Operational Quantum Gravity Prevents the Vacuum Catastrophe: A Full Response-Level Theorem for Homogeneous Vacuum Shifts

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Abstract

The vacuum catastrophe is the radiative local-source problem that arises when arbitrary field-independent offsets in a renormalized Standard Model Lagrangian are promoted into an enormous gravitational source. Operational Quantum Gravity (OQG) instead defines gravitational dynamics from normalized Standard Model stress response evaluated with a fixed volume measure. At fixed physical preparation, nonidentity couplings, integration domain, and boundary conditions, we prove that adding any homogeneous c-number to the matter Lagrangian leaves the normalized closed-time-path generating functional exactly unchanged. Every source derivative is therefore invariant, including the projected mean stress, retarded and noise kernels, the full contact-completed Hessian, and all higher response vertices. The result propagates through the transverse-traceless derivative stiffness, its compliance, the Weyl-transverse-diffeomorphism (WTDiff) curvature coefficient, and the invariant Einstein source coefficient. In the macroscopic trace-free equations, the shift cancels identically and only relabels the stress-cosmological-datum pair used in an Einstein-form representative, leaving its invariant global branch datum unchanged. The proof precedes perturbation theory and therefore holds to all orders in Standard Model interactions when renormalization preserves the fixed-measure source definition. It does not remove vacuum polarization, fluctuations, boundaries, condensate differences, anomalies, or phase-transition dynamics, and it does not determine the invariant global branch datum. OQG thereby prevents the radiative local-source vacuum catastrophe while leaving the separate cosmological value problem explicit.

Keywords: Operational Quantum Gravity; cosmological constant problem; vacuum energy; stress response; WTDiff gravity; closed-time-path formalism

Mathematics Subject Classification (2020): 81T20; 83C05; 83C45

1. Introduction

The cosmological constant problem begins with a mismatch between two uses of energy. In nongravitational quantum theory, adding the same constant to every energy level changes no transition frequency, matrix element, normalized probability, or scattering amplitude. In the conventional Einstein equation, however, the corresponding constant in the matter Lagrangian shifts the stress tensor by a term proportional to the metric. That term has precisely the algebraic form of a cosmological source. Standard Model loop corrections, condensate conventions, and changes of renormalization prescription can therefore appear to demand repeated cancellations against a bare cosmological term. This radiative instability is the central content of the vacuum catastrophe [22].

Three questions are frequently compressed into that one name. The first is the radiative-sensitivity problem: can an arbitrary homogeneous c-number generated or reassigned in the renormalized matter theory enter the local gravitational source? The second is the value problem: what selects the residual cosmological datum after the unobservable energy origin has been removed? The third is the observational problem: which branch and matter-response history fit cosmological data? A calculation that answers the first question does not automatically answer the second or third. Keeping them separate makes it possible to state a strong theorem without hiding its boundary conditions.

Weyl-transverse-diffeomorphism gravity and unimodular gravity already provide the classical architecture needed for this separation. Their fixed volume form produces trace-free equations [1, 7, 8, 10], while stress conservation integrates the missing trace into a cosmological constant rather than a prescribed local coupling [2, 11, 18]. The associated matter weights and linear gauge structure are treated in Refs. [3, 4]. Quantum analyses further show that a field-independent volume term behaves differently from curvature-dependent terms when the measure is fixed before variation [5, 6, 19, 21]. These results establish the relevant precedent. This paper closes the specific response chain required by OQG.

OQG is formulated here as a candidate matter-first theory in which geometry is a compact representation of matter response. A separate companion calculation identifies the physical transverse-traceless derivative stiffness in the full contact-completed Standard Model stress Hessian and matches it to an Einstein-equivalent WTDiff infrared action. This paper asks whether an arbitrary vacuum offset contaminates any stage of that construction. It does not. The fixed measure makes the offset independent of the source before any functional derivative is taken. On a normalized doubled time contour its phase cancels exactly, so the entire response hierarchy is blind to the shift, including the local contact terms that a commutator-only argument would miss.

This establishes a full response-level theorem rather than a cancellation at one loop or in one projected equation. The theorem then propagates through the coefficient chain used in the companion paper: the Euclidean transverse-traceless stiffness, its causal compliance, the WTDiff coefficient, and the invariant combination G/c^4 are all unchanged. At macroscopic order the trace-free source is likewise invariant, while the cosmological integration datum changes only as the label needed to describe the same solution in Einstein form. The result gives the phrase "prevents the vacuum catastrophe" a precise meaning: the arbitrary zero of Standard Model energy cannot become a connected local source of OQG response.

The scope is equally precise. Position-dependent energy, differences between states, connected vacuum fluctuations, boundary stresses, polarization, running couplings, curvature response, defects, and nonequilibrium transition products remain physical. The theorem neither predicts the invariant global cosmological datum nor calculates the reference-state intercept of the total stiffness. It proves that an arbitrary identity-operator coefficient drives neither quantity. That boundary is maintained throughout the calculation.

2. Matter-first source definition

Let $q_{\mu\nu}$ be an unconstrained, nonsingular Lorentzian source of fixed signature, let $q^{\mu\nu}$ denote its matrix inverse, let $q \equiv \det q_{\mu\nu}$, and let ϖ be a fixed positive scalar density. In four dimensions, define the composite source metric $\hat{g}_{\mu\nu}$ by

$$\hat{g}_{\mu\nu}(q,\varpi) = \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2} q_{\mu\nu}, \qquad \sqrt{-\hat{g}} = \varpi. \tag{1}$$

The determinant identity follows algebraically throughout this fixed-signature source domain. The density ϖ specifies the integration measure; it does not prescribe lengths, null cones, curvature, or a pre-existing gravitational field. Those relations enter only through the response carried by the unimodular tensor $\hat{g}_{\mu\nu}$. This ordering implements the OQG statement that matter supplies the physical response and geometry records it.

The complete renormalized Standard Model action is coupled to the source only through the composite metric,

$$S_{\mathrm{SM}}[\Phi, q; \varpi] = \int_{\Omega} d^4x\, \varpi\, \mathcal{L}_{\mathrm{SM}}\big(\Phi, \hat{g}(q,\varpi); g_i(\mu)\big). \tag{2}$$

Here Φ denotes all Standard Model fields, $g_i(\mu)$ denotes the renormalized nonidentity couplings at scale μ, and Ω is the fixed integration domain. The matter action may contain interactions, masses, gauge fixing, ghosts, improvement terms, and the counterterms needed to define the stress response. Equation (2) imposes only one restriction relevant to the theorem: the source variation is performed at fixed ϖ rather than through a variable determinant.

The transformation to be tested is a homogeneous change of the energy origin,

$$\mathcal{L}_{\mathrm{SM}} \to \mathcal{L}_{\mathrm{SM}} - \rho_0, \qquad S_{\mathrm{SM}} \to S_{\mathrm{SM}} - \rho_0 \mathcal{V}_{\varpi}, \qquad \mathcal{V}_{\varpi} \equiv \int_{\Omega} d^4x\, \varpi. \tag{3}$$

The constant ρ_0 multiplies the identity operator and is independent of spacetime, fields, state labels, and sources. The physical preparation, nonidentity couplings, domain, measure, topology, and boundary conditions are held fixed. A change in any of those data is a different physical comparison and is not represented by Eq. (3).

Fixing the measure before source variation is essential. If one first constructs an ordinary diffeomorphism-invariant functional of an unrestricted metric and only afterward imposes a determinant condition, the vacuum term has already acquired a metric derivative. Equation (2) instead defines a different off-shell source map. Its Weyl redundancy removes the trace direction where the response functional is defined, rather than imposing a gauge choice on a completed Einstein theory [7, 10, 11, 18].

The same distinction appears in the first variation. From Eq. (1), a general source displacement obeys

$$\delta\hat{g}_{\alpha\beta} = \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2}\left(\delta q_{\alpha\beta} - \frac{1}{4}q_{\alpha\beta}q^{\mu\nu}\delta q_{\mu\nu}\right). \tag{4}$$

The expression in parentheses is trace free with respect to $q_{\mu\nu}$. Consequently, $q_{\mu\nu}$ couples to the projected stress response rather than to an independent determinant mode. This projection will reappear at macroscopic order as the trace-free WTDiff source.

3. Full response-level vacuum-shift theorem

The simplest part of the proof is already visible at the level of a normalized state. For a static regulated system, let $V_{\varpi}^{(3)}$ be its fixed spatial volume. Equation (3) changes the Hamiltonian by

$$H' = H + \rho_0 V_{\varpi}^{(3)}\mathbf{1}. \tag{5}$$

Every energy eigenvalue receives the same displacement, while eigenvectors, energy differences, transition frequencies, and matrix elements remain unchanged. For a thermal preparation, $Z_\beta \equiv \mathrm{Tr}e^{-\beta H}$, $\hat{\rho}_\beta \equiv e^{-\beta H}/Z_\beta$, and $\beta = (k_\mathrm{B}T)^{-1}$, where k_B is Boltzmann's constant and T is the temperature.

$$Z'_\beta = e^{-\beta\rho_0 V_{\varpi}^{(3)}}Z_\beta, \qquad \hat{\rho}'_\beta = \frac{e^{-\beta H'}}{Z'_\beta} = \frac{e^{-\beta H}}{Z_\beta} = \hat{\rho}_\beta. \tag{6}$$

A normalized pure state acquires only a common phase. Thus the offset changes an unnormalized partition function or vacuum amplitude, but no normalized expectation value. This cancellation is necessary but not sufficient for gravity, because gravitational response is obtained by differentiating with respect to a source. The decisive step is to establish the cancellation before those derivatives are taken. Hereafter, $\hbar$ denotes the reduced Planck constant.

Let q_+ and q_- denote independent sources on the forward and backward branches of a closed-time-path (CTP) contour, let $U[q]$ be the corresponding Standard Model time-evolution operator, and let $\hat{\rho}_i$ be a normalized initial density operator. With the same fixed measure and domain on both branches, define the normalized Schwinger-Keldysh generating functional by [20]

$$Z_{\mathrm{CTP}}[q_+, q_-] \equiv \mathrm{Tr}\big(U[q_+]\hat{\rho}_i U^\dagger[q_-]\big), \qquad Z_{\mathrm{CTP}}[q, q] = 1. \tag{7}$$

Equivalently, the path-integral representation contains the initial kernel $\langle\Phi_+^i|\hat{\rho}_i|\Phi_-^i\rangle$, integrates over a common final configuration satisfying $\Phi_+(t_f) = \Phi_-(t_f)$, and uses the same fixed measure and domain on both branches. Under Eq. (3), $U[q_+]$ and $U^\dagger[q_-]$ acquire opposite phases. Neither phase depends on q_+ or q_-, and both branches contain the same fixed four-volume, so

$$Z'_{\mathrm{CTP}}[q_+, q_-] = \exp\left[-\frac{i\rho_0}{\hbar}(\mathcal{V}_\varpi - \mathcal{V}_\varpi)\right]Z_{\mathrm{CTP}}[q_+, q_-] = Z_{\mathrm{CTP}}[q_+, q_-]. \tag{8}$$

This identity is exact for independent branch sources. It is not obtained by setting $q_+ = q_-$, truncating a loop expansion, or discarding disconnected diagrams after differentiation. The initial density operator is held fixed; for the thermal example its invariance follows from Eq. (6). No hidden normalization factor survives at the initial end of the contour.

Define the connected CTP generating functional by

$$W_{\mathrm{CTP}}[q_+, q_-] = -i\hbar \ln Z_{\mathrm{CTP}}[q_+, q_-]. \tag{9}$$

Equation (8) gives ${W'}_{\mathrm{CTP}} = W_{\mathrm{CTP}}$. Therefore, for every integer $n \geq 1$, every choice of branch labels $a_j \in \{+,-\}$, and every set of spacetime points,

$$\frac{\delta^n {W'}_{\mathrm{CTP}}}{\delta q_{a_1,\mu_1\nu_1}(x_1) \cdots \delta q_{a_n,\mu_n\nu_n}(x_n)} = \frac{\delta^n W_{\mathrm{CTP}}}{\delta q_{a_1,\mu_1\nu_1}(x_1) \cdots \delta q_{a_n,\mu_n\nu_n}(x_n)}. \tag{10}$$

Theorem 1. Let the renormalized Standard Model be coupled to an unconstrained symmetric source only through the fixed-measure composite metric in Eq. (1). Hold the normalized physical preparation, nonidentity couplings, integration domain, measure, topology, and boundary conditions fixed. Then any spacetime-independent identity-operator shift of the form in Eq. (3) leaves the normalized CTP generating functional invariant. Consequently, the projected mean stress, every retarded and fluctuation kernel, the full contact-completed Hessian, and every higher response vertex are invariant. The theorem holds before perturbative or derivative expansion and hence to all orders in Standard Model interactions in a renormalization scheme that preserves the fixed-measure source definition.

It is useful to contrast Eq. (8) with an ordinary metric source. A conventional vacuum term contributes to the doubled action as

$$\Delta S_{\mathrm{Diff}}[g_+, g_-] = -\rho_0 \left[\int d^4x \sqrt{-g_+} - \int d^4x \sqrt{-g_-}\right]. \tag{11}$$

This expression vanishes at coincident sources, but its derivative with respect to the difference source does not. Define $g_r = (g_+ + g_-)/2$ and $g_a = g_+ - g_-$. Differentiating at fixed g_r and then taking the physical limit $g_a = 0$, for which $g_+ = g_- = g$, gives

$$\left.\frac{\delta \Delta S_{\mathrm{Diff}}}{\delta g_{a,\mu\nu}(x)}\right|_{g_a=0} = -\frac{1}{2}\rho_0 \sqrt{-g}\, g^{\mu\nu} \neq 0. \tag{12}$$

The doubled contour by itself therefore does not solve the problem. A common phase cancels at the value of the functional, while the variable determinant retains a nonzero response. OQG's fixed measure removes that derivative before the contour identity is used. The two ingredients—normalized doubling and a source-level fixed measure—are jointly responsible for the full theorem.

4. Complete Hessian and shear channel audit

The second derivative of a matter generating functional contains more than an operator commutator or a separated-point stress correlator. Because the stress operator itself depends on the source, a second variation also produces local seagull terms, improvement terms, gauge-fixing contributions, and the contacts required by Ward identities. A proof based only on the fact that the identity operator commutes with everything would establish invariance of connected separated-point correlators, but it would leave the local part of the Hessian untested [15, 16].

For branch labels $a, b \in \{+,-\}$, write the complete source Hessian schematically as

$$\mathcal{H}_{ab}^{\mu\nu,\rho\sigma}(x,y) \equiv \frac{\delta^2 W_{\mathrm{CTP}}}{\delta q_{a,\mu\nu}(x)\delta q_{b,\rho\sigma}(y)} = \mathcal{H}_{ab,\mathrm{conn}}^{\mu\nu,\rho\sigma}(x,y) + \mathcal{H}_{ab,\mathrm{contact}}^{\mu\nu,\rho\sigma}(x,y). \tag{13}$$

Equation (10) proves the invariance of the entire left-hand side, which automatically includes both terms on the right. More directly, the shifted action in Eq. (3) is independent of q_+ and q_-. Its first variation is zero and its second variation is zero, so it generates neither a mean projected stress nor a local seagull. This is stronger than cancelling disconnected vacuum bubbles after they have been calculated.

The shear source used in the companion calculation gives a transparent component-level check. Work in a local Cartesian Euclidean background with undeformed diagonal entries equal to unity and fixed density $\varpi = 1$. Introduce an off-diagonal perturbation in the x-y block of the ordinary source matrix, with z the Cartesian coordinate along which the shear varies and k its conjugate scalar wave number:

$$q_{xy} = q_{yx} = h(z), \qquad \mathrm{det}q = 1 - h^2, \qquad \sqrt{\mathrm{det}q} = 1 - \frac{1}{2}h^2 + \mathcal{O}(h^4). \tag{14}$$

If a vacuum term is coupled through the ordinary determinant, Eq. (14) produces a quadratic local contribution proportional to $\rho_0 h^2$. In momentum space that contribution is independent of the wave number. It can alter the algebraic k^0 contact but cannot alter the k^2 derivative slope. With the exact fixed-measure source, however,

$$\hat{g}_{\mu\nu} = (\mathrm{det}q)^{-1/4} q_{\mu\nu}, \qquad (\mathrm{det}q)^{-1/4} = 1 + \frac{1}{4}h^2 + \mathcal{O}(h^4), \qquad \sqrt{\mathrm{det}\hat{g}} = 1. \tag{15}$$

The compensating diagonal terms in Eq. (15) remove even the k^0 vacuum contact. Thus the general functional proof and the explicit shear algebra agree.

Let $Z_E[h]$ be the Standard Model Euclidean partition function and $W_E[h] \equiv -\ln Z_E[h]$ its connected equilibrium generating functional. Define $G_E^{xy,xy}$ as the full second shear derivative of W_E, including the local source variations and contact terms described above. Assuming the analytic small-momentum expansion in this channel and prescription, the response is

$$G_E^{xy,xy}(0,k) = G_E^{xy,xy}(0,0) + \frac{\kappa}{2}k^2 + \mathcal{O}(k^4), \qquad A_{2,E} \equiv \frac{\kappa}{4}. \tag{16}$$

The companion observable subtracts the equilibrium block and retains the positive derivative stiffness. Even with the ordinary diagnostic source, a homogeneous vacuum shift could enter only $G_E^{xy,xy}(0,0)$ and would disappear from the subtracted k^2 slope. With the exact composite source it enters neither term. In both descriptions,

$$\frac{\partial A_{2,E}}{\partial \rho_0} = \frac{1}{4}\frac{\partial}{\partial \rho_0}\frac{\partial^2 G_E^{xy,xy}(0,k)}{\partial k^2}\bigg|_{k=0} = 0. \tag{17}$$

This explains why the WTDiff-to-Einstein construction did not require the full theorem established here: its defined k^2 coefficient was already insensitive to a momentum-independent k^0 contact. The present paper proves the stronger statement that, with the physical fixed-measure source, the complete normalized Hessian and every higher response function are invariant before the k^0 subtraction is performed.

5. Invariance of the OQG coefficient chain

Let p denote Euclidean four-momentum. After the background and zero mode have been treated, the transverse-traceless Euclidean inverse response at nonzero p and its inverse on the physical TT subspace are defined at low momentum by

$$\mathcal{K}_{TT,E}(p) = A_{2,E}^{\mathrm{tot}} p^2 + \mathcal{O}(p^4), \qquad \mathcal{D}_{TT,E}(p) = \mathcal{K}_{TT,E}^{-1}(p) = \frac{1}{A_{2,E}^{\mathrm{tot}} p^2} + \mathcal{O}(p^0). \tag{18}$$

Here $A_{2,E}^{\mathrm{tot}}$ is the total derivative stiffness of the chosen reference state plus any calculated state increment. The superscript is important. The detailed Standard Model calculation determines a thermal-minus-reference increment; it does not determine the state-independent intercept. The vacuum-shift theorem removes dependence on the arbitrary identity coefficient from both the increment and the intercept, but it does not supply the intercept's numerical value.

To make the normalization self-contained, let $X^{\mu} = x^{\mu}/(\hbar c)$ denote inverse-energy coordinates and define C_W by the positive dimensionless Euclidean WTDiff quadratic action on the TT block. In this paragraph, $z \equiv X^z$, k is its energy-dimension conjugate, and $V_4 \equiv \int d^4X$. For the real mode $h_{xy} = h_{yx} = h_k e^{ikz} + h_{-k} e^{-ikz}$, with $h_{-k} = h_k^*$, the tensor contraction counts both off-diagonal components. Applying the same source to Eq. (18) gives the two quadratic forms in Eq. (19). Matching them and restoring the fixed-measure Einstein action $S_W = [c^3/(16\pi G)] \int d^4x\, \varpi R$, where G is Newton's coupling, yields

$$\begin{aligned} \Gamma^{(2)}_{W,E}[h^{TT}] &= \frac{C_W}{4} \int d^4X\ \partial_\lambda h^{TT}_{\mu\nu}\, \partial_\lambda h^{\mu\nu}_{TT} = C_W V_4 k^2 h_k h_{-k}, \\ \Gamma^{(2)}_{\mathrm{OQG},E} &= 2A^{\mathrm{tot}}_{2,E} V_4 k^2 h_k h_{-k}, \qquad C_W = 2A^{\mathrm{tot}}_{2,E} = \frac{\hbar c^5}{16\pi G}, \\ A^{\mathrm{tot}}_{2,E} &= \frac{\hbar c^5}{32\pi G}, \qquad \frac{G}{c^4} = \frac{\hbar c}{32\pi A^{\mathrm{tot}}_{2,E}}. \end{aligned} \tag{19}$$

No natural-unit convention is used. The quantity c is the locally realized causal signal speed of the matter response. An identity shift changes no normalized state, transition threshold, pole position, or retarded kernel; it therefore changes neither c nor the derivative coefficient. Equations (10), (17), and (19) give the response-chain identity

$$\frac{\partial A^{\mathrm{tot}}_{2,E}}{\partial \rho_0} = \frac{\partial \mathcal{K}_{TT,E}}{\partial \rho_0} = \frac{\partial \mathcal{D}_{TT,E}}{\partial \rho_0} = \frac{\partial C_W}{\partial \rho_0} = \frac{\partial}{\partial \rho_0}\left(\frac{G}{c^4}\right) = 0. \tag{20}$$

Equation (20) is the OQG-specific closure of the vacuum-shift argument. It begins at the normalized Standard Model functional, includes the local completion of the stress Hessian, and ends at the invariant coefficient multiplying the macroscopic source. There is no stage at which a cutoff-sized homogeneous offset is converted into an enormous gravitational response.

The theorem does not make all matter-dependent corrections to gravitational response vanish. Curvature susceptibilities, nonlocal polarization, thermal corrections, density changes, and other nonidentity operators can alter $A^{\mathrm{tot}}_{2,E}$ and hence the effective response. Those are measurable changes in state or dynamics. Equation (20) removes only the arbitrary common zero.

6. Renormalization and anomalies

The radiative stability of the result can be stated without assigning a cutoff value to the vacuum energy. In a symmetry-preserving renormalization scheme, the matter effective action coupled to the fixed-measure source has the schematic derivative expansion

$$\Gamma_{\mathrm{SM}}[\hat{g};\mu] = \int d^4x\, \varpi \left[-\rho_{\mathrm{vac}}(\mu) + f_1(\mu) R[\hat{g}] + \sum_i \alpha_i(\mu) \mathcal{R}_i^2 + \cdots \right] + \Gamma_{\mathrm{nonlocal}}[\hat{g};\mu]. \tag{21}$$

The first term multiplies the identity operator and contains no source dependence because ϖ is fixed. Changes of regulator, subtraction prescription, or loop order may change the bookkeeping coefficient $\rho_{\mathrm{vac}}(\mu)$, but source derivatives cannot detect that change. The coefficient f_1 multiplies the two-derivative curvature scalar, $\mathcal{R}_i^2$ denotes a basis of local quadratic and higher curvature invariants with coefficients α_i, and $\Gamma_{\mathrm{nonlocal}}$ is the nonlocal remainder. These sectors depend on $\hat{g}_{\mu\nu}$ and may renormalize the derivative stiffness or generate higher-order response.

This separation is visible in heat-kernel language. The leading volume coefficient contributes only to the fixed four-volume, whereas subsequent coefficients contain curvature and field strengths. Carballo-Rubio showed that the absence of longitudinal diffeomorphisms prevents the volume term from becoming a dynamical source in a fixed-measure theory [10]. One-loop calculations in unimodular gravity likewise find that the cosmological sector is not renormalized as an ordinary metric volume term, while the remaining effective action retains quantum corrections [5, 19]. The result is radiative insulation, not the absence of quantum response.

The renormalization-group statement follows directly. Differentiating Eq. (21) with respect to the source removes $\rho_{\mathrm{vac}}(\mu)$ before any scale dependence is considered. The source-dependent response still obeys its Callan-Symanzik equation, including beta functions, anomalous dimensions, operator mixing, and permitted local anomaly or contact terms [9]. The identity coefficient can run in a chosen bookkeeping scheme without entering that equation. Thus two symmetry-preserving schemes that differ only by a homogeneous c-number produce identical normalized response functionals.

A trace anomaly does not reverse the conclusion. Running nonidentity couplings and curvature invariants can produce a nonzero trace response, and those contributions must be retained in a complete calculation. They do not make the fixed integral $\int d^4x\,\varpi$ depend on $q_{\mu\nu}$. Some formulations describe the Weyl redundancy of unimodular gravity as nonanomalous when the matter fields and functional measure are defined compatibly [6], but the present theorem does not require the stronger assertion that every trace anomaly vanishes. It requires only that an allowed anomaly be carried by source-dependent operators rather than by the identity coefficient.

The all-orders claim is also limited to the stated matter-first theory. Equation (8) is exact for arbitrary Standard Model interactions because it precedes a loop expansion. It is not an all-orders theorem for an additional, independently quantized fundamental graviton sector. In OQG, the infrared tensor field is the compliance representation of matter response, so no separate microscopic graviton measure is assumed here. Any extension that adds such a sector would need a new source, measure, and anomaly analysis.

7. Einstein-equivalent trace-free limit

The same vacuum-shift blindness appears in the nonlinear infrared representation. Let $R_{\mu\nu}$ be the Ricci tensor associated with the composite metric. Define $R \equiv \hat{g}^{\mu\nu}R_{\mu\nu}$, and let $\hat{\nabla}$ be the Levi-Civita derivative of $\hat{g}_{\mu\nu}$. Write $T \equiv \hat{g}^{\mu\nu}T_{\mu\nu}$. The full-metric tensor $T_{\mu\nu}$ is a representative whose trace-free projection is the actual fixed-measure source. With $\kappa_G \equiv 8\pi G/c^4$, variation of the fixed-measure WTDiff action gives

$$R_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}R = \kappa_G\left(T_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}T\right), \qquad \kappa_G \equiv \frac{8\pi G}{c^4}. \tag{22}$$

Under the homogeneous matter shift, the stress tensor and its trace transform as

$$T'_{\mu\nu} = T_{\mu\nu} - \rho_0\hat{g}_{\mu\nu}, \qquad T' = T - 4\rho_0, \qquad T'_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}T' = T_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}T. \tag{23}$$

The local field equation is therefore exactly invariant. This is not a cancellation between two large quantities. The identity direction is absent from the source space of Eq. (22).

For a conserved total stress tensor, the contracted Bianchi identity gives

$$\hat{\nabla}^{\mu}T_{\mu\nu} = 0 \quad \Rightarrow \quad \hat{\nabla}_{\nu}(R + \kappa_G T) = 0 \quad \Rightarrow \quad R + \kappa_G T = 4\Lambda, \tag{24}$$

where Λ is constant on each connected solution branch. Substitution into Eq. (22) yields the Einstein-form representative

$$R_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}R + \Lambda\hat{g}_{\mu\nu} = \kappa_G T_{\mu\nu}. \tag{25}$$

Equations (22)-(25) reproduce the standard local Einstein dynamics for conserved sources, but they assign a different status to the cosmological term [2, 7, 10, 11, 18]. It is an integration datum rather than the coefficient of a local matter vacuum operator. If the representative stress tensor is shifted while the physical solution is held fixed, Eq. (24) requires

$$\Lambda' = \Lambda - \kappa_G\rho_0. \tag{26}$$

The separate symbols $T_{\mu\nu}$ and Λ therefore depend on the chosen zero of matter energy, while their local physical content does not. The invariant Einstein representation is the equivalence class of pairs

$$\left[(T_{\mu\nu}, \Lambda)\right] \equiv \left\{(T_{\mu\nu} - \rho\hat{g}_{\mu\nu}, \Lambda - \kappa_G\rho) : \rho \in \mathbb{R},\ \partial_\alpha\rho = 0\right\}. \tag{27}$$

To name the invariant branch explicitly, write $T_{\mu\nu} = T_{\mu\nu}^{(0)} - \rho_{\mathrm{id}}\hat{g}_{\mu\nu}$, where $T_{\mu\nu}^{(0)}$ contains no homogeneous identity term, and define $\Lambda_{\mathrm{inv}} \equiv \Lambda + \kappa_G\rho_{\mathrm{id}}$. This quantity is constant across the class in Eq. (27). The identity-operator part of Standard Model loop corrections cannot force a retuning of the local gravitational equation, because it changes only the representative of that class. The theorem does not select Λ_{inv}. The zero branch means $\Lambda_{\mathrm{inv}} = 0$, equivalently $\Lambda = 0$ only in an identity-free representative.

8. Physical scope and cosmological relation

Vacuum-shift blindness removes one redundant direction in operator space, not the physical quantum vacuum. A Casimir force compares configurations with different boundaries. A Lamb shift changes an energy difference. Vacuum polarization changes propagators and couplings. Connected stress fluctuations produce a noise kernel, while curvature-dependent terms change the derivative response. None of these effects is a common identity displacement, and all survive Eqs. (8) and (10). This distinction is consistent with the operational treatment of vacuum phenomena throughout quantum electrodynamics [17].

Choose a projected component $\tau(x)$ of the tensor $\tau^{\mu\nu}$ defined in Appendix A, and set $\delta\tau \equiv \tau - \langle\tau\rangle$. With θ the Heaviside function and with no additional factor of $\hbar$ absorbed into the noise definition, the retained retarded kernel G_R and symmetrized noise kernel N are

$$G_R(x,x') = -\frac{i}{\hbar}\theta(t-t')\langle[\tau(x),\tau(x')]\rangle, \qquad N(x,x') = \frac{1}{2}\langle\{\delta\tau(x),\delta\tau(x')\}\rangle. \tag{28}$$

Adding a multiple of the identity changes neither the commutator nor the centered operator $\delta\tau$. More generally, Eq. (10) protects the contact-completed versions obtained by source differentiation. Spectral densities, damping widths, thresholds, and causal poles therefore remain physical. The theorem prevents a disconnected constant from masquerading as response; it does not make the vacuum empty.

Phase transitions require similar care. A common displacement of an entire effective potential, $V(\phi) \to V(\phi) + C$, leaves its derivatives, extrema, tunneling-exponent differences, and normalized transition probabilities unchanged. That displacement is covered by the theorem; a transition between physically distinct configurations is not. Let ϕ be a canonical scalar field and define $(\partial\phi)^2 \equiv \hat{g}^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi$. Its trace-free source contains

$$T^{(\phi)}_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}T^{(\phi)} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{4}\hat{g}_{\mu\nu}(\partial\phi)^2. \tag{29}$$

The metric-proportional potential term cancels locally, while gradients, kinetic energy, defects, radiation, latent-heat products, and the transition history entering the integrated relation remain. For a genuine transition, a single common shift cannot remove the energy difference between distinct endpoints. With the same invariant global branch datum, that difference and the transition products can alter the Einstein-form curvature. Equation (26) applies only to a common redefinition of the energy origin, not to separate endpoint redefinitions. This paper therefore does not claim the stronger global sequestering result that every phase-transition contribution is automatically made small [14].

Boundaries and global data provide another controlled limit. Equation (8) assumes that both contour branches have the same fixed domain, measure, topology, and compatible boundary conditions. Varying a boundary embedding, changing total four-volume, or comparing inequivalent topologies can produce boundary work or a different integration datum. Such effects are physical comparisons; they do not restore sensitivity to an arbitrary bulk zero within one fixed problem.

The relation to cosmology is sequential. The response theorem first removes the radiative reason to expect an enormous local source from the Standard Model energy origin. The separate OQG in Practice analysis studies the branch that can be stated invariantly as $\Lambda_{\mathrm{inv}} = 0$; in an identity-free representative this is written $\Lambda = 0$. Its expansion history is represented by evolving matter standards rather than a dark-energy component [13]. That calculation reports consistency with cosmic-chronometer data and, without refitting, with the six DESI Data Release 2 baryon-acoustic-oscillation observables tested there. The DESI measurements remain compatible with flat ΛCDM and show their strongest preference for an evolving dark-energy equation of state only when combined with other datasets [12]. They do not independently select OQG.

Accordingly, the $\Lambda_{\mathrm{inv}} = 0$ OQG branch has survived the stated DESI consistency test and is not excluded by those observables. The cited comparison is not a covariance-complete likelihood or a model-selection result against

ΛCDM and dynamical-dark-energy alternatives. Cosmological data can test which allowed global branch and response history nature realizes; they cannot restore the radiative local-source dependence proved absent here.

9. Discussion and conclusion

Vacuum-shift blindness itself is established in the prior WTDiff and unimodular literature [7, 10, 11, 18]. The contribution here is the OQG response-chain closure: an exact functional identity that begins with normalized Standard Model matter response and ends at the Einstein-equivalent source coefficient.

The proof is established for independent CTP branch sources before differentiation. It therefore covers the projected mean stress, separated-point correlators, seagulls, Ward contacts, and higher vertices in one statement. The explicit shear calculation shows that an ordinary variable-volume diagnostic retains only a k^0 contact, whereas the fixed-measure source removes that contact as well. Equations (18)-(20) then carry the invariance through $A_{2,E}^{\mathrm{tot}}$, $\mathcal{K}_{TT,E}$, $\mathcal{D}_{TT,E}$, C_W, and G/c^4.

This chain supplies a direct answer to the vacuum catastrophe. The conventional problem treats a freely adjustable identity coefficient as though it were a measurable local source and then asks why its inferred curvature is not enormous. In OQG the identity coefficient lies outside the connected source map. No ultraviolet estimate of ρ_0 is needed, no bare term is tuned against successive loop corrections, and no cancellation between large local sources occurs. The shift is absent because normalized matter response cannot measure it.

The remaining results are separate. The theorem does not determine the reference-state intercept of $A_{2,E}^{\mathrm{tot}}$ and therefore does not independently calculate the observed value of G. It does not select Λ_{inv}, prove that every ultraviolet completion preserves the fixed-measure source symmetry, quantize an additional fundamental geometric field, or replace a cosmological likelihood analysis. None of these limitations reintroduces a source derivative of the homogeneous identity term.

The theorem is falsifiable at its assumptions. A demonstrated source dependence of the supposedly fixed measure, a failure of normalized CTP closure, a symmetry-preserving counterterm that converts the identity operator into a nontrivial $q_{\mu\nu}$ functional, or an inconsistency in the companion coefficient matching would narrow or invalidate the chain. Physical boundaries, gradients, phase changes, and nonidentity curvature terms do not constitute such failures because the theorem expressly retains them.

Within its stated domain, the conclusion is exact. A homogeneous shift of the Standard Model Lagrangian changes no normalized state, complete stress-response vertex, transverse-traceless stiffness, causal compliance, or invariant Einstein source coefficient. In the Einstein-equivalent representation it only relabels $(T_{\mu\nu}, \Lambda)$ while leaving Λ_{inv} unchanged. Operational Quantum Gravity therefore prevents the radiative local-source vacuum catastrophe while preserving the experimentally established structure of the quantum vacuum and leaving the invariant global value problem explicit.

Appendix A. Functional derivative conventions

The unconstrained source $q_{\mu\nu}$ carries ten formal components, but Eq. (1) makes one direction redundant. Parentheses on tensor indices denote unit-weight symmetrization, $A_{(\mu\nu)} \equiv (A_{\mu\nu} + A_{\nu\mu})/2$. Define

$$\Omega_q^2 \equiv \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2}, \qquad \mathcal{P}_{\alpha\beta}^{\ \ \mu\nu} \equiv \delta^{\mu}_{(\alpha}\delta^{\nu}_{\beta)} - \frac{1}{4} q_{\alpha\beta} q^{\mu\nu}, \qquad \delta\hat{g}_{\alpha\beta} = \Omega_q^2 \mathcal{P}_{\alpha\beta}^{\ \ \mu\nu} \delta q_{\mu\nu}. \tag{A1}$$

The projector satisfies $q^{\alpha\beta}\mathcal{P}_{\alpha\beta}^{\ \ \mu\nu} = 0$. If the stress convention is fixed by $\delta S_{\mathrm{SM}} = 1/2 \int d^4x\, \varpi T^{\alpha\beta} \delta\hat{g}_{\alpha\beta}$, the source conjugate is

$$\tau^{\mu\nu} \equiv \frac{2}{\varpi}\frac{\delta S_{\mathrm{SM}}}{\delta q_{\mu\nu}} = \Omega_q^2 \mathcal{P}_{\alpha\beta}^{\ \ \mu\nu} T^{\alpha\beta}, \qquad q_{\mu\nu}\tau^{\mu\nu} = 0. \tag{A2}$$

For CTP calculations, introduce average and difference sources $q_r = (q_+ + q_-)/2$ and $q_a = q_+ - q_-$. The physical limit is $q_a = 0$. Mean response and retarded response are obtained from derivatives with respect to the difference source followed by derivatives with respect to the average source. A representative convention is

$$\langle \tau^{\mu\nu}(x) \rangle = \frac{2}{\varpi(x)} \frac{\delta W_{\mathrm{CTP}}}{\delta q_{a,\mu\nu}(x)}\bigg|_{q_a=0}, \qquad G_R \sim \frac{\delta^2 W_{\mathrm{CTP}}}{\delta q_a \, \delta q_r}\bigg|_{q_a=0}. \tag{A3}$$

Overall factors in the second expression depend on the source normalization and tensor symmetrization, but the vacuum-shift result does not. Since $W'_{\mathrm{CTP}} = W_{\mathrm{CTP}}$ as a functional of both independent branch sources, every derivative in any consistent r/a convention agrees exactly. Ward identities derived from the same functional are unchanged for the same reason.

Appendix B. Shear source algebra

For the Euclidean off-diagonal shear used in Eq. (14), the nontrivial block and its determinant are

$$q_{ij} = \begin{pmatrix} 1 & h \\ h & 1 \end{pmatrix}, \qquad \det q_{ij} = 1 - h^2, \qquad (\det q)^{-1/4} = 1 + \frac{1}{4}h^2 + \mathcal{O}(h^4). \tag{B1}$$

Multiplication by the compensating factor gives, through quadratic order,

$$\hat{g}_{xx} = \hat{g}_{yy} = 1 + \frac{1}{4}h^2 + \mathcal{O}(h^4), \qquad \hat{g}_{xy} = h + \mathcal{O}(h^3), \qquad \sqrt{\det \hat{g}} = 1. \tag{B2}$$

For an ordinary variable-volume Euclidean functional, a constant density contributes

$$\Delta W_E^{\mathrm{ordinary}} = \rho_0 \int d^4x \sqrt{\det q} = \rho_0 \int d^4x \left(1 - \frac{1}{2}h^2 + \mathcal{O}(h^4)\right). \tag{B3}$$

Its quadratic part is local. If h_k denotes the Fourier amplitude of $h(z)$, the transform produces a term proportional to $h_k h_{-k}$ with no factor of k^2. For the fixed-measure source, the corresponding functional is

$$\Delta W_E^{\mathrm{fixed}} = \rho_0 \int d^4x \, \varpi, \tag{B4}$$

which contains no h dependence at any order. Equations (B3) and (B4) separate two useful statements: the derivative stiffness in Eq. (17) is blind even under the ordinary shear diagnostic because the residual term is k^0, while the exact OQG source removes the residual contact altogether.

Appendix C. Renormalization ledger

After the Standard Model fields have been integrated out, denote the renormalized effective action by $\varGamma_R \equiv \varGamma_{\mathrm{SM}}$. Let $\mathcal{O}_0 = \mathbf{1}$ be the identity and let $\mathcal{O}_A[\hat{g}]$ denote the local nonidentity operators admitted by the renormalized theory. The coefficient $c_0(\mu)$ multiplies the identity sector, the $c_A(\mu)$ are renormalized operator coefficients, and $\varGamma_{\mathrm{state}}[\hat{g}]$ collects the remaining state-dependent and nonlocal terms. The action can be organized as

$$\boxed{\varGamma_{\mathrm{SM}} = c_0(\mu)\int d^4x\,\varpi + \sum_A c_A(\mu)\,\mathcal{I}_A[\hat{g}] + \varGamma_{\mathrm{state}}[\hat{g}], \qquad \mathcal{I}_A[\hat{g}] \equiv \int d^4x\,\varpi\,\mathcal{O}_A. \qquad \text{(C1)}}$$

For every $n \geq 1$, the identity sector satisfies

$$\boxed{\frac{\delta^n}{\delta q_{\mu_1\nu_1}(x_1)\cdots\delta q_{\mu_n\nu_n}(x_n)}\left[c_0(\mu)\int d^4x\,\varpi\right] = 0, \qquad n \geq 1. \qquad \text{(C2)}}$$

The derivatives of $\mathcal{I}_A$ and $\varGamma_{\mathrm{state}}$ need not vanish. Vacuum polarization, anomalies, curvature counterterms, boundary operators, and state susceptibilities belong to this second class. A renormalization prescription preserves the theorem if counterterm mixing respects the fixed-measure source symmetry, so that the identity sector remains proportional to the source-independent fixed volume.

The source-dependent renormalization-group equation is therefore written in its complete form as

$$\boxed{\begin{aligned} \mathcal{D}_{\mathrm{CS}}\varGamma_R^{(n)} &= \mathcal{A}_R^{(n)}, \\ \varGamma_R^{(n)} &\equiv \frac{\delta^n \varGamma_R}{\delta q_{\mu_1\nu_1}(x_1)\cdots\delta q_{\mu_n\nu_n}(x_n)}, \qquad n \geq 1. \end{aligned} \qquad \text{(C3)}}$$

Here $\mathcal{D}_{\mathrm{CS}}$ is the full Callan-Symanzik operator, including $\mu\,\partial_\mu$, all beta functions, anomalous dimensions, and operator-source mixing, while $\mathcal{A}_R^{(n)}$ denotes allowed local anomaly or contact terms [9]. Source-dependent vertices obey their appropriate homogeneous or anomalous equations after contact completion. The identity coefficient contributes to neither side after one source derivative because its fixed-volume functional is $q_{\mu\nu}$ independent. Thus c_0 may be scheme dependent while the OQG response remains scheme independent.

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Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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Data availability. No new experimental datasets were generated for this theoretical study. The DESI measurements discussed in Sect. 8 are publicly available through Ref. [12]. The OQG cosmology calculation and its supporting numerical materials are identified in Ref. [13].

Code availability. No new numerical code was required for the analytical theorem proved in this paper.

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