Technical Appendices A-C

Companion to Operational Quantum Gravity in Practice: Spectral Stiffness, Matter Scaling, Equivalence, and Cosmology without Dark Energy

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These appendices provide the detailed normalization, state bookkeeping, loading, causal-response, and cosmology checks used by the main paper. Each calculation begins with the question it answers, defines every symbol needed to read it, and ends with the physical conclusion it establishes. The black-hole singularity program is intentionally excluded and reserved for its own dedicated paper.

Appendix A. OQG Interface, Spectral Stiffness, and State-Normalization Checks

A.1 Inherited results and their use here

These appendices retain intermediate checks that would interrupt the educational flow of the main paper. The more extensive closed-time-path construction, complete Standard-Model matching through $O(g^2)$, finite-temperature retarded kernel, and fixed-measure WTDiff completion remain in the foundational OQG manuscript and its Technical Appendices A-H [6]. The present companion starts from those inherited results and shows how they are used in practice.

Table A1 separates inherited response results from the practical deductions developed in the present paper. The middle column identifies where the underlying result is established. The final column states exactly what the present paper and these appendices use from it.

Table A1. Interface between the foundational OQG manuscript and the present paper.

Input used here	Established in the foundational OQG manuscript [6]	Use in the present paper and appendices
Complete conserved Standard-Model tensor and source normalization	Main Sections 2-3; Technical Appendix A	Main Equations (1), (9), and (16)-(25): complete-body energy, inertial mass, and the leading long-range source coefficient

Input used here	Established in the foundational OQG manuscript [6]	Use in the present paper and appendices
Ward/contact-completed TT spectrum and Kubo stiffness	Main Sections 4-5; Technical Appendices B-E	Main Equations (2)-(8) and Appendix A: causal inversion, baseline composition, and $A_{2,E}[\rho]$ normalization
Retarded finite-frequency and finite-wave-number continuation	Main Sections 6-7; Technical Appendices F-G	Main Equations (26)-(55) and Appendix B: preparation, loading, clocks, rulers, unloading, tides, signals, and protocols
Complete-body form factor and common asymptotic pole	Main Section 9; Technical Appendix H	Main Equations (16)-(25): equality of inertial, passive, and source mass and uniqueness of the leading inverse-distance branch
Minimal fixed-measure WTDiff infrared completion	Main Section 8; Technical Appendix H	Main Equations (10)-(11) and (65)-(88), plus Appendix C: Einstein-equivalent bookkeeping, the zero-integration-datum branch, and the cosmological reconstruction

The inherited calculation is not repeated here. Appendix A checks the input normalization, the distinction between the full spectral kernel and its static stiffness, the content of the baseline, the nested state labels, the complete-body limit, and the common-scale reduction. Appendix B checks loading, clocks, rulers, causal unloading, signal accounting, and the special-relativistic branch. Appendix C checks homogeneous evolution and the observational reconstruction.

A.2 Projected inverse, spectral variables, and the long-range pole

The first question is how a physical stress enters the response calculation. Let $q_{\mu\nu}$ denote the operational disturbance conjugate to the complete Standard-Model stress tensor $T^{\mu\nu}$. Their source coupling is

$$S_{\rm src} = \frac{1}{2}\int d^4x\, q_{\mu\nu}T^{\mu\nu}. \tag{A1}$$

The factor $1/2$ prevents the two symmetric index orderings from being counted twice. Equation (A1) therefore fixes the normalization used when a variation of $q_{\mu\nu}$ is converted into a stress response. In engineering language, it fixes the units and sign convention at the input port before the compliance is calculated.

Energy-momentum conservation creates null directions in the response: some apparent tensor variations are bookkeeping changes rather than independent physical deformations. The completed retarded response $\Pi_{\rm phys^R}$ must therefore be projected onto the physical subspace before it is inverted. Its inverse D^R is defined by

$$\Pi_{\rm phys^R} * D^R = \mathcal{P}_{\rm phys}\,\delta^{(4)}. \tag{A2}$$

The star denotes integration and index contraction over the intermediate event. The projector $\mathcal{P}_{\rm phys}$ is the mathematical filter that removes the conservation-forbidden directions, and the four-dimensional delta function is the identity operation on what remains. Equation (A2) is consequently not an unrestricted component-by-component matrix inverse. It says that applying the completed stiffness and then its causal compliance returns the original *physical* disturbance. The superscript R imposes retarded boundary conditions, so the output cannot precede the input.

The transverse-traceless sector must next be kept in its genuinely spectral form. Its two independent spectral coordinates are the frequency ω and wave number k (or the momentum-energy variable $Q = \hbar ck$). The state is carried separately by the density operator ρ:

$$\rho_2(\omega, k; \rho) = -2\,\mathrm{Im}\Pi_{TT}^R(\omega + i0, k; \rho), \qquad A_{2,E}[\rho] = -\frac{\partial \Pi_{TT,E}(0,Q;\rho)}{\partial Q^2}\bigg|_{Q=0} \tag{A3}$$

Equation (A3) is the notation rule that prevents several different ideas from being hidden in one symbol. $\Pi_{TT}^R(\omega, k; \rho)$ is the full frequency- and wave-number-dependent response. $A_{2,E}[\rho]$ is the static two-derivative coefficient obtained after setting $\omega = 0$ and taking the $k \to 0$ slope. The spectral dependence has been used to calculate the coefficient; it is not an unshown residual argument of $A_{2,E}$. Position, source mass, temperature, composition, loading, and history enter through the state ρ. The Euclidean coefficient and local gradient expansion apply only where the state admits the stated static limit and continuation. For driven or nonstationary states, use the full retarded response and specify the order of limits; this static extraction is not automatic.

The same coefficient separates into the Ward-completed local part and the accumulated connected TT spectral weight:

$$A_{2,E}[\rho] = A_{\mathrm{loc}}[\rho] + \int_0^\infty \frac{d\epsilon}{\epsilon}\,\mathcal{B}_{TT}(\epsilon; \rho), \qquad A_{2,E}[\rho(E_s, X)] = E_s^2 a_{TT}(X) \tag{A4}$$

Equation (A4) makes the physical baseline explicit. $A_{\mathrm{loc}}[\rho]$ contains the completed local and Ward-contact contribution. The spectral integral contains connected excitations and fluctuations of all Standard-Model fields in the declared state. The dimensionless data X include couplings, particle and antiparticle content, gauge fields, the Higgs sector, condensates, binding, polarization, thermal occupations, chemical potentials, correlations, and spectral shape. Measurable vacuum polarization and connected vacuum fluctuations are part of this fabric. An arbitrary homogeneous, state-independent vacuum-energy offset is not a connected local source and is not counted as an additional material component.

On the state-subtracted massless branch, the static inverse kernel begins at second order in momentum:

$$K_{TT,E}(0, Q; \rho) = A_{2,E}[\rho]Q^2 + O(Q^4), \qquad Q = \hbar ck \tag{A5}$$

The leading Q^2 term is the long-range one. Its inverse is proportional to $1/Q^2$, whose three-dimensional Fourier transform falls as $1/r$. The $O(Q^4)$ terms describe higher-gradient, shorter-range, or finite-size corrections. They do not belong inside the definition of $A_{2,E}[\rho]$.

The dimensional check is

$$\left[A_{2,E}[\rho]\right] = \text{energy}^2, \qquad [Q] = \text{energy}, \qquad \left[\frac{\hbar c^5}{A_{2,E}[\rho]}\right] = \mathrm{m^3\,kg^{-1}\,s^{-2}} = [G] \tag{A6}$$

Equation (A6) verifies that the stiffness has the units needed to generate a macroscopic compliance without inserting a second dimensional constant. Including the tensor projector and the source convention fixed by Equation (A1) gives

$$G_{\mathrm{comp}}[\rho] = \frac{\hbar c^5}{16\pi A_{2,E}[\rho]} \tag{A7}$$

Equation (A7) is a constitutive conversion. At fixed signal scale, a larger Standard-Model stiffness gives a smaller long-wavelength compliance. The logical direction is $\rho \to \Pi_{TT}^R(\omega, k; \rho) \to A_{2,E}[\rho] \to G_{\mathrm{comp}}[\rho]$; the measured value of gravity is not inserted to manufacture the microscopic spectrum.

The spatial form follows by Fourier inversion:

$$\frac{1}{A_{2,E}[\rho](\hbar c)^2}\left(-\nabla^2\right)^{-1} \;\to\; \frac{1}{4\pi A_{2,E}[\rho](\hbar c)^2 r} \tag{A8}$$

The inverse Laplacian is the familiar inverse-distance Green function. The tensor projector and the source normalization supply the conventional coefficient already collected in $G_{\mathrm{comp}}[\rho]$. Local changes inside a finite body can change form factors and multipoles, but they cannot create a second asymptotic $1/r$ pole unless another independently coupled massless channel or another asymptotic state is introduced.

A.3 State arguments and nested baselines

The same stiffness functional is evaluated on physical states admitting the stated static expansion. Whenever the state is generated by explicit parameters, those parameters should be displayed rather than left implicit:

$$A_{2,E}(r; M, T, \dots) \equiv A_{2,E}[\rho(r; M, T, \dots)] \tag{A9}$$

Here r can label position in the matter-source coordinates, M the source or complete-body mass that helps prepare the state, and T temperature. The ellipsis can contain composition, pressure, chemical potential, boundary traction, acceleration history, or any other state variable required by the calculation. Equation (A9) does not make $A_{2,E}$ a new independent function of arbitrary labels; it exposes how those labels define ρ.

The four baselines used in the main paper are evaluations of that one functional:

$$\begin{gathered}
A_{2,E}^{\mathrm{epoch}}(t) \equiv A_{2,E}\left[\rho_{\mathrm{epoch}}(t)\right], \\
A_{2,E}^{\mathrm{ambient}}(x, t \mid S) \equiv A_{2,E}[\rho_{\mathrm{ambient}}(x, t \mid S)], \\
A_{2,E}^{\mathrm{prepared}}(x, t \mid S, B) \equiv A_{2,E}\left[\rho_{\mathrm{prepared}}(x, t \mid S, B)\right], \\
A_{2,E}^{\mathrm{unloaded}}(\tau) \equiv A_{2,E}[\rho_{\mathrm{unloaded}}(\tau)]
\end{gathered} \tag{A10}$$

The epoch state supplies the homogeneous background. The source data S establish a local ambient state relative to that epoch. Boundary data B then prepare a supported, driven, heated, confined, or otherwise loaded state. The unloaded state is the causal result after the additional preparation has been removed and its transient has propagated. These superscripts name evaluations; they are not four unrelated stiffness coefficients.

On a solved common scalar branch, successive baseline ratios compose exactly:

$$\frac{A_{2,E}^{\mathrm{prepared}}}{A_{2,E}^{\mathrm{epoch}}} = \frac{A_{2,E}^{\mathrm{ambient}}}{A_{2,E}^{\mathrm{epoch}}}\frac{A_{2,E}^{\mathrm{prepared}}}{A_{2,E}^{\mathrm{ambient}}}, \qquad \Delta\lambda_{\mathrm{total}} = \Delta\lambda_{\mathrm{source}} + \delta\lambda_{\mathrm{load}} \tag{A11}$$

Equation (A11) is an identity because the numerator of one ratio becomes the denominator of the next. It is also a warning: directional loading remains tensorial until the boundary-value problem has justified a scalar reduction. A source contribution, a local load response, and a signal-transfer factor must never be inserted twice under different names.

A.4 Complete-body boundary term

This check explains why internal pressure and stress do not create a second mass coefficient for a stationary complete body. In local Cartesian coordinates, stationarity and conservation imply

$$\partial_k T^{kj} = 0. \tag{A12}$$

Equation (A12) says that the spatial stresses are in mechanical balance: there is no unbalanced flow of the j component of momentum through the body. It does not say that the local stresses vanish. A compressed region, a tensioned wall, and a binding field may all carry nonzero T^{ij} while their complete distribution remains balanced.

Multiply Equation (A12) by the position x^i and integrate over a volume V containing the body. Integration by parts gives

$$0 = \int_V d^3\, x\, x^i\, \partial_k T^{kj} = \oint_{\partial V} d\, A\, x^i n_k T^{kj} - \int_V d^3\, x\, T^{ij}. \tag{A13}$$

This equation separates the balance into a surface term and a volume term. The surface term measures momentum traction crossing the chosen boundary. It vanishes only if the integration volume encloses every wall, support field, and binding stress needed to make the object complete, and if the exterior stress falls off sufficiently rapidly. Under those conditions Equation (A13) reduces to the von Laue balance condition

$$\int d^3x\, T^{ij} = 0. \tag{A14}$$

Equation (A14) is a whole-body cancellation. Positive pressure in one part of the system is balanced by tension or binding stress elsewhere. It must not be applied to a selected constituent or to an open subsystem whose boundary is carrying uncounted force.

The leading stationary source contains both energy density and the trace of spatial stress,

$$M_{\mathrm{src}} = \frac{1}{c^2} \int d^3x \left(T^{00} + T^{ii}\right). \tag{A15}$$

The repeated spatial index i is summed. Equation (A14) removes the *integrated* stress term from Equation (A15), leaving the complete conserved rest energy divided by c^2. That is the same quantity that supplies inertial mass in main Equations (16)-(25). The result fails if a container wall, binding field, or external traction is omitted, because the omitted part is exactly what completes the stress balance.

A.5 Common scale and no-double-counting convention

On the homogeneous self-similar branch, one Standard-Model spectral scale E_s carries the common dimensional evolution while dimensionless construction data X are held fixed. On that restricted state path ρ_λ, the stiffness, a clock transition, and a ruler length are

$$A_{2,E}[\rho_\lambda] = E_s^2 a_{TT}(X), \qquad \Delta E_a[\rho_\lambda] = E_s f_a(X), \qquad L_a[\rho_\lambda] = \frac{\hbar c}{E_s} \ell_a(X) \tag{A16}$$

The functions a_{TT}, f_a, and ℓ_a contain the dimensionless information about the state and the selected construction. Equation (A16) does not assert equal sensitivity under arbitrary support, shear, heating, or acceleration. Those changes alter X and must be calculated with the full tensor kernels.

The declared covariant comparison branch relates the effective signal scale to the same state scale:

$$\frac{c[\rho_\lambda]}{c[\rho_{\lambda_0}]} = \left(\frac{E_s}{E_{s,0}}\right)^2 \tag{A17}$$

Substitution into Equation (A16) gives the common endpoint ratios

$$\frac{\omega_a[\rho_\lambda]}{\omega_a[\rho_{\lambda_0}]} = \frac{L_a[\rho_\lambda]}{L_a[\rho_{\lambda_0}]} = \frac{E_s}{E_{s,0}} = \left(\frac{A_{2,E}[\rho_\lambda]}{A_{2,E}[\rho_{\lambda_0}]}\right)^{1/2} \tag{A18}$$

Equation (A18) is the common-scale relation used in main Equation (40) and in the cross-epoch cosmology of main Equations (71)-(81). It applies only to the declared homogeneous path. Directional stress, temperature, composition, binding thresholds, and construction-specific response remain in X and in the tensor kernels of main Section 6.

A calibrated observable can finally be separated into three accounting factors:

$$O_{\text{obs}} = O_{\text{local residual}} \times O_{\text{common comparison}} \times O_{\text{signal/protocol}} \tag{A19}$$

Equation (A19) is an accounting identity, not three automatic physical effects. The local residual is the construction-specific change left after the common comparison has been assigned. The common contribution may be represented by the Einstein-equivalent map or by the integrated matter-standard relation. The last factor records the actual signal and receiver protocol. A contribution placed in one factor cannot be inserted again in another.

Appendix B. Loading, Matter Standards, Causal Unloading, and Relativistic Checks

B.1 Detailed loading and equivalence calculation

Main Section 5 states the physical equivalence result compactly. This subsection supplies the balance equations behind it. Begin by treating the body as an open subsystem while an external apparatus acts on it. A bulk four-force density and a boundary stress change the body four-momentum according to

$$\partial_\mu T_{\text{body}}{}^{\mu\nu} = f_{\text{bulk}}{}^{\nu}, \qquad \frac{dP_{\text{body}}{}^{\nu}}{dt} = \int d^3x\, f_{\text{bulk}}{}^{\nu} - \oint_{\partial V} d\,A\, n_j T_{\text{body}}{}^{j\nu} \equiv F_{\text{ext}}{}^{\nu}. \tag{B1}$$

The first term on the right represents force applied throughout the volume. The surface integral represents force transmitted through a floor, tether, pressure, or actuator at the boundary. Their sum is the external force on the chosen subsystem. Equation (B1) explains why specifying only the center-of-mass acceleration is insufficient for an extended body: different force distributions can produce the same total force while preparing different internal stresses.

The local spatial traction exerted across a surface with normal n_j is

$$t^i = n_j T_{\text{SM,tot}}{}^{ji}. \tag{B2}$$

Traction is the force per unit area actually transmitted through the material boundary. When the body, support, actuator, and relevant fields are enlarged into one closed system, their complete Standard-Model tensor is conserved. Equations (B1)-(B2) simply identify how momentum is exchanged between the parts before that enlargement is made.

A convenient dimensionless loading number is

$$\chi_L = \frac{F_L \xi}{E} \simeq \frac{a_L \xi}{c^2} \sim \frac{\sigma_L}{\varepsilon}. \tag{B3}$$

Here E is the energy of the responding domain, ξ its characteristic size or correlation length, a_L the corresponding acceleration scale, σ_L a representative applied stress, and ε the energy density. Equation (B3) estimates whether the load is weak on the selected domain. It is only a diagnostic magnitude; it cannot replace the direction, shear, or boundary information in the full tensor.

In a small stationary laboratory, local force balance takes the familiar continuum form

$$\partial_j \sigma^{ij} + f_{\text{eff}^i} = 0, \qquad f_{\text{eff}^i} \simeq \frac{\varepsilon}{c^2} a_{\text{eff}^i}. \tag{B4}$$

The divergence of the material stress balances the effective force density. The factor ε/c^2 is the local mass density associated with the complete energy density. Equation (B4) is the bridge between the acceleration applied to the body and the internal stress state that its clocks and rulers actually experience.

For a vertical column of height H whose upper surface is free, the same equation becomes

$$\frac{d\sigma_{zz}}{dz} = -\rho(z) a_{\text{eff}}, \qquad \sigma_{zz}(z) = \int_z^H d\, z' \,\rho(z') a_{\text{eff}}. \tag{B5}$$

Each lower layer carries the load of the material above it. A column supported from below is compressed; the same column suspended from above is placed in tension, so the sign and boundary conditions change even when the acceleration magnitude is the same. This elementary example is why OQG equivalence is stated as equality of the *complete prepared state*, not merely equality of g and a at one point.

Let the superscripts (g) and (a) label a supported gravitational laboratory and a nongravitationally accelerated laboratory. If their density operators and complete stress tensors agree point by point,

$$\rho_{\text{SM}^{(g)}}(x) = \rho_{\text{SM}^{(a)}}(x), \qquad \left(T_{\text{SM,tot}^{\mu\nu}}(x)\right)_g = \left(T_{\text{SM,tot}^{\mu\nu}}(x)\right)_a, \tag{B6}$$

then every response calculated from that state must also agree:

$$\left(\Pi_{\text{comp}^R}\right)_g = \left(\Pi_{\text{comp}^R}\right)_a, \qquad \left(\Pi_{\text{comp,E}}\right)_g = \left(\Pi_{\text{comp,E}}\right)_a, \qquad (D^R, \mathcal{R}_{\text{matter}})_g = (D^R, \mathcal{R}_{\text{matter}})_a. \tag{B7}$$

Equation (B6) states the physical matching condition; Equation (B7) states its unavoidable consequence. The retarded kernel, Euclidean stiffness, compliance, and clock/ruler response are functionals of the prepared Standard-Model state. Equal inputs therefore give equal outputs. This is the extended-body content of the equivalence principle used in main Equations (26)-(30).

B.2 Construction-specific clock and ruler response

Subsection B.1 established what must be matched: the complete physical loading of the body. This section identifies what is measured. A clock is a repeatable transition or oscillation, and a ruler is an equilibrium separation or correlation length. Neither is an abstract coordinate interval. The engineering procedure is to specify the clock transition, specify the ruler's equilibrium condition, prepare the complete applied stress, and calculate the resulting change through the material response.

Different clock and ruler constructions need not have identical sensitivities to temperature, pressure, shear, or applied force. They nevertheless respond within the same completed Standard-Model state. Mechanical strain, binding stress, and environmental sensitivity are already part of that state; none can be omitted and then added again as an independent OQG contraction.

For clock construction a, let $E_{a,+}[\rho]$ and $E_{a,-}[\rho]$ be the two energy levels used to count cycles in state ρ. Its intrinsic angular frequency is

$$\omega_a[\rho] = \frac{E_{a,+}[\rho] - E_{a,-}[\rho]}{\hbar}. \tag{B8}$$

Equation (B8) says exactly what the clock is: a selected Standard-Model energy difference expressed as a cycle rate. When loading changes the complete stress tensor, the corresponding first-order clock shift can be written

$$\delta \ln \omega_a(x) = \int d^4y\, \mathcal{K}^R_{a,\mu\nu}(x,y)\, \delta T_{\mathrm{SM,tot}}{}^{\mu\nu}(y). \tag{B9}$$

The retarded clock kernel $\mathcal{K}^R_{a,\mu\nu}$ tells how strongly that particular transition responds to each component of applied stress and how quickly the response arrives. Its microscopic construction combines the transition's stress sensitivity, obtained through the Feynman-Hellmann relation [11], with the completed causal compliance D^R from Appendix A. An actual calculation or calibration must therefore supply the chosen transition, its state, and its stress-sensitivity coefficients. Equation (B9) is not a universal numerical shift for every clock.

A ruler is treated just as concretely. Let $U_a(R, \hat{n}; \rho)$ be the complete binding or free energy of two fiducial features separated by R in direction $\hat{n}$. The equilibrium length L_a occurs at the minimum of that energy, and a small load moves the minimum according to

$$\left.\frac{\partial U_a}{\partial R}\right|_{R=L_a} = 0, \qquad \delta L_a(\hat{n}) = -\left(\left.\frac{\partial^2 U_a}{\partial R^2}\right|_{L_a}\right)^{-1} \left.\frac{\partial\, \delta U_a}{\partial R}\right|_{L_a}, \qquad \frac{\delta L(\hat{n})}{L} = n_i n_j \epsilon_{ij}. \tag{B10}$$

The curvature $\partial^2 U_a / \partial R^2$ is the ruler's restoring stiffness. A shallow minimum moves more than a steep one under the same perturbation. The final expression projects the material strain tensor ϵ_{ij} onto the ruler's direction, making directional response explicit.

The corresponding causal ruler response is

$$\epsilon_{ij}(x) = \int d^4y\, C^R_{ij,\mu\nu}(x,y)\, \delta T_{\mathrm{SM,tot}}{}^{\mu\nu}(y). \tag{B11}$$

The material kernel $C^R_{ij,\mu\nu}$ contains the binding-energy response, elastic compliance, pressure sensitivity, polarizability, and any other environmental dependence of the selected ruler. A quantitative experiment must supply or measure those material coefficients. The kernel does not replace ordinary elasticity; it organizes those Standard-Model effects into one causal tensor response.

For a material that is initially isotropic in its local ambient equilibrium state, the static ruler response to an additional load separates naturally into a volume-like part and a directional shear part:

$$\epsilon_{ij} = C_0 \frac{\delta T_{kk}}{3} \delta_{ij} + C_2 \left(\delta T_{ij} - \frac{\delta T_{kk}}{3} \delta_{ij}\right). \tag{B12}$$

The coefficient C_0 measures the response to the spatial trace, which changes all directions equally. The coefficient C_2 measures the response to the traceless stress, which distinguishes one direction from another. The local ambient material may therefore be isotropic even though a directional applied force produces an anisotropic deviation from it.

For a uniaxial stress $\delta T_{ij} = \sigma m_i m_j$, the lengths parallel and perpendicular to the applied direction change by

$$\frac{\delta L_\parallel}{L} = \frac{\sigma}{3}(C_0 + 2C_2), \qquad \frac{\delta L_\perp}{L} = \frac{\sigma}{3}(C_0 - C_2). \tag{B13}$$

Equation (B13) makes an important point visible without additional algebra. Longitudinal-only contraction is not automatic. It occurs only for the special material condition $C_0 = C_2$; otherwise a transverse response accompanies the longitudinal one. Directionality comes from the applied stress and the tensor compliance, not from assigning a preferred direction to the local ambient state.

A clock can be decomposed by the same physical logic. Its first-order local response may depend separately on energy density, volume-like stress, and directional stress:

$$\delta \ln \omega_a = K_{a,E}\, \delta T^{00} + K_{a,0} \frac{\delta T_{kk}}{3} + K_{a,2} Q_a^{ij} \left(\delta T_{ij} - \frac{\delta T_{kk}}{3} \delta_{ij} \right). \tag{B14}$$

The tensor Q_a^{ij} describes the orientation sensitivity of the selected transition. The coefficients $K_{a,E}$, $K_{a,0}$, and $K_{a,2}$ are construction dependent; atomic, molecular, optical, nuclear, and mechanical clocks need not assign the same residual weight to each stress component.

When a supported laboratory and an accelerated laboratory have the same applicable ambient state and identical complete loading profiles, the same kernels act on the same tensor deviation. Their clock and ruler responses relative to that ambient state are therefore equal:

$$\left. \frac{\omega_a}{\omega_{a,amb}} \right|_g = \left. \frac{\omega_a}{\omega_{a,amb}} \right|_a, \qquad \left. \frac{L(\hat{n})}{L_{amb}} \right|_g = \left. \frac{L(\hat{n})}{L_{amb}} \right|_a \quad \text{for every } \hat{n}. \tag{B15}$$

Equation (B15) is the observable consequence of the loading equivalence derived in Subsection B.1. Here the subscript amb denotes the same local ambient clock or ruler reference for the two matched preparations. The equation applies to the same construction in matched ambient and loaded states. It does not claim that all clocks or all materials have identical residual sensitivities, or that equal proper acceleration removes a difference between two genuinely different ambient source states.

B.3 Causal unloading, tides, and retained history

Support release removes a prepared boundary condition; it does not delete the source-established ambient state. The uniform support terms vanish only after the unloading disturbance has crossed the body:

$$a_{\mathrm{CM,ff}^\mu} = 0, \qquad t_{\mathrm{support}^i} = 0, \qquad \Delta T_{\mathrm{uniform\ support}^{\mu\nu}} = 0. \tag{B16}$$

The late statement in Equation (B16) must not be read as instantaneous action across an extended object. The actual state passes through a causal transient,

$$T_{\mathrm{supported}^{\mu\nu}} \overset{\text{release}}{\rightarrow} T_{\mathrm{transient}^{\mu\nu}}(t) \overset{t \gg \tau_{\mathrm{relax}}}{\rightarrow} T_{\mathrm{ff}^{\mu\nu}}. \tag{B17}$$

The relaxation time τ_{relax} includes elastic-wave travel, internal oscillation, and damping. If the release creates heat, damage, radiation, or permanent deformation, those changes remain in the final density operator even though the external support has vanished.

After the transient has relaxed, the most general local tensor needed at this order is

$$\begin{aligned} T_{\mathrm{late}^{\mu\nu}} &= T^{\mu\nu}_{ambient} + \Delta T_{\mathrm{tide}^{\mu\nu}} + \Delta T_{\mathrm{hist}^{\mu\nu}}, \\ \Delta T_{\mathrm{hist}^{\mu\nu}} &= 0 \quad \text{on the ideal reversible branch.} \end{aligned} \tag{B18}$$

The reference tensor is the complete equilibrium state established locally by the specified source within the current epoch, not an empty tensor and not a universal state at infinity. The tidal term is the finite-size response to the source gradient. The history term retains any heat, excitation, radiation loss, plastic strain, fracture, or other irreversible change produced before or during release. Unloading removes the applied support; it does not erase the ambient response or a Standard-Model state that has genuinely changed.

Across a finite freely falling laboratory, the first gravitational load that cannot be removed at the center is the field gradient. To first order in position,

$$\Delta a^i(\mathbf{x}) = \mathcal{T}_{ij} x^j + O(|\mathbf{x}|^2), \qquad \mathcal{T}_{ij} \equiv \left.\frac{\partial a^i}{\partial x^j}\right|_0. \tag{B19}$$

The tensor $\mathcal{T}_{ij}$ is the tidal gradient. Its opposite signs in different directions can stretch one axis while compressing another. Two useful control parameters are

$$\epsilon_{\text{grad}} \sim \frac{\|\mathcal{T}\|\ell}{a_{\text{eff}}} \ll 1, \qquad \chi_{\text{tide}} \sim \frac{\|\mathcal{T}\|\ell^2}{c^2}. \tag{B20}$$

The first number asks whether the acceleration varies little across a supported laboratory of size ℓ; when it is small, the uniform-field approximation is reliable. The second asks how strong the irreducible tidal loading is on a freely falling domain. Equations (B1)-(B20) establish the detailed result used in main Sections 5-7: a force is a force, but extended-body equivalence requires the same force distribution, traction, boundary conditions, and complete Standard-Model state. Equal center-of-mass acceleration alone is not enough.

Choose local matter coordinates X^i centered on the body center of energy. For a bound system with local energy density $\varepsilon(X)$, the leading tidal force density and quasistatic stress balance are

$$f_{\text{tide},i}(X) = \frac{\varepsilon(X)}{c^2} \mathcal{T}_{ij} X^j, \qquad \partial_j \sigma_{ij} + f_{\text{tide},i} = 0. \tag{B21}$$

Equation (B21) says that binding stress must oppose the differential force if the body is to remain intact. A freely falling dust cloud can change its separations without supporting that stress; a solid, molecule, or atom must develop a response because its binding interactions resist the relative motion.

The force monopole vanishes at the center of energy, but the first spatial moment remains:

$$\int d^3X\, f_{\text{tide},i} = 0, \qquad \int d^3X\, X^k f_{\text{tide},i} = \frac{1}{c^2} \mathcal{T}_{ij} I_E^{kj}, \qquad I_E^{kj} \equiv \int d^3X\, \varepsilon X^k X^j. \tag{B22}$$

The first relation explains why the center remains in free fall. The second identifies a nonzero force dipole that can stretch, compress, or torque the internal distribution. The energy moment I_E^{kj} plays the role of a size-and-shape tensor weighted by the complete energy density.

For characteristic size ℓ, the order of the induced stress and the dimensionless control parameter are

$$\sigma_{\text{tide}} \sim \frac{\varepsilon}{c^2} \parallel \mathcal{T} \parallel \ell^2, \qquad \chi_{\text{tide}} \equiv \frac{\|\mathcal{T}\|\ell^2}{c^2} \tag{B23}$$

The dimensionless number χ_{tide} controls the expansion of the local two-point response. It is not a universal fracture threshold. Ordinary binding and yield scales can be far below the complete relativistic energy density, so a macroscopic material may fail while $\chi_{\text{tide}} \ll 1$.

When the gradient changes with time or excites internal modes, the response is causal:

$$\delta T_{\mathrm{tide}^{\mu\nu}}(x)=\int d^4y\,\mathcal{G}_i^{R\,\mu\nu}(x,y)f_{\mathrm{tide},i}(y),\qquad \mathcal{G}^R(x,y)=0\quad \text{for } x^0<y^0. \tag{B24}$$

Equation (B24) is the dynamical form of the balance: the force gradient excites the body's completed Standard-Model response only after the disturbance arrives. Resonance, damping, heat, and damage belong to this retarded material evolution. Main Equation (47) then converts the completed tidal stress into the response of a specified clock or ruler.

The completed tidal tensor is then passed through the same construction-specific kernels used for any other load:

$$\Delta T_{\mathrm{tide}^{\mu\nu}}\to\begin{pmatrix}\delta\ln\omega_a\\ \epsilon_{\hat{\imath}\hat{\jmath}}\end{pmatrix}_{\mathrm{tide}}=\begin{pmatrix}\mathcal{K}^R_{a,\mu\nu}\\ \mathcal{C}^R_{\hat{\imath}\hat{\jmath},\mu\nu}\end{pmatrix}*\Delta T_{\mathrm{tide}^{\mu\nu}}. \tag{B25}$$

Equation (B25) does not assign one universal tidal strain to every material. The response depends on the density profile, binding strength, construction, exposure time, resonances, and boundary conditions. It also prevents the tidal strain from being counted once through the completed tensor response and then added again as a separate mechanical correction.

For two otherwise identical devices compared at the same location after relative-motion and signal effects have been removed, the source-established ambient baseline cancels. The released device approaches that reference, whereas the supported device retains the additional support response:

$$\begin{aligned}\Delta_{\mathrm{drop-supp}}\ln\omega_a &\quad\to-\delta\ln\omega_{a,\mathrm{supp}}+O(\chi_{\mathrm{tide}},\epsilon_{\mathrm{tr}},\delta_{\mathrm{hist}}),\\ \Delta_{\mathrm{drop-supp}}\ln L_a(\hat{n}) &\quad\to-\delta\ln L_{a,\mathrm{supp}}(\hat{n})+O(\chi_{\mathrm{tide}},\epsilon_{\mathrm{tr}},\delta_{\mathrm{hist}}).\end{aligned} \tag{B26}$$

Equation (B26) is construction specific, and its sign becomes definite only after the support response of that clock or ruler has been calculated or calibrated. It does not predict a universal release jump equal to a complete source-to-reference comparison factor. On a branch where the additional support preparation slows the chosen clock and shortens the chosen ruler, the freely falling device differs from its supported counterpart by that residual amount before impact or capture applies a new load. The common local ambient baseline remains present in both states and is not counted again.

The complete sequence is now explicit. The epoch supplies the underlying baseline; the source establishes the local ambient state; support adds a prepared tensor deviation; release removes that deviation through a retarded Standard-Model transient; and the late body retains the local ambient state, finite tides, and any genuine irreversible history.

B.4 Signal transfer and protocol accounting

Subsection B.2 defined the intrinsic response of a physical clock or ruler to its prepared Standard-Model state. A remote observation adds other operations that must be kept distinct. First, the emitter's local state determines the frequency or length it physically produces. Second, the clock or ruler history determines any accumulated phase or stored record. Third, a signal transfers information between locations. Finally, the receiver applies a specified comparison protocol using its own local standards. These four contributions form one measurement; none is a second physical deformation of the matter.

The distinction matters because the Einstein-equivalent metric already summarizes the completed long-wavelength comparison. Once that representation has supplied the signal or common relativistic factor, the same factor cannot also be inserted as an additional local change of the emitter or receiver. Construction-specific shifts caused by temperature, pressure, loading, composition, or damage remain genuine local matter effects and are retained separately.

For a clock of construction a, the local state ρ_e fixes the rate at which its phase is produced:

$$\frac{d\phi_a}{d\tau_e} = \omega_a[\rho_e]. \tag{B27}$$

The variable τ_e is the time counted locally by the emitter. Equation (B27) concerns the clock mechanism itself. It says nothing yet about how rapidly the emitted cycles arrive somewhere else.

Signal transfer is described by the same phase viewed along its propagation path. If k_μ is the gradient of the signal phase and u_e^μ and u_r^μ are the emitter and receiver four-velocities, the two endpoints measure

$$\omega_{\text{emit}} = -\left(u_e^\mu k_\mu\right)_e, \qquad \omega_{\text{rec}} = -\left(u_r^\mu k_\mu\right)_r. \tag{B28}$$

The contraction $u^\mu k_\mu$ converts one propagating phase into the frequency counted by a particular observer. The null-path description is the Einstein-equivalent infrared bookkeeping map; it does not assert that a geometrical medium is the microscopic carrier.

After an explicit reference state ρ_{ref} has been selected, define the intrinsic matter factor of clock a by $\mathcal{M}_a[\rho] \equiv \omega_a[\rho]/\omega_a\left[\rho_{ref}\right]$. For a coherent carrier in a calibrated propagation mode, the completed comparison is

$$\frac{\omega_{\text{rec}}}{\omega_a[\rho_r]} = \frac{\mathcal{M}_a[\rho_e]}{\mathcal{M}_a[\rho_r]}\,\mathcal{P}_{\text{sig}}, \qquad \mathcal{P}_{\text{sig}} \equiv \frac{(u_r\cdot k)_r}{(u_e\cdot k)_e}. \tag{B29}$$

Equation (B29) has a direct physical reading. The first ratio compares the two endpoint clock constructions in their actual local states. The second factor describes one calibrated signal transfer between them. Absorption, dispersion, or instrumental delay must either be negligible or included in that calibration. A source-established ambient comparison may be represented through the endpoint matter-standard ratio or through the Einstein-equivalent propagation map. Those are two representations of the same common contribution and must not be multiplied together. After that common contribution has been assigned once, $\mathcal{M}_a$ contains only construction-specific residual loading or state effects not already represented by $\mathcal{P}_{sig}$.

Collinear recession provides a familiar example. For constant relative speed v in flat propagation,

$$\mathcal{P}_{1\text{w}} = \sqrt{\frac{1-\beta}{1+\beta}}, \qquad \beta = \frac{v}{c}, \qquad \frac{\omega_{\text{return}}}{\omega_{\text{send}}} = \mathcal{P}_{1\text{w}^2} = \frac{1-\beta}{1+\beta}. \tag{B30}$$

The one-way factor gives the complete relativistic arrival-rate comparison for that protocol. A coherent reflection or calibrated transponder provides the round-trip relation shown in the same equation. Once the transfer has been calibrated, Equation (B29) isolates any residual difference between the endpoint clock states without introducing another velocity factor.

Present clock rate and accumulated clock history are different observables. A clock carried along a physical state history $\mathcal{H}_a$ records

$$N_a = \frac{1}{2\pi} \int_{\mathcal{H}_a} \omega_a \, [\rho_a] \, d\tau_a, \qquad \Delta N = N_A - N_B. \tag{B31}$$

The number N_a is the accumulated cycle count. Two clocks may be reunited at the same location, unloaded to the same local ambient state, and restored to the same present rate while retaining a nonzero ΔN, because their earlier histories were different. No continuing remote signal is required to read that stored phase difference.

Ruler measurements require the same separation between the object and the protocol. If $L_{\mathrm{mat}}(\hat{n}; \rho)$ is the ruler's intrinsic equilibrium length and $\mathcal{P}_L$ is the transfer factor defined by a particular measurement,

$$L_{\mathrm{inf}} = \mathcal{P}_L[\text{protocol}, u_e, u_r, k, \hat{n}] \, L_{\mathrm{mat}}(\hat{n}; \rho). \tag{B32}$$

A photograph, radar range, simultaneous endpoint assignment, and direct comoving comparison do not use the same $\mathcal{P}_L$. A local comparison has $\mathcal{P}_L = 1$; a radar measurement uses the receiver's clock and calibrated signal speed; an equal-time assignment compares two endpoint events selected by the chosen frame. The inferred length must therefore identify its protocol before it can be compared with the ruler's local material state.

The complete accounting can be summarized without assigning any effect twice:

$$\begin{gathered} \text{prepared Standard-Model state} \\ \rightarrow \text{intrinsic clock or ruler response and accumulated phase} \\ \rightarrow \text{one calibrated signal transfer} \rightarrow \text{receiver protocol} \rightarrow \mathcal{O}_{\mathrm{observed}}. \end{gathered} \tag{B33}$$

This organization preserves the tested mathematics of frequency shift, radar distance, accumulated phase, and relativistic endpoint comparison. It also identifies the cleanest experimental controls. Co-located comparisons set the transfer factor to unity, coherent round trips calibrate the transfer independently, and reunited clocks compare accumulated histories after both instruments have returned to the same local state.

B.5 Force, work, inertia, and the special-relativistic branch

The purpose of this section is to separate what physically happens while a force acts from what remains in a later relativistic comparison. The clean special-relativistic construction is first stated in free space, where the applicable ambient state is the current-epoch baseline. An external apparatus transfers energy and momentum to matter. Its local force distribution creates transient tensor stress, strain, heating, and clock response, while the total four-momentum determines the work required to reach a new velocity. After an ideal reversible acceleration ends and the body unloads, its local clocks and rulers return to that applicable ambient inertial state. The accumulated clock phase, momentum, and equal-time endpoint comparisons still retain the standard Lorentz relations because the completed histories differ.

The two descriptions are therefore complementary. The work-and-loading ledger describes the physical deviation while acceleration is transmitted. The phase-and-comparison ledger describes the relational result after the body has returned to its local ambient inertial state. Constant velocity is not treated as a continuing material load. Near a gravitating source, the same inertial calculation describes a further preparation relative to the source-established ambient baseline rather than replacing that baseline.

For the clean reversible branch, the body's invariant mass M_0 is unchanged. Standard-Model Poincare symmetry gives the energy-momentum relation

$$E^2 = p^2c^2 + M_0^2c^4. \tag{B34}$$

Using the group velocity $\mathbf{v} = \partial E / \partial \mathbf{p}$, Equation (B34) gives

$$E = \gamma M_0 c^2, \qquad \mathbf{p} = \gamma M_0 \mathbf{v}, \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{B35}$$

These quantities describe the complete body's energy and momentum relative to the original laboratory. They do not say that its invariant rest mass has become γM_0.

The energy must be supplied physically. Force changes momentum, power transfers energy, and integration gives the work accumulated from rest:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}, \qquad \frac{dE}{dt} = \mathbf{F} \cdot \mathbf{v}, \qquad W = \int \mathbf{F} \cdot d\mathbf{x} = (\gamma - 1) M_0 c^2. \tag{B36}$$

If the acceleration also leaves heat, excitation, radiation loss, or permanent deformation, those channels belong in the energy ledger and change the final invariant mass according to main Equation (25). Equation (B36) is the reversible result after those residual changes have been excluded.

The resistance to an additional velocity change is directional. Differentiating momentum gives

$$\mathcal{M}^{\text{in}}{}_{ij} \equiv \frac{\partial p_i}{\partial v_j} = \gamma M_0 \left(\delta_{ij} + \gamma^2 \frac{v_i v_j}{c^2} \right), \qquad M_{\parallel \text{in}} = \gamma^3 M_0, \qquad M_{\perp \text{in}} = \gamma M_0. \tag{B37}$$

The longitudinal and transverse quantities are slopes of momentum with respect to an additional velocity change. They express the growing force required to keep accelerating; they are not three different rest masses.

For a finite longitudinal force,

$$v \to c \quad \Rightarrow \quad E, W, M_{\parallel \text{in}} \to \infty, \qquad a_\parallel = \frac{F_\parallel}{\gamma^3 M_0} \to 0. \tag{B38}$$

The limiting speed is therefore approached only through unbounded accumulated work. This result follows from the Standard-Model energy-momentum relation before any clock or ruler comparison is made.

It is useful to normalize each work increment to the body's instantaneous energy. The accumulated coordinate and its reduced comparison factor are

$$d\Lambda_E \equiv \frac{dE}{E} = \frac{\mathbf{F} \cdot d\mathbf{x}}{E}, \qquad \Lambda_E(v) = \ln \frac{E}{E_0} = \ln \gamma, \qquad b_{\text{SR}} \equiv e^{-\Lambda_E} = \frac{1}{\gamma}. \tag{B39}$$

The scalar b_{SR} is a completed comparison ratio, not a replacement for the tensor loading while the force acts. Its clock meaning follows from Standard-Model phase. Along a stable mass eigenstate,

$$d\varphi = \frac{1}{\hbar} p_\mu dx^\mu = -\frac{M_0 c^2}{\hbar} d\tau, \qquad \frac{d\tau}{dt} = \frac{M_0 c^2}{E} = \frac{1}{\gamma} = b_{\text{SR}}. \tag{B40}$$

For two nearby internal energy levels transported along the same center trajectory, the difference of Equation (B40) gives the familiar accumulated clock-phase ratio. This is a signal-free comparison between phases accumulated over different event histories. If a force is still acting and also shifts the intrinsic transition, that construction-specific shift is included once through the Subsection B.2 clock kernel.

The corresponding ruler relation follows from the boost transformation of the endpoint events. For relaxed comoving separations $\Delta X_\parallel, \Delta X_\perp$,

$$\Delta X_{\parallel} = \gamma(\Delta x_{\parallel} - v\Delta t), \qquad \Delta X_{\perp} = \Delta x_{\perp}, \qquad \left.\frac{L_{\parallel}}{L_0}\right|_{\Delta t=0} = \frac{1}{\gamma}, \qquad \left.\frac{L_{\perp}}{L_0}\right|_{\Delta t=0} = 1. \tag{B41}$$

The final two ratios use endpoint events chosen at equal time in the original laboratory. They are protocol-defined correlations, not a claim that the unloaded comoving ruler has acquired a permanent local deformation.

The distinction between loading and comparison can now be stated in one causal sequence:

$$\begin{aligned} \Delta T_L^{\mu\nu} \neq 0 \quad &\Rightarrow \quad \text{intrinsic clock and ruler response} \\ &\qquad \text{while force is transmitted,} \\ \Delta T_L^{\mu\nu} \to 0 \quad &\Rightarrow \quad \omega_{a,\mathrm{local}} \to \omega_{a,amb}, \quad L_{\mathrm{mat}}(\hat{n}) \to L_{amb}, \\ &\qquad (E, \mathbf{p}) = (\gamma M_0 c^2, \gamma M_0 \mathbf{v}). \end{aligned} \tag{B42}$$

Residual heat, damage, excitation, or radiation would remain in the final state and must be included separately. In Equation (B42), $\omega_{a,amb}$ and L_{amb} are the local ambient standards applicable after the force has been removed. On the ideal branch, local standards return to those ambient values, while accumulated phase and equal-laboratory-time comparisons retain $b_{SR} = 1/\gamma$.

The accomplishment is the ordinary special-relativistic branch recovered from Standard-Model energy, momentum, work, phase, and boost covariance. Physical acceleration requires force, performs work, and loads the body while that force is transmitted. Inertial coasting after unloading requires no continuing local deformation relative to the applicable ambient state. In free space that state approaches the current-epoch baseline; near a source it is the source-modified local baseline. The same framework nevertheless retains Einstein's tested phase, momentum, limiting-speed, and ruler-comparison mathematics.

When support, drive, heating, confinement, and other forces act together, they prepare one complete state before any scalar comparison is made:

$$\begin{aligned} \rho_{\mathrm{ambient}} \xrightarrow{f_{\mathrm{support}} + f_{\mathrm{drive}} + f_{\mathrm{other}},\, t_{\mathrm{tot}}} \rho_{\mathrm{tot}} \to T^{\mu\nu}_{\mathrm{SM,tot}}[\rho_{\mathrm{tot}}], \\ b_{\mathrm{tot}}[\rho_{\mathrm{tot}}; B, P] \neq b_{\mathrm{support}} b_{\mathrm{drive}} b_{\mathrm{other}} \quad \text{in general} \end{aligned} \tag{B43}$$

The boundary conditions and measurement protocol are part of the reduction. Equation (B43) is the final safeguard against double counting: named causes are not independently multiplied after they have already combined in the complete tensor state. Additivity or factorization is allowed only when the solved boundary-value problem derives it.

Appendix C. Homogeneous State Evolution and Cosmology Checks

C.1 WTDiff integration datum and vacuum-shift blindness

The first check separates two statements that are often conflated: a constant vacuum offset does not act as a local connected source, while the global integration constant must still be selected by a boundary condition. The fixed-measure WTDiff completion begins with the trace-free infrared equation

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = \kappa \left(T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T \right), \qquad \kappa(t) \equiv \frac{8\pi G_{\mathrm{comp}}[\rho_{\mathrm{epoch}}(t)]}{c^4(t)} \tag{C1}$$

Subtracting one quarter of the trace removes the part proportional to $g_{\mu\nu}$ from each side. The left-hand side is the Einstein-equivalent bookkeeping tensor with its trace removed; the right-hand side contains only the trace-free part of the complete Standard-Model stress. On the declared covariant branch, c^4/G_{comp} and therefore κ are constant.

Take the covariant divergence of Equation (C1). Conservation of the complete stress tensor and the contracted Bianchi identity give

$$\nabla_\nu (R + \kappa T) = 0. \tag{C2}$$

Equation (C2) says that the scalar combination $R + \kappa T$ cannot vary from point to point on one connected settled branch. It is therefore an integration datum rather than a local dynamical field. Write its constant value as

$$R + \kappa T = 4\Lambda_{\mathrm{int}}. \tag{C3}$$

The factor four is conventional in four spacetime dimensions. Substitution of Equation (C3) back into the trace-free equation restores the Einstein form

$$G_{\mu\nu} + \Lambda_{\mathrm{int}} g_{\mu\nu} = \kappa T_{\mu\nu}. \tag{C4}$$

Equation (C4) has the tested tensor structure of Einstein gravity. In OQG it is the long-wavelength comparison equation produced by the completed Standard-Model response. The constant Λ_{int} appears because the trace-free equation had to be integrated; it is not generated by a local vacuum loop in this step.

Now add a state-independent constant B to the stress tensor:

$$T_{\mu\nu} \to T_{\mu\nu} + B g_{\mu\nu}, \qquad T \to T + 4B. \tag{C5}$$

In Equation (C1), the added terms cancel exactly: $B g_{\mu\nu} - (1/4) g_{\mu\nu}(4B) = 0$. A constant shift therefore cannot drive a local connected response on the trace-free branch. In the Einstein-equivalent form, changing how the same equation is split between "source" and "integration constant" amounts to

$$\Lambda_{\mathrm{int}} \to \Lambda_{\mathrm{int}} + \kappa B. \tag{C6}$$

Equation (C6) is a relabeling of the split, not a measurable local force caused by B. Vacuum-shift blindness removes the constant offset from the local source, but the mathematics alone does not force the global integration datum to vanish. The physical branch used throughout the paper is selected separately by the minimal no-external-curvature boundary condition

$$\boxed{\Lambda_{\mathrm{int}} = 0.} \tag{C7}$$

No nonzero value is fitted to chronometers, DESI, or the RC response. Equations (C1)-(C7) are the detailed justification for main Equations (10)-(11): the Einstein-equivalent tensor law survives, the constant vacuum offset is locally invisible, and the selected cosmological branch contains no physical cosmological term.

C.2 Fixed-state-data enthalpy tangent

The next check identifies the Standard-Model quantity that responds to an infinitesimal homogeneous change of scale. Define

$$\lambda \equiv \ln \frac{E_s}{E_{s,0}}, \qquad \mathfrak{A} \equiv e^{-\lambda}, \qquad d\varepsilon = (\varepsilon + p)\frac{dn}{n}, \qquad n \propto \mathfrak{A}^{-3} = e^{3\lambda}. \tag{C8}$$

Here E_s is the common state scale, $\mathfrak{A}$ is its inverse comparison scale, and n is the number density of a conserved matter-label cell. The thermodynamic identity states that compressing or dilating that cell changes its energy density through $\varepsilon + p$, not through energy density alone. Because three spatial dimensions make n proportional to $e^{3\lambda}$, differentiation at fixed dimensionless state data Y gives

$$\left(\frac{\partial \varepsilon}{\partial \lambda}\right)_Y = 3(\varepsilon + p) = 3h. \tag{C9}$$

The enthalpy density h is therefore the homogeneous scale tangent. Equation (C9) does not insert a new fluid; it identifies which part of the existing Standard-Model state changes when the common scale coordinate changes while composition and dimensionless ratios are held fixed.

For a locally equilibrated extensive state, the same enthalpy can be written

$$h = \varepsilon + p = Ts + \sum_a \mu_a n_a. \tag{C10}$$

The right-hand side separates thermal entropy and chemical or compositional contributions. A state-independent vacuum offset cancels from $\varepsilon + p$, but temperature, particle abundances, binding, and chemical potentials remain. This is why cooling and bound-state formation can change the operative clocks and rulers even though a constant vacuum energy cannot drive the local tangent.

The actual cosmic trajectory need not hold Y fixed. Its total derivative is

$$\frac{d\varepsilon}{d\lambda} = 3h + \sum_I \frac{\partial \varepsilon}{\partial Y_I}\frac{dY_I}{d\lambda}. \tag{C11}$$

The first term is the common homogeneous tangent; the sum records changes of composition, thresholds, state populations, binding, correlations, and spectral shape. Equation (C11) separates the exact common-scale contribution at fixed Y from dimensionless state evolution. In the cosmological reconstruction, that evolution enters the residual response and effective time constant. It does not change the covariant scaling weights on the declared common-scale branch.

C.3 Conserved power ledger, stiffness-defined capacity, and the effective RC time

Main Section 10 distinguishes total conserved energy from the inventory still available to support the earlier equilibrium scale. On the declared common covariant branch, the driving energy carries one power of the common scale and the epoch TT stiffness carries two. Dividing each quantity by its reference value therefore gives the exact scaling relation:

$$\frac{E_{\text{drive}}(t)}{E_{\text{drive}}(t_0)} = \left(\frac{A_{2,E}[\rho_{\text{epoch}}(t)]}{A_{2,E}[\rho_{\text{epoch}}(t_0)]}\right)^{1/2} \tag{C12}$$

Equation (C12) follows from covariant scaling. Here E_{drive} denotes the common scale-supporting available energy inventory, rather than the total energy of the universe. Reference normalization fixes the constant of proportionality; the covariant weights fix the ratio throughout the declared branch. The relaxation is entropic in nature within this OQG description: irreversible processes reduce availability while conserving total energy. The effective time constant and its microscopic origin are separate dynamical quantities.

The availability-destruction power, contraction rate, and effective response time are then

$$\begin{aligned} P_{\text{loss}}(t) \quad &\equiv -\dot{E}_{\text{drive}}(t) = T_{\text{ref}}(t)\dot{S}_{\text{prod}}(t) \geq 0, \qquad \dot{E}_{\text{tot}} = 0, \\ H_L(t) \quad &\equiv -\frac{\dot{L}}{L} = \frac{P_{\text{loss}}}{E_{\text{drive}}} = -\frac{1}{2}\frac{d}{dt}\ln A_{2,E}[\rho_{\text{epoch}}(t)], \\ \tau_{\text{eff}} \quad &= \frac{E_{\text{drive}}}{P_{\text{loss}}} = \frac{1}{H_L} = R_{\text{eff}}C_{\text{eff}} \end{aligned} \tag{C13}$$

The cosmological reconstruction determines the effective RC product. OQG also fixes the canonical scale dependence of its capacity-like factor. On the homogeneous self-similar branch, the completed stiffness carries two powers of the common Standard-Model spectral scale, while a charge-normalized capacity-like response carries the inverse first power. The normalized capacity therefore follows directly from the already-derived stiffness:

$$\frac{C_A(t)}{C_A(t_0)} = \left(\frac{A_{2,E}[\rho_{\text{epoch}}(t_0)]}{A_{2,E}[\rho_{\text{epoch}}(t)]}\right)^{1/2} \tag{C13a}$$

Writing the effective response time as a product then isolates the remaining dimensionless entropy-production and transport response:

$$\frac{\tau_{\text{eff}}(t)}{\tau_{\text{eff}}(t_0)} = \frac{R_A(t)}{R_A(t_0)}\frac{C_A(t)}{C_A(t_0)} \tag{C13b}$$

Equation (C13a) fixes the canonical capacity scaling but not its absolute normalization. Equation (C13b) therefore separates what is already determined by the OQG stiffness from the residual dimensionless Standard-Model entropy-production and transport response. Occupations, binding, thresholds, correlations, transport, and departure from detailed balance belong in that residual factor. The observational reconstruction determines the product, while the theory fixes its canonical scale split. The rate history is reconstructed from observations; the microscopic channels and coefficients that generate it are not calculated here.

For a spatially resolved nonequilibrium state, let $\sigma_S(\mathbf{x}, t)$ be the local entropy-production density. The total produced entropy and corresponding availability-destruction power are

$$\dot{S}_{\text{prod}}(t) \equiv \int d^3x\, \sigma_S(\mathbf{x}, t) \geq 0, \qquad P_{\text{loss}}(t) \equiv \int d^3x\, T_{\text{ref}}(\mathbf{x}, t)\sigma_S(\mathbf{x}, t) \tag{C14}$$

The same local ledger gives the fractional matter-scale rate directly:

$$H_L(t) = \frac{P_{\text{loss}}(t)}{E_{\text{drive}}(t)} = \frac{\int d^3x\, T_{\text{ref}}(\mathbf{x},t)\sigma_S(\mathbf{x},t)}{E_{\text{drive}}(t)} \tag{C15}$$

Equations (C14)-(C15) give the OQG discharge description a local entropy ledger. The nonnegative density $\sigma_S(\mathbf{x},t)$ records entropy production; its reference-temperature weighting gives availability destruction, with reversible work, transport, and reference-state changes accounted for in the complete ledger. Division by the remaining available inventory gives the fractional contraction rate on the declared common-scale branch. The relaxation is entropic in nature, while covariance fixes the energy–stiffness ratio. The second law alone does not determine every TT spectral moment: microscopic dynamics must supply the rate at which the epoch stiffness $A_{2,E}\left[\rho_{\text{epoch}}(t)\right]$ evolves along this branch.

A future microscopic calculation of the effective time constant has the schematic form

$$\dot{A}_{2,E}[\rho_{\text{SM}}] = \mathcal{P}_A\, \mathcal{C}_{TT}^{\text{SM}}[\rho_{\text{SM}}] \tag{C16}$$

The collision-and-correlation functional $\mathcal{C}_{TT}{}^{\text{SM}}$ evolves the complete Standard-Model density operator, and the projection $\mathcal{P}_A$ extracts the part that changes the static TT stiffness where the stated static expansion exists. At exact detailed balance with stationary external parameters, the net functional vanishes, so $P_{\text{loss}} = 0$ and $\mathcal{H}_L = 0$. The run-down branch requires the additional physical result $P_{\text{loss}} > 0$. Equation (C16) is the future first-principles target for predicting the RC history. This task concerns the microscopic origin of the effective time constant; the common-scale identity in Equation (C12) already follows from covariance.

C.4 Redshift, deceleration, and jerk identities

The observational identities in this subsection translate a matter-standard history into the conventional kinematic quantities without changing their successful mathematics. On the common homogeneous branch, comparing the emission epoch t_e with the reception epoch t_0 gives

$$1 + z = \frac{E_S(t_e)}{E_S(t_0)} = \frac{L(t_e)}{L(t_0)} = \left(\frac{A_{2,E}\left[\rho_{\text{epoch}}(t_e)\right]}{A_{2,E}\left[\rho_{\text{epoch}}(t_0)\right]}\right)^{1/2} \tag{C17}$$

Equation (C17) compares the explicitly named emission and reception epoch states. The two standards need not coexist; the signal carries the phase record between them. Once this endpoint ratio has been assigned, no second geometrical redshift is multiplied into the same observation.

Separate the leading self-similar scale from the changing dimensionless Standard-Model state by defining

$$\mathcal{R}_{\text{ent}}(z) \equiv \frac{r_{\text{ent}}[Y(z)]}{r_{\text{ent}}(Y_0)}, \qquad \mathcal{R}_{\text{ent}}(0) = 1. \tag{C18}$$

The response $\mathcal{R}_{\text{ent}}$ is the actual fractional contraction rate divided by the rate obtained from the leading scale dependence. It is a dimensionless state-response ratio, not a density of Dark Energy. The normalized Hubble function is

$$E(z) \equiv \frac{H_{\text{op}}(z)}{H_0} = (1+z)\mathcal{R}_{\text{ent}}(z). \tag{C19}$$

The factor $1 + z$ is the self-similar baseline. All observed curvature relative to that baseline is carried by $\mathcal{R}_{\text{ent}}$.

The conventional deceleration parameter is the kinematic identity

$$q_{\mathrm{op}} = -1 + (1+z)\frac{d\ln H_{\mathrm{op}}}{dz}. \tag{C20}$$

Substituting Equation (C19) cancels the derivative of the explicit $1+z$ factor and leaves

$$q_{\mathrm{op}} = \frac{d\ln \mathcal{R}_{\mathrm{ent}}}{d\ln(1+z)}. \tag{C21}$$

Equation (C21) is the key interpretation: apparent acceleration measures the logarithmic slope of the matter-state response. A negative q_{op} does not require a material substance that accelerates space; it means that the normalized contraction response decreases in the specified direction along the comparison history.

The stiffness-defined capacity makes this interpretation more specific. Combining Equation (C13a) with the homogeneous cross-epoch scale relation gives

$$\frac{A_{2,E}[\rho_{\mathrm{epoch}}(z)]}{A_{2,E}[\rho_{\mathrm{epoch}}(0)]} = (1+z)^2, \qquad \frac{C_A(z)}{C_A(0)} = \frac{1}{1+z} \tag{C21a}$$

Because the normalized effective time is the product of the residual response and the canonical capacity, the remaining dimensionless factor is

$$\frac{R_A(z)}{R_A(0)} = (1+z)\theta(z) = \frac{1}{\mathcal{R}_{\mathrm{ent}}(z)} \tag{C21b}$$

Substituting Equation (C21b) into Equation (C21) gives

$$q(z) = -\frac{d\ln[R_A(z)/R_A(0)]}{d\ln(1+z)} \tag{C21c}$$

Thus the self-similar branch has a unit residual response, and $q=0$ marks an extremum of that response. Apparent acceleration is the logarithmic evolution of the residual Standard-Model entropy-production and transport response factor after the canonical $A_{2,E}$ capacity scaling has been removed. This introduces no new fit parameter and does not alter the reconstructed cosmological curve; it identifies which part of the RC product the low-energy Standard-Model calculation must explain.

For completeness, let $x = \ln(1+z)$, $r = \ln\mathcal{R}_{\mathrm{ent}}$, and let a prime denote d/dx. The deceleration and jerk become

$$q_{\mathrm{op}} = r', \qquad j_{\mathrm{op}} = q_{\mathrm{op}}\left(2q_{\mathrm{op}}+1\right) + q_{\mathrm{op}}{}' = r'' + r' + 2(r')^2. \tag{C22}$$

The jerk is the next derivative of the same response history. At the present epoch,

$$r'_0 = q_0, \qquad r''_0 = j_0 - q_0 - 2q_0^2. \tag{C23}$$

Equations (C22)-(C23) are conversion identities. They allow an independently calculated or reconstructed $\mathcal{R}_{\mathrm{ent}}$ to be compared with conventional cosmographic parameters; they do not supply those parameters as inputs.

For strict self-similarity, $Y = Y_0$ and $\mathcal{R}_{\mathrm{ent}} = 1$, so

$$\frac{H_{\mathrm{op}}}{H_0} = 1+z, \qquad q_{\mathrm{op}} = 0. \tag{C24}$$

Equation (C24) is the parameter-free OQG baseline. The chronometer and DESI departures displayed in the main paper specify the curvature that the low-energy Standard-Model calculation in Equation (C16) must eventually reproduce.

C.5 Homogeneous field-equation consistency

The scalar matter-standard history and the Einstein-equivalent tensor equation are two descriptions of the same physical state and must agree. In the receiver-unit representation and on the branch $\Lambda_{\text{int}} = 0$,

$$T_{\mu\nu}^{\text{SM,hom}}[\rho_{\text{SM}}(t)] = \frac{c^4(t)}{8\pi G_{\text{comp}}[\rho_{\text{epoch}}(t)]} G_{\mu\nu}[A(t)] = \frac{c_0^4}{8\pi G_0} G_{\mu\nu}[A(t)] \tag{C25}$$

The second equality uses the covariant branch condition $c^4(t)/G_{\text{comp}}\big[\rho_{\text{epoch}}(t)\big] = c_0^4/G_0$. Equation (C25) therefore does not permit the scalar history to be chosen independently of the complete homogeneous stress tensor. Both must arise from the same density operator.

The associated conservation conditions are

$$\nabla^{\mu} T_{\mu\nu}^{\text{SM,hom}}[\rho_{\text{SM}}(t)] = 0, \qquad \nabla^{\mu} G_{\mu\nu}[A(t)] = 0 \tag{C26}$$

Defining an effective tensor after choosing $A(t)$ would make Equation (C25) a bookkeeping identity. The nontrivial OQG test is stronger: an independently evolved Standard-Model density operator must produce the same conserved tensor, the same $A_{2,E}\big[\rho_{\text{epoch}}(t)\big]$, and the same $R_{\text{ent}}(z)$. Equations (C25)-(C26) state that consistency requirement explicitly.

C.6 Visible-data and reproducibility audit

The main paper keeps the observations visible rather than hiding the result inside a code archive. Table C1 maps each main-paper item to its public input and the theory quantity plotted.

Table C1. Visible-data inputs and plotted theory quantities in the main paper.

Main-paper item	Public input	Theory quantity plotted or tabulated
Figure 1	32-point cosmic-chronometer compilation [10]	$E(z) = H_{\text{op}}(z)/H_0$ for constant, self-similar, linear-RC, quadratic-RC, and chronometer-fitted flat-ΛCDM histories
Figure 2 and Table 1	Six DESI DR2 anisotropic BAO ratios [5]	$F_{\text{AP}}(z) = D_M/D_H$ predicted without refitting from the chronometer-fitted histories
Figure 3	Chronometer reconstruction and fitted RC parameters	$R_{\text{ent}}(z)$, the implied $q_{\text{op}}(z)$, and $\theta(u) = H_0\tau_{\text{eff}}(u)$
Table 2	Diagnostics calculated from the two public data sets	Chronometer $\chi^2_{\text{diag}}/\text{dof}$ and no-refit DESI χ^2_{diag} for the nested response laws

For any normalized rate function $E(z)$, the flat receiver-unit Alcock-Paczynski prediction used in Figure 2 is

$$F_{\text{AP}}(z) = \frac{D_M(z)}{D_H(z)} = E(z)\int_0^z \frac{dz\prime}{E(z\prime)}. \tag{C27}$$

The instantaneous rate appears outside the integral, while the accumulated reciprocal rate appears inside it. This makes F_{AP} especially useful for testing curvature: changing $E(z)$ changes both factors in a correlated way. The six displayed pulls divide each observed-minus-predicted difference by its published marginal error. They are transparent diagnostics, not substitutes for the full DESI covariance.

The v0.3 reproducibility package is defined to contain the machine-readable input rows, fitted and fixed quantities, model curves, figure-source tables, analysis script, environment-independent formulas, and checksums. It reproduces the visible figures and tables; it does not replace them.

C.7 Quadratic RC reconstruction and independent slope check

The RC reconstruction begins in physical time rather than by assuming a convenient polynomial directly in redshift. Define dimensionless lookback u, logarithmic redshift x, and the normalized effective time constant θ by

$$u \equiv H_0(t_0 - t), \qquad x \equiv \ln(1+z), \qquad \theta(u) \equiv H_0\tau_{\mathrm{eff}}(t). \tag{C28}$$

Because $\tau_{\mathrm{eff}} = 1/H_{\mathrm{op}}$, the discharge law and the observable rate satisfy

$$\frac{du}{dx} = \theta(u), \qquad E(z) = \frac{1}{\theta[u(z)]}, \qquad \mathcal{R}_{\mathrm{ent}}(z) = \frac{1}{(1+z)\theta[u(z)]}. \tag{C29}$$

Equation (C29) fixes the order of the calculation. First specify the effective Standard-Model response time as a function of physical lookback. Then integrate to obtain the mapping between time and redshift. Only after that step are $H(z)$, $\mathcal{R}_{\mathrm{ent}}(z)$, distances, and DESI observables calculated. An arbitrary redshift curve need not satisfy this physical discharge relation.

The nested test uses the lowest-order time dependences capable of revealing what the data require:

$$\theta_{\mathrm{const}}(u) = 1, \qquad \theta_{\mathrm{lin}}(u) = 1 - \alpha u, \qquad \theta_{\mathrm{quad}}(u) = 1 - \alpha u - \tfrac{1}{2}\beta u^2. \tag{C30}$$

A constant effective RC time predicts a constant Hubble rate. The linear law can be integrated exactly:

$$\begin{gathered}\theta_{\mathrm{lin}}(u) = (1+z)^{-\alpha}, \qquad E_{\mathrm{lin}}(z) = (1+z)^{\alpha},\\ \mathcal{R}_{\mathrm{ent,lin}}(z) = (1+z)^{\alpha-1}, \qquad q_{\mathrm{op}} = \alpha - 1.\end{gathered} \tag{C31}$$

The parameter-free self-similar OQG baseline is the special linear case $\alpha = 1$. It has $\theta = 1 - u$, $E = 1 + z$, $\mathcal{R}_{\mathrm{ent}} = 1$, and $q_{\mathrm{op}} = 0$. This is the useful physical result of the RC analogy: the baseline is already a discharge law whose effective time shortens linearly into the past. A fitted linear law can change the constant slope, but it cannot make that slope turn.

The quadratic term is the first addition capable of producing a turnover. Differentiating it gives

$$q_{\mathrm{op}}(z) = -1 + \alpha + \beta u(z), \qquad u_{\mathrm{turn}} = \frac{1-\alpha}{\beta}, \qquad q_{\mathrm{op}}(z_{\mathrm{turn}}) = 0. \tag{C32}$$

The coefficient α fixes the present slope of the effective response time; β determines how that slope changes into the past. Positive β permits the present slow-contraction branch to turn continuously toward faster contraction at earlier epochs.

With $H_0 = 69\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ fixed as an external dimensional datum, the chronometer parameters minimize

$$\chi^{2\,\mathrm{CC,diag}}(\alpha,\beta) = \sum_{i=1}^{32}\left[\frac{H_i - H_0E(z_i;\alpha,\beta)}{\sigma_{H_i}}\right]^2. \tag{C33}$$

The subscript “diag” states that the displayed point uncertainties are used without a covariance matrix. The chronometer-only result is

$\alpha = 0.510 \pm 0.186, \qquad \beta = 1.103 \pm 0.552, \qquad \mathrm{corr}(\alpha,\beta) = -0.962, \qquad \chi^{2}{}_{\mathrm{CC,diag}} = 11.84 \quad (30\ \mathrm{dof}).$ (C34)

The strong anti-correlation means that the two coefficients must be varied together when an uncertainty band or turnover interval is calculated. Treating their quoted errors as independent would overstate or misdirect the uncertainty. Propagating the correlation gives

$$q_0 = -0.490 \pm 0.186, \qquad z_{\mathrm{turn}} = 0.70^{+0.32}_{-0.14}, \qquad q_{\mathrm{op}}(2.33) = 0.346 \quad \text{at the best fit.} \tag{C35}$$

Thus the reconstructed response has negative present curvature, crosses $q_{op} = 0$ near $z = 0.70$, and is on the opposite side of the turnover by the Ly-α epoch. Varying the fixed Hubble datum from 67 to $73\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ moves the best turnover only from about 0.67 to 0.72, even though the individual coefficients change.

The turnover therefore has a second, equivalent reading. By Equation (C21c), the $q = 0$ crossing is the extremum of the residual dimensionless entropy-production-to-availability response. The chronometer-fitted effective-time history is unchanged; the stiffness-defined capacity split identifies which factor carries the observed curvature.

No RC parameter is refitted to DESI. The transfer diagnostic is

$$\chi^{2}{}_{\mathrm{DESI,diag}} = \sum_{i=1}^{6}\left[\frac{F_{\mathrm{AP},i}{}^{\mathrm{obs}} - F_{\mathrm{AP}}(z_i;\hat{\alpha},\hat{\beta})}{\sigma_{F_i}}\right]^2 = 6.60. \tag{C36}$$

The hats mark the chronometer-fitted values from Equation (C34). Every marginal DESI residual is below 1.55 standard deviations, including Ly-α at 1.41. The corresponding chronometer-fitted flat-ΛCDM reference gives $\chi^{2}{}_{\mathrm{DESI,diag}} = 6.55$. These nearly equal diagnostics show that the measured curvature can be represented by a changing matter-response time without inserting a physical Dark Energy density. They do not make the two physical interpretations identical.

The former main-text cosmographic reconstructions are retained in Table C2 as an independent present-slope comparison. They do not determine the RC fit.

Table C2. DESI DR2 cosmographic reconstructions of the present slope. Source: Mishra et al. [18], Table 1; the final column is calculated here.

Reconstruction	H_0 [$\mathrm{km\,s^{-1}\,Mpc^{-1}}$]	q_0	$\Xi_{2,0} = 1 + q_0$
Taylor, $z \le 1$	$66.1^{+1.8}_{-2.1}$	-0.41 ± 0.23	0.590 ± 0.230
Padé (2,1)	67.8 ± 1.3	$-0.49^{+0.12}_{-0.10}$	$0.510^{+0.120}_{-0.100}$
Padé (2,2)	67.13 ± 0.91	$-0.397^{+0.089}_{-0.077}$	$0.603^{+0.089}_{-0.077}$
Chebyshev	$68.74^{+0.82}_{-0.73}$	-0.472 ± 0.052	0.528 ± 0.052

The RC result $q_0 = -0.490 \pm 0.186$ is consistent with these independent present-slope reconstructions. The statistical boundary remains unchanged: the RC family was proposed after an earlier monotonic response failed against high-redshift DESI, and the diagnostics use displayed diagonal uncertainties. The result is therefore an observational feasibility reconstruction and a no-refit cross-dataset consistency test, not a prospectively blind discovery significance or a covariance-complete likelihood.

The physical boundary is equally important. The quadratic effective-time history is an observational target for the microscopic Standard-Model evolution in Equation (C16), and Equations (C13a)-(C13b) and (C21a)-(C21c) separate its canonical stiffness-defined capacity from the residual dimensionless entropy-production and transport response. Over the observed range, the reconstruction therefore gives a definite low-energy Standard-Model target without extrapolating through recombination or primordial inflation. Cooling through particle, nuclear, and binding thresholds created the durable clocks and rulers that define the later operational branch and set its initial conditions. Predicting the effective time constant and residual response across those thresholds is a subsequent microscopic test, beyond the present fit and its exact common-scale relations.

Declarations

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Competing Interests

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Author contributions. Todd J. Desiato conceived the OQG framework, developed the physical interpretation and research program, directed the theoretical synthesis, reviewed the derivations and literature, and approved the final manuscript.

Data availability. No new experimental datasets were generated for this theoretical study. The detailed derivations and proof layers are provided as Online Resource 1 (Technical Appendices A–C). The numerical reproducibility package, including Python code, benchmark inputs, and verification outputs, is provided as Online Resource 2 and available via Dropbox (v0.3 numerical reproducibility package). Empirical results discussed in the manuscript are taken from the cited literature.

Code availability. The Python source code and benchmark inputs required to reproduce the reported numerical checks are supplied as Online Resource 2 and available via Dropbox (v0.3 numerical reproducibility package). The package requires Python 3.11 or later and has no third-party Python dependencies.

Ethics statement. Not applicable; this theoretical study involved no human participants, animals, or identifiable human data.

Use of generative artificial intelligence. The author used OpenAI ChatGPT, including GPT-5.5, GPT-5.6 Sol and GPT-6 Astra model configurations, as an AI-assisted research and writing tool for literature discovery, mathematical cross-checking, organization, drafting, and editing during development of the manuscript. The author reviewed the scientific arguments, equations, references, and final text and takes full responsibility for the content.

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Reference numbers match the main manuscript. Only works cited in these appendices are listed here.

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