels the hard contact pole in Eq. (13) directly. The operator poles are instead distributed among the hard region, the electric and magnetostatic regions, and the local-curvature counterterm. Let $P_E = -0.001079676881389$ be the electric operator pole coefficient, let $P_M = 0.000949664887569$ be the perturbative magnetostatic pole coefficient, and let P_R be the local-curvature counterterm coefficient. Ultraviolet finiteness requires

$$-p_H\xi_H + p_H\xi_H = 0, \qquad P_{\text{hard,op}} + P_E + P_M + P_R = 0, \qquad P_R = -0.000133251996674.$$

The second equality defines the pole subtraction of the renormalized local operator; it is a consistency requirement, not an independently predicted finite number. Stating the ledger this way avoids the incorrect impression that the electric theory alone cancels Eq. (11).

For each reduced-theory block, D denotes the coefficient of its explicit logarithmic derivative with respect to μ_3. The subscripts $E3$, $E2$, and EY label color-electric, weak-electric, and hypercharge-electric operator terms, while $M3$ and $M2$ label the color and weak magnetostatic terms. Their values are $D_{E3}^{\text{op}} = -0.002999279576$, $D_{E2}^{\text{op}} = -0.001199069961$, $D_{EY}^{\text{op}} = -0.000120357988$, $D_{M3} = +0.002999279576$, and $D_{M2} = +0.000799379974$. In this pole convention, $P_M = (D_{M3} + D_{M2})/4$. Define $D_{\text{soft,op}}$ as their sum. It is

$$D_{\text{soft,op}} = -\frac{g_Y^2 + 3g_2^2}{192\pi^2}. \tag{17}$$

The reduced Higgs condensate contributes $+0.003120287851698\,\xi_H$ to the scale derivative. Together with the hard Higgs-contact derivative $-0.002639119906948\,\xi_H$, it yields $+0.000481167944750\,\xi_H$, exactly the coefficient in Eq. (15). Define the dimensionless matching coefficient $c_\xi \equiv (g_Y^2 + 3g_2^2)/[2(4\pi)^2] = 0.003120287851698$. The finite Higgs matching is

$$\frac{\delta A_{H,\text{matched}}^{(g^2)}}{\Theta^2} = c_\xi \xi_H \left[\ln\frac{\bar{\mu}}{2m_H} + \frac{1}{4}\right]. \tag{18}$$

The massive electric scalar-gauge graphs also contain an absolute finite threshold. Let m denote the massive scalar screening mass in one charged block, and let Γ denote the Euler gamma function. In dimensional regularization, the combined exchange-plus-seagull curvature term is proportional to $\Gamma(3-d)(m^2)^{d-3}I(d)$. To specify the dimensionless Feynman-parameter integral, take $x \in [0,1]$, $u \in [0,\infty)$, and define $a_x \equiv x(1-x)$. Then

$$N_d(x,u) \equiv a_x(d + 4a_x) + d(4a_x + 1)u + (5d - 32a_x - 2)u^2 + 4(d-6)u^3, \quad I(d) \equiv -\frac{1}{24}\int_0^1 d\,x \int_0^\infty d\,u\, \frac{N_d(x,u)}{[a_x + u]^{d/2+2}}.$$

Here d is the continued spatial dimension defined above, and a prime denotes differentiation with respect to d. Evaluation gives $I(3) = -\pi$ and $I'(3)/I(3) = 2\ln 2 - 5/9$. Defining $c_{sg}^{(m)}$ as the finite massive scalar-gauge threshold relative to the logarithm, one obtains

$$c_{sg}^{(m)} = -\frac{I'(3)}{2I(3)} = \frac{5}{18} - \ln 2 = -0.415369402782167. \tag{19}$$

For a charged-scalar block indexed by i, let m_i be its screening mass and D_i its logarithmic coefficient. That block contributes $D_i\left[\ln(\mu_3/m_i) + c_{sg}^{(m)}\right]$. The three coefficients required here are $D_{A_0^{(3)}} = -24g_3^2/(192\pi^2)$ for the color-electric adjoint, $D_{A_0^{(2)}} = -6g_2^2/(192\pi^2)$ for the weak-electric adjoint, and $D_{Hg} = -(g_Y^2 + 3g_2^2)/(192\pi^2)$ for the Higgs-gauge block. Their masses are m_{D3}, m_{D2}, and m_H, respectively. At $\mu_3 = g_2^2\Theta$, the three completed blocks are 0.004253519982, 0.001124469063, and 0.000291448990, which sum to 0.005669438036. This exposes both the unequal mass logarithms and the common threshold in Eq. (19).

A temporary magnetic regulator mass m_M provides a direct Ward-identity check on the separation. The current-exchange and covariant-kinetic seagull terms each contain a contribution proportional to $1/m_M$, but their coefficients are equal and opposite in the completed metric variation. Their sum therefore has no powerlike magnetic term. The regulator is removed after this cancellation and does not enter any reported coefficient.

Finally, let C_R^{3D} denote the dimensionless local three-dimensional curvature matching coefficient and define $D_{\mathrm{soft}} \equiv D_{\mathrm{soft,op}} = -0.000520047975283$. The constant part of the four-dimensional renormalization condition fixes, rather than predicts, the diagonal logarithmic coefficient:

$$D_{R,\mathrm{diag}} \equiv D_{\mathrm{required}}\Big|_{\xi_H=0} - D_{\mathrm{hard,op}} - D_{\mathrm{soft}} = -0.000613202644155.$$

It is the derivative obtained when $\bar{\mu}$ and μ_3 are varied together, not a third independent beta function. Renormalization-group equations determine the logarithms but leave an additive finite local-R convention. We define that constant to vanish at $\bar{\mu} = 2\pi\Theta$ in the thermal-subtracted $\overline{\mathrm{MS}}$ matching convention. With this definition, the two-scale bridge is

$$C_R^{3D} = D_{\mathrm{soft}} \ln\frac{\bar{\mu}}{\mu_3} + D_{R,\mathrm{diag}} \ln\frac{\bar{\mu}}{2\pi\Theta}, \tag{20}$$

At $\mu_3 = g_2^2\Theta$ this gives $C_R^{3D} = -0.001671619098126$. A different finite local-R convention would shift this coefficient and the quoted susceptibility by the same local amount, so numerical comparisons must use the convention just stated. At fixed $\bar{\mu}$, the μ_3 derivative of the electric, magnetostatic, and curvature terms vanishes; this is an independent reduced-theory closure. At fixed μ_3, the constant equality $0.001053055961979 - 0.000520047975283 - 0.000613202644155 = -0.000080194657459$ verifies the numerical assembly of the imposed four-dimensional matching condition, but it is not a second prediction. By contrast, the ξ_H coefficient closes against Eq. (15) after the hard and reduced condensate derivatives are evaluated.

Define the perturbatively matched part by $a_{\mathrm{pert}} \equiv \left(A_{2,E} - A_M^{\mathrm{NP}}\right)/\Theta^2$. With $p_0 \equiv 0.263713376086$ and $p_\xi \equiv 0.066624158194$, the endpoint before the nonperturbative magnetostatic boundary is

$$a_{\mathrm{pert}} = p_0 + p_\xi \xi_H. \tag{21}$$

Table 3 exposes the numerical assembly. Each row has a distinct origin, so agreement of only the final sum would be insufficient. The pole ledger, reduced-theory scale cancellation, four-dimensional running condition, and row-by-row arithmetic are tested separately. The subtotal is evaluated from the unrounded analytic expressions; the displayed row values are rounded audit summaries.

Table 3. Assembly of the matched coefficient.

Contribution	**Constant part**	**Coefficient of ξ_H**
Free plus ring	+0.261940621924	+0.060106452558
Strict-hard operator	-0.001975390570075	0
Hard Higgs contact	0	-0.002835981753442
Three-dimensional potential	-0.000249674206	+0.001159341402
Matched finite Higgs contact	0	+0.008194345988
Massive electric loop	+0.005669438036	0
Local-curvature bridge	-0.001671619098	0
Through electric/local matching	**+0.263713376086**	**+0.066624158194**

4. Magnetostatic remainder as a lattice observable

After the hard and electric scales are integrated out, the spatial SU(3) and SU(2) zero modes form confining pure three-dimensional Yang-Mills theories. This magnetostatic label refers to the unscreened spatial gauge sector; it does not refer to an externally applied magnetic field. There is no corresponding confining Abelian contribution at this order.

Let $G = \mathrm{SU}(N_c)$ denote either non-Abelian group. Its adjoint Casimir and adjoint dimension are $C_A \equiv N_c$ and $d_G \equiv N_c^2 - 1$. Let g_G denote the corresponding four-dimensional gauge coupling. Define the three-dimensional gauge coupling by $g_{M,G}^2 \equiv g_G^2\Theta$ and its intrinsic dimensionful scale by $\lambda_G \equiv C_A g_{M,G}^2$. Let $A_{M,G}^{\mathrm{NP}}$ be the contribution of group G to the magnetostatic remainder, let $D_{M,G}$ be the known coefficient of its logarithm, and let c_G^{NP} be the finite, scheme-defined nonperturbative boundary constant. Dimensional analysis and the ultraviolet logarithm then imply

$$\frac{A_{M,G}^{\mathrm{NP}}(\mu_3)}{\Theta^2} = D_{M,G}\left[\ln\frac{\mu_3}{\lambda_G} + c_G^{\mathrm{NP}}\right]. \tag{22}$$

For SU(3) and SU(2), respectively, $D_{M,3} = 0.002999279576$ and $D_{M,2} = 0.000799379974$. At the benchmark choice $\mu_3 = g_2^2\Theta$, define the known logarithmic offset $b_0 \equiv -0.003657169408$, and define the two weights $b_3 \equiv D_{M,3}$ and $b_2 \equiv D_{M,2}$. The two-group result is

$$\frac{A_M^{\mathrm{NP}}}{\Theta^2} = b_0 + b_3 c_3^{\mathrm{NP}} + b_2 c_2^{\mathrm{NP}}, \tag{23}$$

Combining Eqs. (21) and (23), define $a_0 \equiv 0.260056206678$, $a_\xi \equiv 0.066624158194$, $a_3 \equiv 0.002999279576$, and $a_2 \equiv 0.000799379974$. The central result can then be written compactly as

$$\frac{A_{2,E}}{\Theta^2} = a_0 + a_\xi \xi_H + a_3 c_3^{\mathrm{NP}} + a_2 c_2^{\mathrm{NP}}, \tag{24}$$

The c_G^{NP} are not fitting coefficients and are not adjusted to reproduce a gravitational measurement. They are local responses of pure Yang-Mills theory in the same $\overline{\mathrm{MS}}$ convention as the perturbative matching. The SU(3) and SU(2) weights carry 78.956% and 21.044% of the known magnetostatic logarithmic coefficient, respectively; this split does not assume that the two finite constants are equal.

To determine the constants, let g_{ij} be a weak Euclidean spatial metric with determinant $g \equiv \det(g_{ij})$, inverse g^{ij}, and shear component $g_{12} = g_{21} = h(z)$. Let F_{ij}^a be the Yang-Mills field strength, with spatial indices $i, j, k, l \in \{1,2,3\}$ and adjoint color index a; repeated spatial and color indices are summed. For one group, abbreviate $g_{M,G}$ to g_M. The continuum action is

$$S_G[g] = \frac{1}{4g_M^2}\int d^3x\,\sqrt{g}\,g^{ik}g^{jl}F_{ij}^a F_{kl}^a, \tag{25}$$

The first derivative with respect to h defines $T_{12} \equiv F_{13}^a F_{23}^a / g_M^2$. The second derivative gives the local Yang-Mills Lagrangian density $\mathcal{L}_{\mathrm{YM}} \equiv F_{ij}^a F_{ij}^a/(4g_M^2)$ on the flat background. Consequently, define the completed positive-correlator response by

$$\mathcal{G}_G(k) = \int d^3x\, e^{ikz}\langle T_{12}(x)T_{12}(0)\rangle_c - \langle \mathcal{L}_{\mathrm{YM}}\rangle. \tag{26}$$

The second term is the microscopic contact required by the Ward identity. Since it is momentum independent on the flat background, it cancels from momentum differences, but it is retained in the definition so that the lattice and continuum Hessians have the same normalization. For the shear metric used here, the geometric normalization is $\int d^3x\sqrt{g}\,R = -1/2\int d^3x(\partial_z h)^2 + O(h^3)$ up to a periodic total derivative, where R is the three-dimensional Ricci scalar. Define $C_G^{\overline{\mathrm{MS}}}(\mu_3)$ as the renormalized coefficient of k^2 in the small-momentum expansion,

$$\mathcal{G}_G^{\overline{\mathrm{MS}}}(k;\mu_3) = \mathcal{G}_G(0) + C_G^{\overline{\mathrm{MS}}}(\mu_3)k^2 + O(k^4). \tag{27}$$

The perturbative ultraviolet result fixes the running of this slope, and dimensional reduction fixes its conversion to the four-dimensional susceptibility:

$$\frac{\partial C_G^{\overline{\mathrm{MS}}}}{\partial \ln\mu_3} = \frac{d_G\lambda_G}{96\pi^2}, \qquad \frac{A_{M,G}^{\mathrm{NP}}}{\Theta^2} = \frac{C_G^{\overline{\mathrm{MS}}}(\mu_3)}{2\Theta}.$$

The first equality reproduces $D_{M,G}$ after the second is differentiated, independently fixing the factor of two. Combining these relations with Eq. (22) and solving for the finite boundary constant gives

$$c_G^{\mathrm{NP}} = \frac{96\pi^2}{d_G \lambda_G} C_G^{\overline{\mathrm{MS}}}(\mu_3) - \ln\frac{\mu_3}{\lambda_G}. \tag{28}$$

On a flat cubic lattice, let a_L be the lattice spacing, let L be the linear physical extent, and let $\mathcal{G}_G^L(k; a_L, L)$ be the completed Hessian obtained from the same two source derivatives of the lattice action. An absolute lattice slope contains an additive local coefficient $C_R^L(a_L)$. It need not be computed separately. For a momentum k along the third lattice axis, define the improved lattice momentum $\hat{k} \equiv (2/a_L)\sin(a_L k/2)$. For any two distinct momenta k_a and k_b—whose subscripts are labels, not powers of the spacing—define the secant

$$\mathcal{S}_G^L(k_a, k_b) = \frac{\mathcal{G}_G^L(k_a) - \mathcal{G}_G^L(k_b)}{\hat{k}_a^2 - \hat{k}_b^2}, \tag{29}$$

Choose k_{i1} and k_{i2} in an infrared window and k_{u1} and k_{u2} in an ultraviolet matching window on the same ensemble. Define the infrared-minus-ultraviolet difference

$$\Delta\mathcal{S}_G^L = \mathcal{S}_G^L(k_{i1}, k_{i2}) - \mathcal{S}_G^L(k_{u1}, k_{u2}). \tag{30}$$

Both $C_R^L(a_L)$ and the momentum-independent contact cancel algebraically because they are common to the two secants. Continuum perturbation theory supplies the ultraviolet anchor in the window $\lambda_G \ll k_u \ll a_L^{-1}$, where k_u denotes either ultraviolet momentum, and confinement supplies the analytic infrared anchor. Let $\mathcal{S}_{G,\mathrm{pert}}^{\overline{\mathrm{MS}}}$ denote the secant computed from the renormalized continuum ultraviolet response, and let $k_i \to 0$ mean that both members of the infrared pair approach zero at fixed ratio. The absolute coefficient is

$$C_G^{\overline{\mathrm{MS}}}(\mu_3) = \mathcal{S}_{G,\mathrm{pert}}^{\overline{\mathrm{MS}}}(k_{u1}, k_{u2}; \mu_3) + \lim_{k_i \to 0} \lim_{L\to\infty} \lim_{a_L \to 0} \Delta\mathcal{S}_G^L. \tag{31}$$

The order of limits is operational: at fixed physical momenta take the continuum limit $a_L \to 0$, then the infinite-volume limit $L \to \infty$, and only then send the infrared pair to zero. Reversing these limits can mix discretization or finite-volume effects with the curvature slope.

For the ultraviolet anchor, define $\ell(k) \equiv 8\ln(k^2/\mu_3^2)/(3\pi^2)$, $u_1 \equiv -1 + 20/(3\pi^2)$, and $u_2 \equiv 431/768 - 37/(9\pi^2)$. The ellipsis below denotes terms of higher order in λ_G/k. The three-loop ultraviolet response [5] is

$$\mathcal{G}_{G,\mathrm{pert}}^{\overline{\mathrm{MS}}}(k) = \frac{d_G}{512}\{k^3 + \lambda_G k^2[u_1 - \ell(k)] + \lambda_G^2 k u_2 + \cdots\}, \tag{32}$$

A constant ultraviolet contact is omitted because it drops out of the secant. The extraction is deliberately overconstrained: continuum, volume, infrared-momentum, and ultraviolet-window extrapolations must each form a stable plateau; variation of μ_3 must reproduce the known logarithm; and a second shear discretization or gradient-flow energy-momentum tensor [10,16] must give the same continuum result. Agreement among these routes tests both the normalization of the source derivative and the cancellation of the additive lattice counterterm.

Published glueball masses [1] do not supply $C_G^{\overline{\mathrm{MS}}}$. If $A_0(p^2)$ and $A_2(p^2)$ denote the scalar and tensor stress form factors at Euclidean momentum squared p^2, the commonly studied superconvergent combination $[A_0(p^2) + A_2(p^2)]/(p^2)^2$ cancels local terms proportional to p^2 [5]. A gapped spectrum validates the analytic infrared expansion but is intentionally insensitive to the local curvature coefficient isolated by Eq. (31). Equation (31) therefore defines a curvature-response observable that is not determined by the published glueball spectrum.

5. Closure tests and conclusion

Equation (24) gives the renormalized static transverse-traceless susceptibility through $\mathcal{O}(g^2)$ for the symmetric high-temperature Standard Model in the stated top-Yukawa truncation. It combines the free response, ring-resummed zero modes, five strict-hard interaction sectors, the Higgs second-variation contact, the electrostatic effective theory, the massive scalar-gauge threshold, and the local-curvature bridge. In the thermal-subtracted $\overline{\mathrm{MS}}$ convention, these contributions define one matched coefficient rather than a sum of independently adjustable terms.

The closure tests establish the consistency of this coefficient across its effective theories. The hard Higgs-contact poles cancel against the reduced condensate, and the remaining operator poles are removed by the local-curvature counterterm. The dependence on the three-dimensional factorization scale μ_3 cancels at fixed $\bar{\mu}$. The residual four-dimensional scale derivative equals the running required by the Higgs-curvature coupling ξ_H. The analytic massive scalar-gauge threshold, the sector-by-sector hard reductions, and the numerical assembly of Eq. (21) provide additional checks on the finite terms. The quantities P_R and $D_{R,\mathrm{diag}}$ retain their distinct roles as the required pole subtraction and the finite matching condition, while the additive finite local-curvature boundary is fixed by the declared renormalization convention.

The two coefficients c_3^{NP} and c_2^{NP} are the finite magnetostatic responses of pure three-dimensional SU(3) and SU(2) Yang-Mills theory. They are scheme-defined observables rather than fit parameters. Equation (31) expresses each coefficient as a continuum ultraviolet anchor plus an infrared-minus-ultraviolet lattice difference. The common additive curvature counterterm and the momentum-independent contact cancel in that difference. The ordered continuum, infinite-volume, and zero-momentum limits, together with the ultraviolet matching window and operator-normalization comparison, define the measurement completely.

The result applies to the symmetric phase and the stated top-Yukawa truncation. Table 1 specifies a dimensionless benchmark at $\bar{\mu} = 2\pi\Theta$ rather than a unique thermal epoch. Within that domain, every perturbatively calculable term through $\mathcal{O}(g^2)$ is assigned to a hard, electric/Higgs, contact, magnetostatic, or local-curvature sector. The final expression depends on the renormalized coupling ξ_H, the two Yang-Mills response coefficients, and the stated finite local-curvature convention. Matching to phases with different active degrees of freedom defines a different effective-theory problem.

The completed matter metric Hessian therefore has a finite local k^2 coefficient after its variational contacts, counterterms, and scale matching are included. Its perturbative and effective-theory content closes through second order, and all nonperturbative dependence is isolated in two explicit lattice observables. This establishes the static matter-response result independently of any measured gravitational normalization.

Declarations

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Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Author contributions. Todd J. Desiato conceived the study, developed and checked the calculation, analyzed the results, prepared the manuscript, and approved its final form.

Data availability. No new experimental datasets were generated for this theoretical study. The benchmark inputs and verification outputs are supplied in Online Resource 1, version v0.4. The same archive is available from the Online Resource 1 repository. No numerical values for c_3^{NP} or c_2^{NP} are reported because the lattice observables defined here have not been measured.

Code availability. The Python source and benchmark inputs used to reproduce the numerical assembly checks are supplied in Online Resource 1, version v0.4, and at the Online Resource 1 repository. The package requires Python 3.11 or later and has no third-party Python dependencies.

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Use of generative artificial intelligence. The author used OpenAI ChatGPT, including GPT-5.5 and GPT-5.6 Sol model configurations, as an AI-assisted research and writing tool for literature discovery, mathematical cross-checking, organization, drafting, and editing during development of the manuscript. The author reviewed the scientific arguments, equations, references, and final text and takes full responsibility for the content.

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