Operational Quantum Gravity from a Standard Model Stress Hessian to Einstein-Equivalent WTDiff Dynamics

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Abstract

We derive an Einstein-equivalent infrared gravitational representation from the renormalized thermal-minus-vacuum transverse-traceless stress Hessian of electroweak-symmetric high-temperature Standard Model matter. The Hessian determines the coefficient of the physical spin-two derivative block but does not determine its off-shell gauge representation. Removing the zero-derivative equilibrium response and inverting the remaining kernel produces a massless transverse-traceless compliance. A local two-derivative completion with a massless two-helicity branch, no additional scalar, and a fixed volume measure is then represented directly by Weyl and transverse diffeomorphism symmetry, or WTDiff. Evaluation of the same real off-diagonal shear source in both quadratic actions gives the exact normalization $C_W = 2A_{2,E}^{tot}$, where $A_{2,E}^{tot}$ is the total source-normalized stiffness and C_W multiplies the WTDiff curvature action. Restoring all dimensional factors gives $G/c^4 = \hbar c/\left(32\pi A_{2,E}^{tot}\right)$, where G is the gravitational coupling, c is the state-dependent signal speed, and $\hbar$ is the reduced Planck constant; both sides have units of inverse force. Variation of the fixed-measure action yields the trace-free Einstein equation, and stress conservation with the contracted Bianchi identity supplies the missing trace as a cosmological integration datum. This construction proves that the calculated Standard Model stiffness admits an Einstein-equivalent infrared completion. It does not make WTDiff unique, and the thermal subtraction determines a state increment rather than the reference-state intercept of the total gravitational stiffness.

Keywords: Operational Quantum Gravity; Standard Model; stress tensor; transverse-traceless response; WTDiff gravity; Einstein equations

Mathematics Subject Classification 2020: 81T13; 81T20; 83C05; 83C45

1. Introduction

A renormalized matter stress Hessian is a response function, not yet a gravitational field equation. In the transverse-traceless (TT) shear channel, the Standard Model calculation of Ref. [7] gives a finite coefficient of the term quadratic in spatial wave number after the connected stress correlator, the local second metric variation, and the effective-theory matching contributions are combined. The present paper asks whether that coefficient can be carried into a local infrared tensor theory with Einstein-equivalent dynamics.

Three coefficients enter the construction and must remain distinct. The first is the signed hydrostatic curvature susceptibility in the equilibrium matter functional. The second is the positive derivative stiffness defined from the subtracted TT response and used as an inverse propagator. The third multiplies the curvature term in the infrared gravitational action. Their magnitudes are related by the source normalization derived below, but their signs and physical roles follow from different definitions. The same distinction separates a thermal state difference from the total coefficient that includes a reference-state intercept.

Operational Quantum Gravity (OQG) supplies two infrared conditions that are not contained in a static Hessian: the collective branch is massless and luminal at long wavelength, and its settled volume measure is not an independent dynamical response. The healthy local two-derivative representations of a massless symmetric tensor reduce to two scalar-free choices, full diffeomorphism symmetry (Diff) and Weyl symmetry combined with transverse diffeomorphisms (WTDiff) [1–4]. Both reproduce the same on-shell TT block. WTDiff is used here because its Weyl redundancy removes the trace while its transverse diffeomorphisms preserve the fixed measure. The choice is therefore a direct representation of the stated OQG conditions, not a claim that the Hessian uniquely determines a gauge group.

The initial calculation is Euclidean because an equilibrium partition function defines static thermal correlation functions most directly in imaginary time. Euclidean signature is used only to calculate and invert the matter response. It does not postulate a Euclidean spacetime with independent dynamics. The Lorentzian tensor representation is introduced after the static response has been isolated, and geometry then records the long-wavelength relations among matter-defined clocks, rulers, phases, signal paths, and trajectories.

All dimensional factors remain explicit. The symbol $\hbar$ is the reduced Planck constant, c is the locally realized signal speed of the selected state, and G is the gravitational coupling appearing in the Einstein source coefficient. The thermal energy is $\Theta \equiv k_B T$, where k_B is Boltzmann's constant and T is temperature. Greek indices run from 0 to 3, repeated indices are summed, and TT denotes projection transverse to the four-momentum and traceless with respect to the background metric. OQG comparisons allow c to depend on the state while $\hbar$ remains invariant; the resulting invariant is G/c^4, not G considered separately. Figure 1 summarizes the coefficient-level sequence developed below.

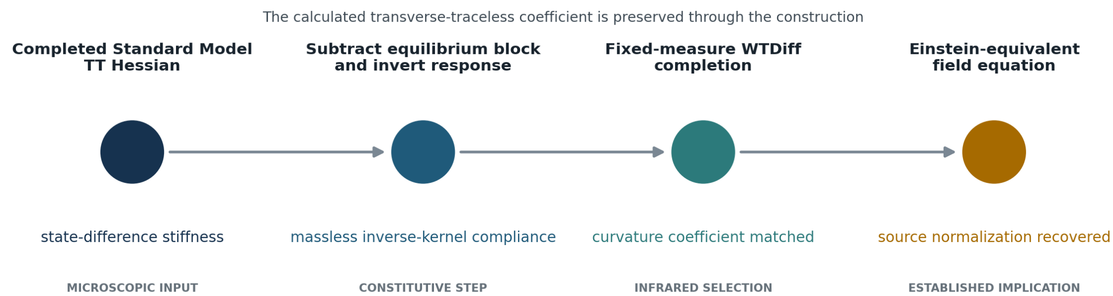

Fig. 1 *The calculated transverse-traceless coefficient is preserved through constitutive inversion, fixed-measure completion, and the Einstein-equivalent field equation*

2. Standard Model stress Hessian

Let X_E^μ denote Euclidean coordinates with dimensions of inverse energy, let $z \equiv X_E^3$, and let k be the real wave number conjugate to z. Write the dimensionless Euclidean metric source as $g_{\mu\nu}^E = \delta_{\mu\nu} + h_{\mu\nu}$, where $\delta_{\mu\nu}$ is the flat Euclidean metric. The real off-diagonal shear source is

$$h_{xy}(z) = h_{yx}(z) = h_k e^{ikz} + h_{-k} e^{-ikz}, \qquad h_{-k} = h_k^*. \tag{1}$$

Here x, y, and z label mutually orthogonal spatial directions, h_k and h_{-k} are Fourier amplitudes, and the asterisk denotes complex conjugation. Because the momentum points along z, the source is transverse and traceless. Let $V_4 \equiv \int d^4X_E$ be the regulated Euclidean four-volume, $Z_E[h]$ the Standard Model partition function, and $W_E[h] \equiv -\ln Z_E[h]$ its connected generating functional. If T_{xy} is the complete Standard Model stress tensor component, $\langle \cdots \rangle_c$ denotes a connected thermal expectation value, and $G_{E,ct}(k)$ denotes the local second metric variation of the microscopic action, then the completed static response is

$$G_E^{xy,xy}(0,k) \equiv -\frac{1}{V_4}\frac{\delta^2 W_E}{\delta h_k \delta h_{-k}}\bigg|_{h=0} = \int d^4X_E\, e^{ikz} \langle T_{xy}(X_E) T_{xy}(0)\rangle_c - G_{E,ct}(k). \tag{2}$$

The zero in the first argument denotes zero Euclidean frequency. All thermodynamic and renormalized parameters are held fixed when differentiating with respect to k. Rotational symmetry makes the response even in k, so its local expansion defines κ and the source-normalized susceptibility $A_{2,E}$ by

$$\kappa \equiv \left. \frac{\partial^2 G_E^{xy,xy}(0,k)}{\partial k^2} \right|_{k=0}, \qquad G_E^{xy,xy}(0,k) = G_E^{xy,xy}(0,0) + \frac{\kappa}{2}k^2 + \mathcal{O}(k^4), \qquad A_{2,E} \equiv \frac{\kappa}{4}. \tag{3}$$

The remainder $\mathcal{O}(k^4)$ begins at fourth order in the small-wave-number expansion. The factor $1/4$ in Eq. (3) follows from the symmetric tensor-source normalization. Defining the TT polarization kernel $\Pi_{TT,E}$ gives

$$\Pi_{TT,E}(k) \equiv -\frac{1}{2} G_E^{xy,xy}(0,k), \qquad -\left[\Pi_{TT,E}(k) - \Pi_{TT,E}(0)\right] = A_{2,E}k^2 + \mathcal{O}(k^4). \tag{4}$$

On the stable branch, Eq. (4) is the positive derivative block used for constitutive inversion. The subtracted quantity $G_E^{xy,xy}(0,0)$ is the zero-derivative equilibrium response. It acts as a preload rather than a massless stiffness and is not included in the propagating kernel.

The Standard Model result is defined as a thermal-minus-vacuum difference at the same renormalized parameters. Denote the thermal state by $\mathcal{S}$ and its reference vacuum by $\mathcal{S}_0$. Their difference is $A_{2,E}^{\Delta} \equiv A_{2,E}(\mathcal{S}) - A_{2,E}(\mathcal{S}_0)$, also written $A_{2,E}^{\Delta}(\mathcal{S};\mathcal{S}_0)$ when the state labels are needed. The calculation is complete through second order in the Standard Model interaction couplings in the thermal-subtracted $\overline{\text{MS}}$ convention [7]. The four-dimensional modified-minimal-subtraction scale is denoted by $\bar{\mu}$; the dimensionless benchmark is specified at $\bar{\mu} = 2\pi\Theta$. Its gauge couplings are $g_Y = 0.477571$, $g_2 = 0.502460$, and $g_3 = 0.486635$, where the subscripts identify the hypercharge, weak, and strong interactions. The top Yukawa and Higgs quartic couplings are $y_t = 0.401965$ and $\lambda = -0.0335869$, respectively. Bottom and tau Yukawa couplings are omitted in this top-Yukawa truncation. At that benchmark, $a_0 = 0.260056206678$ and $a_\xi = 0.066624158194$. The nonperturbative weights are $a_3 = 0.002999279576$ and $a_2 = 0.000799379974$; the displayed digits are arithmetic audit precision rather than uncertainty estimates. The symbol ξ_H is the renormalized coefficient of the Standard Model operator $RH^\dagger H$, where H is the Higgs doublet, the dagger denotes Hermitian conjugation, and R is the Ricci scalar of the nondynamical source metric. The symbols c_3^{NP} and c_2^{NP} denote dimensionless, finite, scheme-defined nonperturbative responses of pure three-dimensional $SU(3)$ and $SU(2)$ Yang-Mills theory; the superscript NP means nonperturbative. With these definitions, the result is

$$\frac{A_{2,E}^{\Delta}}{\Theta^2} = a_0 + a_\xi \xi_H + a_3 c_3^{NP} + a_2 c_2^{NP}. \tag{5}$$

The two nonperturbative responses are defined lattice observables rather than gravitational fit parameters. Equation (5), together with Eqs. (1)–(4), supplies the complete microscopic input required below.

The same Hessian has a signed hydrostatic interpretation. In the Lorentzian hydrostatic convention of Ref. [9], let f_1 denote the coefficient multiplying R in the equilibrium matter functional. Its Kubo formula is

$$f_1 = -\frac{1}{2} \left. \frac{\partial^2 G_E^{xy,xy}(0,k)}{\partial k^2} \right|_{k=0}. \tag{6}$$

The superscript Δ on f_1 denotes the same thermal-minus-vacuum difference used for $A_{2,E}^{\Delta}$. The two coefficients therefore obey

$$f_1^{\Delta} = -\frac{\kappa}{2} = -2A_{2,E}^{\Delta}. \tag{7}$$

This negative sign belongs to the Lorentzian hydrostatic matter functional. The positive quantity in Eq. (4) is instead the subtracted inverse-response stiffness. The WTDiff coefficient constructed below is matched to that positive stiffness after constitutive inversion; it is not obtained by changing the sign of f_1^{Δ} or by identifying the thermal matter functional with a complete induced Einstein-Hilbert action. Because the subtraction cancels state-independent local terms, Eq. (5) fixes an increment and leaves the reference-state intercept undetermined [9, 10, 12].

3. Constitutive inversion

Let $A_{2,E}^{tot}(\mathcal{S})$ denote the full positive TT stiffness in state $\mathcal{S}$, including the reference-state intercept that cancels from Eq. (5). Let $p = (p_0, \mathbf{p})$ be Euclidean four-momentum, with $p^2 = p_0^2 + |\mathbf{p}|^2$; the static source of Eq. (1) has $p_0 = 0$ and $p^2 = k^2$. If $h_{\mu\nu}^{TT}(p)$ is the TT collective source variable and $\tau_{\mu\nu}^{TT}(p)$ is its conjugate TT stress perturbation, define the subtracted constitutive kernel $\mathcal{K}_{TT,E}$ by

$$\begin{aligned} \tau_{\mu\nu}^{TT}(p) &= \mathcal{K}_{TT,E}(p) h_{\mu\nu}^{TT}(p), \\ \mathcal{K}_{TT,E}(p) &= A_{2,E}^{tot} p^2 + \mathcal{O}(p^4). \end{aligned} \tag{8}$$

The compliance $\mathcal{D}_{TT,E}$ is the inverse of this kernel on the TT subspace:

$$\mathcal{D}_{TT,E}(p) \equiv \mathcal{K}_{TT,E}^{-1}(p) = \frac{1}{A_{2,E}^{tot} p^2} + \mathcal{O}(p^0). \tag{9}$$

The $1/p^2$ term is the massless pole, while $\mathcal{O}(p^0)$ denotes terms finite as $p \to 0$. The pole appears only after the algebraic equilibrium block has been removed; inverting the un-subtracted response would not produce Eq. (9). With $\int_p \equiv \int d^4p/(2\pi)^4$ denoting the Euclidean momentum integral, let $\Gamma_{OQG,E}^{(2)}$ be the corresponding dimensionless quadratic effective action:

$$\Gamma_{OQG,E}^{(2)} = \frac{1}{2} \int_p h_{\mu\nu}^{TT}(-p) A_{2,E}^{tot} p^2 h_{TT}^{\mu\nu}(p). \tag{10}$$

The TT projector is implicit because both fields in Eq. (10) already lie in that subspace. This action fixes the two physical helicity-two components and their normalization. It does not specify the trace, longitudinal variables, gauge redundancy, or nonlinear completion.

The input to Eq. (10) is a zero-frequency spatial slope. A Lorentzian wave equation also requires the temporal residue of the retarded kernel. OQG supplies that causal input by selecting a long-wavelength branch with two helicities and dispersion $\omega^2 = c^2|\mathbf{k}|^2$, where ω is Lorentzian frequency and $\mathbf{k}$ is spatial wave vector. Local analyticity then continues the Euclidean derivative block to the corresponding Lorentzian massless wave operator. This causal condition is logically separate from the equilibrium calculation and from the later choice of tensor gauge representation.

4. WTDiff completion

The local Lorentz-invariant two-derivative actions for a massless symmetric tensor are classified in Refs. [2, 8]. Stability and absence of ghosts require at least transverse diffeomorphism symmetry (TDiff), whose infinitesimal vector parameter is divergence free. A generic TDiff action retains a propagating scalar in addition to the helicity-two sector. The scalar is removed by enhancement either to full diffeomorphism symmetry, Diff, or to TDiff supplemented by a local Weyl symmetry, WTDiff. Diff and WTDiff give the same on-shell TT exchange between conserved sources, although they organize the trace and determinant differently [1–4, 6]. Table 1 summarizes these alternatives and their roles in the present construction.

Table 1. Local infrared representations of the calculated transverse-traceless block.

Completion	Physical TT block	Additional content	Role in this construction
Generic TDiff	Same p^2 helicity-two block	Generally contains a propagating scalar	Excluded by the no-scalar condition
Full Diff	Same p^2 helicity-two block	Uses full diffeomorphism redundancy	Viable and on-shell equivalent in the calculated sector
WTDiff	Same p^2 helicity-two block	Uses Weyl redundancy and transverse diffeomorphisms with a fixed measure	Chosen to implement the OQG fixed-measure condition directly
Massive or higher-derivative tensor	Changes the pole or adds modes	Introduces further scales, poles, or degrees of freedom	Outside the local massless two-derivative construction

The choice of WTDiff is thus economical and explicit. The Hessian was calculated in a traceless shear channel, while OQG independently excludes the volume element as a dynamical response coordinate. WTDiff represents both conditions without adding a scalar. Full Diff remains a legal representation of the same calculated TT sector, so the argument establishes an admissible completion rather than a uniqueness theorem.

To define the fixed measure, let $\varpi(x)$ be a nondynamical positive scalar density with the same density weight as $\sqrt{-q}$, let $q_{\mu\nu}$ be an auxiliary Lorentzian symmetric tensor, and write $q \equiv \det q_{\mu\nu}$. Let $\hat{g}_{\mu\nu}$ denote the composite metric and $\hat{g} \equiv \det\hat{g}_{\mu\nu}$ its determinant. In four dimensions define

$$\hat{g}_{\mu\nu} = \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2} q_{\mu\nu}, \qquad \sqrt{-\hat{g}} = \varpi. \tag{11}$$

Let $\sigma(x)$ denote the scalar parameter of a local Weyl rescaling. The composite metric $\hat{g}_{\mu\nu}$ is invariant under $q_{\mu\nu} \to e^{2\sigma(x)} q_{\mu\nu}$. The density ϖ fixes only the integration measure; it supplies neither curvature nor a causal cone. Around a flat background compatible with the chosen measure, write $q_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ is the Minkowski metric and $h \equiv \eta^{\rho\sigma} h_{\rho\sigma}$. The linear Weyl-invariant perturbation is

$$\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu} h, \qquad h \equiv \eta^{\rho\sigma} h_{\rho\sigma}. \tag{12}$$

Let ξ_μ be a vector gauge parameter constrained by $\partial_\mu \xi^\mu = 0$, where ∂_μ denotes the coordinate derivative. Together with the Weyl parameter σ, its linear gauge transformation is

$$\delta h_{\mu\nu} = 2\,\partial_{(\mu}\xi_{\nu)} + 2\sigma\eta_{\mu\nu}, \qquad \partial_\mu \xi^\mu = 0. \tag{13}$$

Parentheses around indices denote symmetrization with unit weight. The transverse vector supplies three independent gauge functions and the Weyl transformation supplies the fourth, leaving the two helicity-two degrees of freedom.

For coefficient matching, return to the inverse-energy Euclidean coordinates X_E^μ introduced in Section 2. Let C_W be the positive coefficient, with dimensions of energy squared, multiplying the dimensionless quadratic WTDiff action. With Euclidean signs chosen so that the TT kernel is positive,

$$\Gamma_{W,E}^{(2)} = C_W \int d^4 X_E \left[\frac{1}{4}\partial_\lambda \hat{h}_{\mu\nu}\,\partial_\lambda \hat{h}_{\mu\nu} - \frac{1}{2}\partial_\mu \hat{h}_{\mu\nu}\,\partial_\rho \hat{h}_{\rho\nu}\right]. \tag{14}$$

On the TT subspace, $\hat{h}_{\mu\nu} = h_{\mu\nu}^{TT}$ and $\partial_\mu h_{\mu\nu}^{TT} = 0$, so Eq. (14) reduces to

$$\Gamma_{W,E}^{(2)}\Big|_{TT} = \frac{C_W}{4}\int d^4 X_E\ \partial_\lambda h_{\mu\nu}^{TT}\,\partial_\lambda h_{\mu\nu}^{TT}. \tag{15}$$

The exact relation between C_W and $A_{2,E}^{tot}$ follows by applying the real source of Eq. (1) to Eqs. (10) and (15). Because a symmetric tensor contains both h_{xy} and h_{yx}, the contraction gives

$$\partial_\lambda h_{\mu\nu}^{TT}\,\partial_\lambda h_{\mu\nu}^{TT} = 2(\partial_z h_{xy})^2. \tag{16}$$

Integration of the two Fourier modes over V_4 gives

$$\int d^4X_E(\partial_z h_{xy})^2 = 2V_4 k^2 h_k h_{-k}. \tag{17}$$

Equations (15)–(17) therefore yield

$$\Gamma_{W,E}^{(2)} = C_W V_4 k^2 h_k h_{-k}, \tag{18}$$

whereas Eq. (10), with the same two Fourier modes and the same symmetric tensor contraction, yields

$$\Gamma_{OQG,E}^{(2)} = 2A_{2,E}^{tot} V_4 k^2 h_k h_{-k}. \tag{19}$$

Matching the identical TT inverse-propagator blocks fixes

$$C_W = 2A_{2,E}^{tot}. \tag{20}$$

The factor of two is fixed by the real symmetric shear source. The quantity $A_{2,E}$ is normalized to one off-diagonal source pair, whereas C_W multiplies the full symmetric-tensor contraction. For the state difference in Eq. (5), define $\Delta C_W \equiv C_W(\mathcal{S}) - C_W(\mathcal{S}_0)$. Equation (20) gives

$$\frac{\Delta C_W}{\Theta^2} = 2\left(a_0 + a_\xi \xi_H + a_3 c_3^{NP} + a_2 c_2^{NP}\right). \tag{21}$$

The four coefficients in Eq. (21) are therefore 0.520112413356, 0.133248316388, 0.005998559152, and 0.001598759948, respectively. They are the curvature-action normalization of the same calculated state difference, not a second fit or an additional microscopic contribution.

5. Dimensional normalization and force invariant

Let x^μ denote coordinates measured in length, with $x^0 = ct$ and t the Lorentzian time, and let X^μ denote the corresponding inverse-energy coordinates. The global conversion is

$$X^\mu = \frac{x^\mu}{\hbar c}. \tag{22}$$

With the same dimensionless metric field expressed as a function of the rescaled coordinates, $d^4x = (\hbar c)^4 d^4X$ and $R_x = (\hbar c)^{-2} R_X$. Here R_x and R_X are the Ricci scalars formed with derivatives with respect to x^μ and X^μ, respectively. In a fixed-measure chart this gives

$$d^4x\,\varpi R_x = (\hbar c)^2 d^4X\,\varpi R_X. \tag{23}$$

The Einstein-normalized WTDiff action in length coordinates is

$$S_W = \frac{c^3}{16\pi G}\int d^4x\,\varpi R_x. \tag{24}$$

Dividing by $\hbar$ and applying Eq. (23) gives the dimensionless quantum action

$$\frac{S_W}{\hbar} = \frac{\hbar c^5}{16\pi G} \int d^4X\, \varpi R_X \equiv C_W \int d^4X\, \varpi R_X. \tag{25}$$

Combining Eqs. (20) and (25) yields

$$A_{2,E}^{tot} = \frac{\hbar c^5}{32\pi G}. \tag{26}$$

The coefficient multiplying stress-energy in the Einstein equation is therefore

$$\frac{G}{c^4} = \frac{\hbar c}{32\pi A_{2,E}^{tot}}. \tag{27}$$

Both sides of Eq. (27) have units of inverse force. Its reciprocal is

$$\frac{c^4}{G} = \frac{32\pi A_{2,E}^{tot}}{\hbar c}. \tag{28}$$

The left side of Eq. (28) is commonly called the Planck force or superforce. The right side shows the engineering meaning of $A_{2,E}^{tot}$: after division by the action-to-length factor $\hbar c$, the TT stiffness equals the invariant force scale apart from the exact normalization 32π.

The invariance can be checked without assigning an external value to c. Let $K > 0$ be the dimensionless OQG comparison factor relating a baseline settled state, labeled 0, to a second settled state, labeled K. The OQG comparison rule is

$$c_K = c_0 K^{-1}, \qquad G_K = G_0 K^{-4}, \qquad A_{2,E,K}^{tot} = A_{2,E,0}^{tot} K^{-1}, \qquad \hbar_K = \hbar_0. \tag{29}$$

It follows directly that

$$\frac{G_K}{c_K^4} = \frac{G_0}{c_0^4}, \qquad \frac{\hbar_K c_K}{A_{2,E,K}^{tot}} = \frac{\hbar_0 c_0}{A_{2,E,0}^{tot}}, \qquad \frac{c_K^4}{G_K} = \frac{c_0^4}{G_0}. \tag{30}$$

Thus Eqs. (27) and (28) compare states while allowing c, G, and the stiffness to transform together. The thermal subtraction remains a separate issue. Using the state notation defined before Eq. (5),

$$\begin{aligned} C_W(\mathcal{S}) - C_W(\mathcal{S}_0) &= 2A_{2,E}^{\Delta}(\mathcal{S};\mathcal{S}_0), \\ C_W(\mathcal{S}) &= C_W(\mathcal{S}_0) + 2A_{2,E}^{\Delta}(\mathcal{S};\mathcal{S}_0). \end{aligned} \tag{31}$$

The field-theory Hessian fixes the difference in Eq. (31). A reference-state matching condition or an independently derived OQG closure condition is required to determine $C_W(\mathcal{S}_0)$ and hence the total force scale.

6. Einstein-equivalent nonlinear equations

The nonlinear WTDiff representation uses the composite fixed-measure metric of Eq. (11). Let Ψ denote the collection of matter fields, let $S_m[\hat{g},\Psi]$ be their macroscopic action, and let $R[\hat{g}]$ be the Ricci scalar of $\hat{g}_{\mu\nu}$. The two-derivative action is

$$\begin{aligned} S[\hat{g},\Psi] &= \frac{c^3}{16\pi G} \int d^4x\, \varpi R[\hat{g}] + S_m[\hat{g},\Psi], \\ \sqrt{-\hat{g}} &= \varpi. \end{aligned} \tag{32}$$

Define the symmetric macroscopic stress tensor by $\delta S_m = -\frac{1}{2c}\int d^4x\, \varpi T_{\mu\nu}\delta\hat{g}^{\mu\nu}$, as appropriate when $x^0 = ct$, and define its trace by $T \equiv \hat{g}^{\mu\nu}T_{\mu\nu}$. Let $R_{\mu\nu}$ and R denote the Ricci tensor and scalar of $\hat{g}_{\mu\nu}$. Because the fixed determinant permits only traceless metric variations, Eq. (32) gives

$$R_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}R = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}T\right). \tag{33}$$

Let $\hat{\nabla}$ denote the Levi-Civita covariant derivative of $\hat{g}_{\mu\nu}$. For covariantly conserved macroscopic matter,

$$\hat{\nabla}^\mu T_{\mu\nu} = 0. \tag{34}$$

Taking the divergence of Eq. (33) and using the contracted Bianchi identity $\hat{\nabla}^\mu R_{\mu\nu} = 1/2\,\hat{\nabla}_\nu R$ gives

$$\hat{\nabla}_\nu\left(R + \frac{8\pi G}{c^4}T\right) = 0. \tag{35}$$

On each connected spacetime region, define Λ as one quarter of this integration constant. Then

$$R + \frac{8\pi G}{c^4}T = 4\Lambda, \tag{36}$$

Here Λ is a cosmological integration datum. Substitution into Eq. (33) gives

$$R_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}R + \Lambda\hat{g}_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{37}$$

Equation (37) is the Einstein equation with the source normalization fixed by Eqs. (20) and (27). The cosmological term is not a coupling in the fixed-measure action; it enters when stress conservation and the Bianchi identity restore the trace. If ρ_0 is a constant vacuum energy density, the homogeneous shift

$$T_{\mu\nu} \to T_{\mu\nu} - \rho_0\hat{g}_{\mu\nu} \tag{38}$$

cancels identically from the trace-free source in Eq. (33). The same geometry can then be written with the shifted integration datum $\Lambda \to \Lambda - (8\pi G/c^4)\rho_0$. This is a structural consequence of the fixed-measure variation rather than a gauge choice imposed on an already varied Diff action.

7. Discussion

The calculation establishes a coefficient-level path from a Standard Model response to an Einstein-equivalent infrared equation. The microscopic result is the completed, renormalized TT Hessian in Eqs. (1)–(5). Subtraction and inversion produce the compliance in Eq. (9). The OQG causal condition supplies the Lorentzian massless two-helicity branch, and the fixed-measure condition selects WTDiff as a direct off-shell representation. The shear calculation then fixes the otherwise ambiguous action normalization through $C_W = 2A^{tot}_{2,E}$. The final WTDiff-to-Einstein step is the established implication reproduced in Eqs. (33)–(37).

This ordering limits the claim precisely. A static Euclidean slope does not determine a retarded temporal residue, and a TT two-point function cannot distinguish Diff from WTDiff because their on-shell helicity-two kernels agree. The causal branch and fixed measure are therefore stated infrared conditions. Once those conditions are imposed, no further microscopic vertex calculation is needed to exhibit one healthy local completion. A generic TDiff model would add a scalar not present in the construction, while massive and higher-derivative models would change the assumed infrared pole.

The result is related to Sakharov induced gravity but is not an evaluation of a complete induced Einstein-Hilbert coefficient [11, 12]. Conventional induced-gravity calculations generate local curvature terms whose renormalized total coefficient depends on state-independent ultraviolet information. Equation (5) instead gives a finite thermal-minus-vacuum susceptibility in a declared renormalization convention. That subtraction makes the state-dependent contribution calculable while removing the intercept. Constitutive inversion preserves this fact, as Eq. (31) shows.

The fixed density ϖ does not reintroduce a pre-existing gravitational geometry. It specifies the volume form used to integrate the local response but fixes no distance, curvature, geodesic, or null cone. Those relations reside in the unimodular tensor $\hat{g}_{\mu\nu}$, whose physical TT normalization comes from the matter Hessian and whose Lorentzian cone is supplied by the causal branch. In engineering terms, the measure fixes the integration cell count; the constitutive tensor determines how the state responds to shear.

The phrase Einstein equivalent has a defined range here. With conserved sources and suitable boundary data, WTDiff and general relativity give the same classical local spin-two equations, while the cosmological constant enters as an integration datum [2–6]. Quantum measures, anomalies, global sectors, and boundary conditions can retain differences between the formulations [6]. The present two-point matching neither assumes nor establishes quantum equivalence beyond the classical infrared equation derived above.

The construction has direct failure conditions. A surviving low-energy scalar pole would require a generic TDiff or scalar-tensor completion. A nonlocal leading kernel, a graviton mass, or unequal temporal and spatial residues would invalidate the local luminal two-derivative continuation. Failure of macroscopic stress conservation would prevent Eq. (35) and hence the recovery of the Einstein trace. An independent determination of the reference-state intercept inconsistent with Eq. (26) would falsify the OQG normalization while leaving the thermal Standard Model Hessian as a separate field-theory result.

8. Conclusion

The renormalized Standard Model stress Hessian supplies a finite TT derivative coefficient after the equilibrium block is subtracted. Constitutive inversion turns its total positive stiffness into a $1/p^2$ compliance. Under the stated OQG requirements of a local massless two-helicity branch, no additional scalar, and a fixed volume measure, WTDiff provides the direct infrared completion. Full Diff remains an on-shell-equivalent representation of the calculated TT sector.

The real symmetric shear source fixes the normalization $C_W = 2A_{2,E}^{tot}$. Restoring $\hbar$ and the state-dependent signal speed c gives $A_{2,E}^{tot} = \hbar c^5/(32\pi G)$, together with the inverse-force invariant $G/c^4 = \hbar c/\left(32\pi A_{2,E}^{tot}\right)$ and the reciprocal Planck-force relation $c^4/G = 32\pi A_{2,E}^{tot}/(\hbar c)$. The WTDiff variational principle then yields the trace-free Einstein equation, and conservation with the contracted Bianchi identity supplies the cosmological integration datum.

The derivation therefore reaches Einstein-equivalent infrared dynamics without treating geometry as the microscopic source of the calculated coefficient. Its remaining quantitative boundary is the reference-state intercept: the Standard Model result determines the thermal-minus-vacuum increment, while a separate OQG closure must determine the total force scale.

Declarations

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Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Author contributions. Todd J. Desiato conceived the study, developed and checked the calculation, analyzed the results, prepared the manuscript, and approved its final form.

Data availability. No new experimental datasets were generated for this theoretical study. The benchmark inputs and verification outputs underlying the numerical coefficients in Eq. (5) accompany Ref. [7]. The nonperturbative coefficients c_3^{NP} and c_2^{NP} remain defined lattice observables and are not assigned numerical values here.

Code availability. No new numerical code was required for the analytical WTDiff matching. The reproducibility package accompanying Ref. [7] contains the numerical assembly checks for the Standard Model Hessian.

Ethics statement. Not applicable; this theoretical study involved no human participants, animals, or identifiable human data.

Use of generative artificial intelligence. The author used OpenAI ChatGPT as an AI-assisted research and writing tool for literature discovery, mathematical cross-checking, organization, drafting, and editing during development of the manuscript. The author reviewed the scientific arguments, equations, references, and final text and takes full responsibility for the content.

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