Operational Quantum Gravity: From Standard Model Stress Response to Gravity-Induced Entanglement

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ABSTRACT

Gravity-induced entanglement can establish that the gravitational interaction acting on matter is nonclassical, but a static entangling phase does not by itself identify a fundamental quantum metric. We construct the tested infrared channel from the normalized stress response of the interacting Standard Model. Independent tensor sources on the two branches of a closed-time-path functional determine the Ward- and contact-completed retarded stress Hessian; inversion on its physical subspace yields the compliance without assigning microscopic dynamics to the source. The static transverse-traceless susceptibility is evaluated through second order in the Standard-Model couplings. Hard, screened, Higgs-contact, electric, local-curvature, and magnetostatic contributions combine into a finite stiffness whose nonperturbative content is isolated in two explicitly defined three-dimensional Yang–Mills lattice observables. Static inversion produces the long-range compliance characteristic of a massless interaction, while the finite-frequency retarded kernel contains Landau and production continua and a simple underdamped resonance. On a stated, testable massless two-helicity infrared branch, a fixed-measure Weyl-transverse-diffeomorphism representation matches the calculated residue with exact dimensional normalization and yields Einstein-equivalent equations for conserved sources. The same normalized source construction proves that a homogeneous shift of the matter-energy origin leaves every response vertex invariant while preserving physical vacuum polarization, boundary effects, condensate differences, anomalies, phase transitions, and the cosmological integration datum. For two spatially superposed finite bodies, the conserved-source response fixes the four branch phases, the locally invariant Bose–Marletto–Vedral entangling phase, and the ideal concurrence. Equality of the mean and retarded responses guarantees the same coherent phase; matching the symmetrized noise kernel is additionally required for equivalence of the Gaussian reduced map. The absolute stiffness is fixed once the renormalized Higgs-curvature coupling, two lattice observables, and reference-state intercept are supplied; no parameter-free numerical prediction of Newton's constant is claimed. Within these limits, the matter-first response realizes a nonclassical effective gravitational channel without postulating an independently fundamental metric.

Keywords: operational quantum gravity; Standard Model; stress-tensor response; closed-time-path formalism; WTDiff gravity; vacuum energy; gravity-induced entanglement

I. INTRODUCTION

Gravity-induced entanglement (GIE) proposals place two masses in spatial superposition and allow their mutual gravitational interaction to imprint branch-dependent phases [1–3]. In the Bose–Marletto–Vedral (BMV) protocol, the resulting phase can entangle internal or path degrees of freedom [1–2]. Under the protocol's locality, subsystem-independence, and preparation assumptions, a verified entanglement witness excludes any account restricted to local operations and classical communication between the two initially independent systems. Inferring an independently fundamental quantum metric from that nonclassical channel requires additional dynamical assumptions.

A local linearized quantum field of gravity can generate the BMV phase [4–5], but it is not the only quantum description with that coherent limit. Quantum-controlled direct interactions can produce the same long-interaction-time phase without an independent local field subsystem, while retardation, causal ordering, and noise differ [6–8].

Classical-channel and hybrid classical–quantum models likewise show that the label "classical mediator" does not specify a unique dynamics [9–11]. The measured phase therefore constrains the effective interaction channel; it does not by itself select the microscopic ontology.

Operational Quantum Gravity (OQG) addresses this inference from the matter side. Its microscopic system is the renormalized interacting Standard Model (SM), and a symmetric rank-two source is introduced to define the stress functional. The source has neither an independent Hilbert space nor a microscopic equation of motion. After Ward identities and source-contact terms complete the stress response, inversion yields the tensor compliance that represents long-wavelength relations among matter-built clocks, rulers, trajectories, and interferometers. The central question is whether that completed matter response supports an Einstein-equivalent conserved-source channel and the BMV observable.

The derivation first fixes the normalized closed-time-path (CTP) functional, its complete retarded Hessian, and the transverse-traceless (TT) physical subspace. It then evaluates the static TT susceptibility through $\mathcal{O}(g^2)$, where g collectively denotes the Standard-Model gauge, Yukawa, and scalar interaction couplings. Hard, screened, Higgs-contact, electric, local-curvature, and magnetostatic sectors enter one renormalized stiffness.

Finite-frequency continuation identifies the causal poles and continua, and constitutive inversion exposes the static massless compliance. Because the static slope does not determine a Lorentzian temporal residue, the causal continuation is restricted to a stated, testable massless two-helicity infrared branch. A scalar-free fixed-measure Weyl-transverse-diffeomorphism (WTDiff) representation carries the calculated residue into an Einstein-equivalent equation for conserved sources. Geometry therefore enters as the infrared representation of the completed matter response, not as its microscopic premise.

At the same source level, fixed-measure normalization proves that a homogeneous shift of the matter-energy origin changes neither the normalized state nor any response vertex. The arbitrary common energy zero is removed from the local gravitational response while vacuum polarization, boundaries, condensate differences, anomalies, phase transitions, fluctuations, and the global cosmological integration datum remain physical. For two finite bodies in spatial superposition, the completed conserved-source compliance then determines the BMV branch phases and concurrence. It thereby separates two claims: the experiment can establish a nonclassical effective interaction, whereas the static phase cannot decide whether its tensor representation is fundamental or collective.

Within the stated symmetric-phase and top-Yukawa domain, every perturbatively calculable Standard-Model contribution through $\mathcal{O}(g^2)$ is included. The absolute numerical stiffness is determined once the renormalized Higgs-curvature coupling, the two three-dimensional Yang–Mills lattice observables, and the reference-state intercept are specified. Each has an independent physical definition; none is an adjustable gravitational fit parameter. The analytic response is thus fixed up to declared physical inputs; it is not a parameter-free numerical prediction of Newton's constant G.

Throughout, the reduced Planck constant $\hbar$, the locally realized limiting signal speed c, and Boltzmann's constant k_B remain explicit; no natural-unit convention is used. Greek spacetime indices run from 0 to 3, the metric signature is $(-,+,+,+)$, and repeated tensor indices and spacetime arguments are contracted and integrated. Within OQG, geometry denotes the infrared tensor representation of operationally measured relations unless an independently fundamental metric is stated explicitly. The symbol i is the imaginary unit, and ln is the natural logarithm.

II. NORMALIZED STANDARD-MODEL RESPONSE

A. Closed-time-path normalization and matter-first inversion

Causal stress response requires the source history used to perturb the matter state to remain independent of the history used to read out that response until all functional derivatives have been taken. The closed-time-path (CTP), or Schwinger–Keldysh in-in, formalism supplies precisely this ordering: it computes expectation values in one

prepared state rather than transition amplitudes between distinct asymptotic states. Its retarded stress Hessian and inverse compliance will emerge as the two orientations of one constitutive relation.

Let $q_{+}^{\mu\nu}$ and $q_{-}^{\mu\nu}$ be independent symmetric rank-two probe sources on the forward and backward CTP branches, and let $T_{+}^{\mu\nu}$ and $T_{-}^{\mu\nu}$ be the corresponding branch copies of the complete Standard-Model energy-momentum tensor. With d^4x denoting the spacetime volume element, their source action is

$$S_{\mathrm{src}}^{\mathrm{CTP}}=\frac{1}{2}\int d^4x\left[q_{+\mu\nu}(x)T_{+}^{\mu\nu}(x)-q_{-\mu\nu}(x)T_{-}^{\mu\nu}(x)\right]. \tag{1}$$

The factor $1/2$ is the conventional normalization for contraction of two symmetric tensors. It is not a gravitational coupling. It fixes the source coordinate $q_{\mu\nu}$ and therefore the normalization of every response coefficient derived from it.

Let ρ_0 be the normalized initial density operator, U_q the Standard-Model evolution operator in the presence of the source history q, and Tr the trace over the matter Hilbert space. Define the doubled partition functional Z and the dimensionless connected functional W by

$$Z[q_{+},q_{-}]=\mathrm{Tr}\left(U_{q_{+}}\rho_0U_{q_{-}}^{\dagger}\right),\qquad W[q_{+},q_{-}]=-i\ln Z[q_{+},q_{-}]. \tag{2}$$

If the two sources are made equal, the forward and backward evolutions cancel by unitarity,

$$Z[q,q]=1,\qquad W[q,q]=0. \tag{3}$$

Equation (3) is why the contour cannot be collapsed before differentiation. The equality of the two branch sources is the *physical limit*, but the difference between the branches is the coordinate that generates physical expectation values and causal response.

Introduce the average and difference variables

$$q_r=\frac{q_{+}+q_{-}}{2},\qquad q_a=q_{+}-q_{-}, \tag{4}$$

The corresponding difference variables are

$$T_r=\frac{T_{+}+T_{-}}{2},\qquad T_a=T_{+}-T_{-}. \tag{5}$$

The source bilinear then reorganizes exactly as

$$q_{+}T_{+}-q_{-}T_{-}=q_aT_r+q_rT_a, \tag{6}$$

with tensor indices and spacetime integrations understood. Thus the two CTP branches are not two physical geometries. They are the doubled bookkeeping required to distinguish the applied physical perturbation from the variation used to read out its response.

The source-conjugate stress variables follow directly from Eqs. (1)-(6):

$$T_r^{\mu\nu}(x)=2\,\frac{\delta W}{\delta q_{a\,\mu\nu}(x)},\qquad T_a^{\mu\nu}(x)=2\,\frac{\delta W}{\delta q_{r\,\mu\nu}(x)}. \tag{7}$$

At the physical limit,

$$q_a=0,\qquad T_a=0, \tag{8}$$

The variables q_r and T_r become the physical probe coordinate and expectation value in this limit. Equation (7) fixes the normalization used throughout the response calculation: differentiation with respect to the difference source produces one half of the physical stress because the symmetric-tensor source action contains the same factor.

For the following rank-four response equations, capital labels A, B denote condensed symmetric-tensor index pairs, including the associated spacetime arguments when no ambiguity can arise. The retarded susceptibility is the response of the physical stress to the physical average source,

$$\Pi^R_{AB}(x,y) \equiv \left.\frac{\delta T_{r,A}(x)}{\delta q_{r,B}(y)}\right|_{q_a=0} = 2\left.\frac{\delta^2 W}{\delta q_{a,A}(x)\delta q_{r,B}(y)}\right|_{q_a=0}, \tag{9}$$

With the source convention of Eq. (1), and with $\theta(t)$ denoting the Heaviside step function, the completed retarded response is

$$\Pi^R_{AB}(x,y) = -\frac{i}{2}\,\theta(x^0-y^0)\langle[T_A(x), T_B(y)]\rangle + \Pi^{R,\mathrm{contact}}_{AB}(x,y). \tag{10}$$

The contact term in Eq. (10) is local because the stress operator itself depends on the symmetric tensor source. It belongs to the same second functional variation as the separated-point commutator. The object to be inverted is therefore the completed Hessian:

$$\Pi^R = \Pi^R_{\mathrm{conn}} + \Pi^R_{\mathrm{contact}}, \tag{11}$$

The connected correlator alone is therefore not the object to be inverted. Ward identities require the completed Hessian, and its local terms contribute explicitly to the Higgs and dimensionally reduced matching in Sec. III.

For the conserved completed response, let p_μ denote the external four-momentum and let Π_{phys} denote the completed response after projection onto the physical tensor sector. The tangent-state Ward identity then takes the form

$$p_\mu \Pi^{\mu\nu,\rho\sigma}_{\mathrm{phys}}(p) = 0, \tag{12}$$

after the contact terms required by the same metric variation have been included. Equation (12) also explains why the full rank-four tensor Hessian cannot be inverted naively. Conservation and gauge redundancy produce null directions. Let P_{phys} denote the projector onto the non-gauge physical tensor subspace relevant to the response under consideration. The projected susceptibility is

$$\Pi^R_{\mathrm{phys}} = P_{\mathrm{phys}} \Pi^R P_{\mathrm{phys}}, \tag{13}$$

and its inverse is defined wherever this physical block is nonsingular,

$$D^R_{\mathrm{phys}} \equiv \left(\Pi^R_{\mathrm{phys}}\right)^{-1}. \tag{14}$$

The poles of D^R are therefore zeros of the corresponding physical eigenvalues of Π^R. No inverse is assigned to a gauge or Ward-null direction.

The matter-first orientation is now obtained by a genuine doubled Legendre transform, performed before Eq. (8) is imposed. This is the usual CTP effective-action construction applied to the stress/source pair rather than an additional microscopic dynamics. Define

$$\Gamma[T_r, T_a] = W[q_r, q_a] - \frac{1}{2}\int d^4x \left(q_{a\,\mu\nu} T_r^{\mu\nu} + q_{r\,\mu\nu} T_a^{\mu\nu}\right). \tag{15}$$

Using Eq. (7), variation gives the exact conjugacy relations

$$\frac{\delta\Gamma}{\delta T_r^{\mu\nu}(x)} = -\frac{1}{2} q_{a\,\mu\nu}(x), \qquad \frac{\delta\Gamma}{\delta T_a^{\mu\nu}(x)} = -\frac{1}{2} q_{r\,\mu\nu}(x). \tag{16}$$

Only after these derivatives have been defined is the physical limit taken. On that branch the second relation in Eq. (16) reads

$$q_{r,A}(x) = -2\frac{\delta\Gamma}{\delta T_{a,A}(x)}. \tag{17}$$

Differentiating Eq. (17) with respect to the physical stress and using Eqs. (9), (13) and (14) gives

$$\frac{\delta q_r}{\delta T_r} = D^R_{\text{phys}} = \left(\Pi^R_{\text{phys}}\right)^{-1}, \tag{18}$$

Equivalently, the mixed functional curvature is

$$\left.\frac{\delta^2\Gamma}{\delta T_a\,\delta T_r}\right|_{\text{phys}} = -\frac{1}{2} D^R_{\text{phys}}. \tag{19}$$

Equations (18) and (19) are the technical core of the matter-first inversion. They show directly that the mixed curvature of the stress-space functional is the inverse of the same retarded Standard-Model susceptibility that was obtained by differentiating the probe functional. Nothing has been added to the microscopic theory. The Legendre transform changes which member of a conjugate response pair is used as the independent variable.

The corresponding linear constitutive equations can therefore be written in either orientation,

$$\delta T_r = \Pi^R_{\text{phys}}\,\delta q_r, \qquad \delta q_r = D^R_{\text{phys}}\,\delta T_r. \tag{20}$$

Equation (20) is the field-theory analogue of writing a single constitutive law in either direction. The relations $F = kx$ and $x = F/k$ describe the same spring, just as polarization versus applied field and applied field versus specified polarization describe the same dielectric. Likewise, the two forms in Eq. (20) describe one completed Standard-Model tensor response, oriented either from source to stress or from stress to its source-conjugate response coordinate. Stress-space and inverse-correlator constructions provide independent field-theoretic precedent [12].

The physical matter perturbation may now be made explicit. Let the Standard-Model fields and state variables be denoted collectively by Φ. A physical change of state produces

$$\delta T_{r,A}(x) = \int d^4y\, \frac{\delta T_{r,A}(x)}{\delta\Phi_I(y)}\,\delta\Phi_I(y), \tag{21}$$

Substitution into the inverse response therefore gives

$$\delta q_{r,A}(x) = \int d^4y\, D^R_{\text{phys},AB}(x,y)\,\delta T_{r,B}(y). \tag{22}$$

Equation (22) fixes the physical causal order: a change of the matter state changes the complete stress, and the inverse completed susceptibility assigns the corresponding operational tensor response. No source equation imposed after setting the probe to zero is substituted for the ordinary Standard-Model dynamics.

The same result can be written as a reduced matter constitutive functional by composing $\Gamma[T_r, T_a]$ with the stress doublet generated by the Standard-Model fields. To quadratic order, the retarded cross term is

$$\Gamma^{(2)}_{\mathrm{red,ret}} = -\frac{1}{2}\int d^4x\, d^4y\, \delta T_{a,A}(x) D^{R}_{\mathrm{phys},AB}(x,y) \delta T_{r,B}(y) + \cdots, \tag{23}$$

where the ellipsis denotes the symmetric/noise and local pieces required by the complete CTP functional. Varying Eq. (23) reproduces Eq. (22). Thus the matter-only description and the source-response description are not competing constructions; they are Legendre-dual representations of the same causal Hessian.

Three consistency checks follow. First, the reverse CTP branch is not backward physical time; it disappears after the causal differentiations and physical limit. Second, the factor $1/2$ in the retarded commutator is fixed by Eq. (1), so changing it would rescale both Π^R and its inverse and spoil the Kubo-to-stiffness normalization of Sec. III. Third, Ward/contact completion must precede inversion; otherwise the object being inverted lacks the conserved tensor structure of the Standard-Model functional.

The CTP normalization, doubled Legendre transform, and physical projection therefore establish the matter-first constitutive step. The mixed curvature of the stress-space functional is exactly the inverse completed retarded susceptibility, so matter stress may serve as the physical input and its source-conjugate tensor as the response coordinate without adding a microscopic field. A fixed-measure tensor representation is introduced only after this reduction.

B. Transverse-traceless projection and spectral positivity

The TT projection isolates the unitary spin-two content of the completed Hessian. It also separates the positive absorptive spectral moment from the static thermodynamic stiffness and from any independently imposed double-zero condition.

Let $\eta_{\mu\nu}$ denote the local tangent-space Lorentz metric and p_μ an external four-momentum, with $p^2 \equiv \eta^{\alpha\beta} p_\alpha p_\beta$. Away from the null point $p^2 = 0$, define the transverse tensor

$$\theta_{\mu\nu}(p) = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}. \tag{24}$$

In four spacetime dimensions the spin-two projector acting on a symmetric rank-two tensor is

$$P^{(2)}_{\mu\nu,\rho\sigma}(p) = \frac{1}{2}\left(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}\right) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \tag{25}$$

while the transverse scalar projector is

$$P^{(0)}_{\mu\nu,\rho\sigma}(p) = \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}. \tag{26}$$

The spin-two projector is transverse and traceless,

$$p^\mu P^{(2)}_{\mu\nu,\rho\sigma} = 0, \qquad \eta^{\mu\nu} P^{(2)}_{\mu\nu,\rho\sigma} = 0, \tag{27}$$

The projector is idempotent on the physical tensor subspace. Transversality and tracelessness isolate the TT response once the completed stress Hessian obeys the Ward identity derived in Sec. II.A.

For a conserved stress tensor, let $\rho_{\mu\nu,\rho\sigma}(p)$ denote the spectral density of its two-point function. The Källén–Lehmann representation decomposes it into independent spin-two and transverse-scalar densities $\rho_2(p^2)$ and $\rho_0(p^2)$ [13]:

$$\rho_{\mu\nu,\rho\sigma}(p)=\rho_2(p^2)\,P^{(2)}_{\mu\nu,\rho\sigma}(p)+\rho_0(p^2)\,P^{(0)}_{\mu\nu,\rho\sigma}(p), \tag{28}$$

up to convention-dependent overall normalization and contact polynomials. The physical spin-two density is nonnegative in a unitary theory,

$$\rho_2(s) \geq 0. \tag{29}$$

After TT projection, the spectral density is a sum of squared stress-tensor matrix elements over physical intermediate states n. Here $|0\rangle$ is the reference state, m_n is the invariant mass of state n, s is invariant mass squared, and the subscript TT denotes projection onto the spin-two channel:

$$\rho_2(s) \propto \sum_n (2\pi)\ \delta(s\text{-}m_n^2)|\langle 0|T^{TT}|n\rangle|^2. \tag{30}$$

Equation (30) is the positivity statement used throughout the coherent-channel analysis. It applies to the resolved absorptive spin-two spectral response. Local subtraction terms, metric seagulls and trace-sector contacts do not become additional positive spectral states merely because they occur in the same completed Hessian.

For free four-dimensional fields the normalized spin-two stress coefficients are

$$C_T^{(s)}=\frac{1}{120}, \qquad C_T^{(W)}=\frac{1}{40}, \qquad C_T^{(V)}=\frac{1}{10}, \tag{31}$$

for one real scalar, one Weyl fermion and one gauge vector, respectively [13]. Dividing by the scalar unit gives the particularly transparent ratio

$$C_T^{(s)}: C_T^{(W)}: C_T^{(V)}=1: 3: 12. \tag{32}$$

This ratio is not a count of helicities. It is the relative strength with which each free field contributes to the physical spin-two stress correlator in one common normalization. The gauge-vector coefficient already refers to the physical, Ward-completed tensor response rather than to a naive count of gauge-potential components.

The minimal Standard Model contains four real Higgs components, 45 Weyl fermions and 12 gauge vectors. Therefore its free coherent TT weight is

$$\begin{aligned} N_T&=4(1)+45(3)+12(12)\\ &=4+135+144\\ &=283. \end{aligned} \tag{33}$$

Equivalently, in the conventional stress-central-function normalization,

$$C_{T,\mathrm{SM}}=4\left(\frac{1}{120}\right)+45\left(\frac{1}{40}\right)+12\left(\frac{1}{10}\right)=\frac{283}{120}. \tag{34}$$

Several useful conclusions follow immediately from Eq. (33). First, the gauge sector carries slightly more than half of the free coherent TT weight, the Weyl sector carries slightly less than half, and the four Higgs components contribute only a small fraction. Second, all three contributions enter with the same positive spectral sign. Third, 283 characterizes a nonlocal coherent matter moment; it does not by itself determine Newton's constant or the static coefficient $A_{2,E}$.

A superficially similar one-loop heat-kernel coefficient is sometimes written in terms of N_0 real scalars, $N_{1/2}$ Weyl fermions, and N_1 gauge vectors as

$$N_0+N_{1/2}-4N_1. \tag{35}$$

That combination is not the physical TT spectral count. Its negative vector term precludes an interpretation as a sum of the nonnegative matrix-element weights in Eq. (30). It is instead a local proper-time or heat-kernel coefficient that combines gauge, ghost, seagull, subtraction, and contact information in a curvature operator [14]. Because it answers a different question from the resolved TT spectral density, coherent spectral response, local Ward/contact response, and static thermodynamic stiffness remain separate until the completed observable is formed.

The same TT projection also fixes the role of the Higgs curvature improvement. For one scalar,

$$\Delta T_{\mu\nu}^{(\xi)}=\xi\left(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu\right)\phi^2. \tag{36}$$

In momentum space this contribution is proportional to

$$\eta_{\mu\nu}p^2-p_\mu p_\nu. \tag{37}$$

Applying the transverse-traceless projector defined above gives

$$P^{(2)\,\mu\nu,\rho\sigma}\left(\eta_{\rho\sigma}p^2-p_\rho p_\sigma\right)=0, \tag{38}$$

because the metric term is removed by tracelessness and the momentum term by transversality. Hence

$$P^{(2)}\Delta T^{(\xi)}=0. \tag{39}$$

For the Higgs field, ξ becomes the renormalized Higgs-curvature coupling ξ_H. Its resolved physical TT spectral weight is independent of ξ_H, although curved-space Standard-Model renormalization still requires the operator [15]. Its second metric variation contributes to the local/contact and static response calculated in Sec. III. Equation (39) states only that ξ_H neither creates propagating TT spectral states nor changes the $1:3:12$ weight.

Positivity also constrains a proposed double zero. Let t be the Lorentz-invariant momentum variable, let A and B be the two local subtraction coefficients, and let $\rho_2(s)$ be the nonnegative spin-two spectral density. A twice-subtracted inverse response for one unitary Lorentz-invariant TT eigenchannel is

$$K(t)=At+Bt^2+t^2\int_0^\infty ds\,\frac{\rho_2(s)}{s^2(s-t)}, \qquad \rho_2(s)\geq 0. \tag{40}$$

Suppose this isolated positive channel possessed a nondegenerate double zero at a negative invariant point $t=-x$, with $x>0$. Such a zero would have to satisfy both conditions

$$K(-x)=0, \qquad K_t(-x)=0 \tag{41}$$

The derivative condition would give

$$B=-\int_0^\infty ds\,\frac{\rho_2(s)}{s(s+x)^2}<0, \tag{42}$$

The zero condition would independently give

$$A=x^2\int_0^\infty ds\,\frac{\rho_2(s)}{s^2(s+x)^2}. \tag{43}$$

For every $s > 0$, however, positivity implies

$$0 < \frac{x^2}{(s+x)^2} < 1 \tag{44}$$

Consequently, the resolved moment satisfies

$$0 < A < A_{\text{resolved}}, \qquad A_{\text{resolved}} \equiv \int_0^\infty d\, s\, \frac{\rho_2(s)}{s^2}. \tag{45}$$

Equation (45) separates a positive resolved TT moment from a double-zero condition. If one isolated positive channel were required to supply both an enhanced local stiffness and a nondegenerate imaginary-axis double zero, the latter would force the local coefficient below the resolved positive moment. Any closure requiring the opposite ordering is therefore excluded without changing the physical TT count.

The bound distinguishes three independent objects: the positive coherent spectral moment, the state-dependent static variational stiffness, and an extreme critical or self-loading condition. A finite Kubo–Martin–Schwinger (KMS) stiffness is a thermodynamic derivative of a state-dependent Hessian, not a positive Lorentz-invariant absorptive moment, and its scalar contribution may be negative. The ordinary causal branch in Sec. IV is correspondingly a simple underdamped resonance rather than a free-vacuum double zero.

The spectral analysis thus yields a positive physical spin-two sector with free-field weights $1{:}\,3{:}\,12$, the minimal-Standard-Model coefficient $N_T = 283$, and an exactly TT-null Higgs improvement term in the resolved channel. It also prevents a local heat-kernel coefficient or an independent critical closure from being counted as additional physical spectral weight. The complementary static response is now fixed directly from its Kubo slope.

III. STATIC TT SUSCEPTIBILITY THROUGH SECOND ORDER

The massless infrared residue is determined by the zero-frequency, small-wave-number slope of the completed shear response. That slope must include the operator correlator, its source contacts, all hard and screened sectors, and the nonperturbative magnetostatic boundary in one renormalized coefficient.

A. Completed susceptibility and coefficient ledger

Write $\Theta \equiv k_B T$ for the thermal energy. The symbols g_Y, g_2, and g_3 denote the hypercharge, weak, and strong gauge couplings; y_t is the top Yukawa coupling; λ is the Higgs quartic coupling; and ξ_H is the renormalized Higgs-curvature coupling. The top-Yukawa truncation neglects y_b and y_τ at this order and defines $Y_2 \equiv 3y_t^2$. At the benchmark couplings, restoring y_b and y_τ changes Y_2 by 0.038% and the associated ξ_H running target by 0.041%. The leading color-electric, weak-electric, hypercharge-electric, and Higgs screening masses are m_{D3}, m_{D2}, m_{DY}, and m_H. The four-dimensional modified-minimal-subtraction scale is μ. Table 1 and the analytic mass relations below define the dimensionless benchmark at $\mu = 2\pi\Theta$; the displayed digits audit arithmetic and cancellations rather than statistical uncertainty.

Table 1. Benchmark inputs and leading screening masses.

Running coupling	Value	Screening mass	Value divided by Θ
g_3	0.486635	m_{D3}	0.688205817
g_2	0.502460	m_{D2}	0.680334056
g_Y	0.477571	m_{DY}	0.646634191
y_t	0.401965	m_H	0.291877592
λ	-0.0335869		
$Y_2 = 3y_t^2$	0.484727584		

The mass ratios in Table 1 follow the leading symmetric-phase relations

$$\frac{m_{D3}^2}{\Theta^2} = 2g_3^2, \qquad \frac{m_{D2}^2}{\Theta^2} = \frac{11}{6}g_2^2, \qquad \frac{m_{DY}^2}{\Theta^2} = \frac{11}{6}g_Y^2, \qquad \frac{m_H^2}{\Theta^2} = \frac{3g_2^2 + g_Y^2}{16} + \frac{\lambda}{2} + \frac{y_t^2}{4}. \tag{46}$$

The coupling inputs and analytic relations, rather than the rounded mass ratios alone, define the benchmark used in the coefficient ledger.

The static TT stiffness is extracted from a shear that varies along one Cartesian direction. For a thermal state at temperature T, write $\Theta \equiv k_B T$, where k_B is Boltzmann's constant. Let z be the selected Cartesian coordinate, k a real wave number along z, $h_{xy}(z)$ the real off-diagonal shear source, and h_k and $h_{-k} = h_k^*$ its Fourier amplitudes, with $*$ denoting complex conjugation. The source is

$$h_{xy}(z) = h_k e^{ikz} + h_{-k} e^{-ikz}. \tag{47}$$

Because h_{xy} is the only nonzero component and its momentum points along z, the perturbation is transverse and traceless. Let $Z_E[h]$ be the Euclidean partition function, $W_E[h] \equiv -\ln Z_E[h]$ its connected generating functional, and V_4 the Euclidean spacetime volume. Let T_{xy} be the xy component of the complete Standard-Model stress tensor, $\langle \cdots \rangle$ a thermal Euclidean expectation value, the subscript c its connected part, and $G_{E,\mathrm{contact}}(k)$ the local second metric variation of the microscopic action. Every response below is the renormalized thermal value minus its zero-temperature value at the same renormalized parameters. The completed response is therefore

$$G_E(k) \equiv -\frac{1}{V_4}\frac{\delta^2 W_E}{\delta h_k \delta h_{-k}}\bigg|_{h=0} = \int d^4x\, e^{ikz} \langle T_{xy}(x) T_{xy}(0) \rangle_c - G_{E,\mathrm{contact}}(k). \tag{48}$$

The same source action therefore fixes both terms and their relative sign; the contact is not an optional correction added after the correlator is computed.

The superscript xy,xy merely labels the shear channel. At zero Euclidean frequency the channel-specific quantity is the same completed response defined in Eq. (48).

The symbol ∂_k^2 denotes the ordinary second derivative with respect to the scalar wave number while every thermodynamic and renormalized parameter is held fixed. The conventional hydrostatic coefficient κ and the source-normalized susceptibility $A_{2,E}$ are defined by

$$\kappa = \partial_k^2 G_E^{xy,xy}(0,k)\big|_{k=0}, \qquad A_{2,E} = \frac{\kappa}{4}. \tag{49}$$

Equivalently, $G_E(0,k) = G_E(0,0) + (\kappa/2)k^2 + \mathcal{O}(k^4)$ near the origin. The factor $1/4$ follows from the symmetric tensor-source convention $\Pi_{TT,E} \equiv -G_E^{xy,xy}/2$ adopted here; it is only a normalization convention. The notation $\mathcal{O}(k^4)$ collects terms whose leading small-k behavior is fourth order or higher. The susceptibility has mass dimension two, so the dimensionless quantity quoted below is $A_{2,E}/\Theta^2$.

The calculation uses the state-dependent thermal Hessian. A state-independent local Einstein term cancels in the thermal-minus-vacuum subtraction. Modified minimal subtraction is denoted $\overline{\mathrm{MS}}$, while μ and μ_3 are the four- and three-dimensional renormalization scales. The μ_3 dependence must cancel among the reduced-theory sectors; the explicit μ derivative must reproduce the running of the Higgs-curvature coupling defined next. A finite local matching coefficient fixed by that condition is not an independent prediction.

The operator $\xi_H R H^\dagger H$ is essential, where H is the Higgs doublet, the dagger denotes Hermitian conjugation, R is the Ricci scalar, and ξ_H is the renormalized Higgs-curvature coupling. Its first flat-space improvement vanishes in a resolved on-shell TT matrix element, but its second metric variation supplies a local two-derivative contact proportional to the thermal condensate $\langle H^\dagger H\rangle$. The remaining renormalizable Standard-Model action depends algebraically on the background metric: the Yang–Mills field strength contains no Levi-Civita connection, the Dirac kinetic term has no independent two-metric-derivative contact in this channel, and the quartic and Yukawa interactions generate only momentum-independent local seagulls. Those seagulls disappear from the k^2 derivative; the Higgs-curvature contact does not. It is therefore possible for ξ_H to affect a static susceptibility without adding a propagating TT degree of freedom.

Six factorization sectors organize the calculation. Superscripts 0, g, and g^2 label the free, ring-resummed, and second-order terms. The labels hard, E/H, and R denote nonzero Matsubara modes, the screened electrostatic/Higgs theory, and local-curvature matching, respectively. Finally, A_M^{NP} denotes the nonperturbative magnetostatic remainder. The complete coefficient is

$$A_{2,E} = A_{2,E}^{(0)} + A_{2,E}^{(g)} + A_{\mathrm{hard}}^{(g^2)} + A_{E/H}^{(g^2)} + A_R^{(g^2)} + A_M^{\mathrm{NP}}. \tag{50}$$

These labels are factorization sectors, not separately observable forces. Only their completed sum has the claimed scale and contact independence. The organization also prevents double counting: bosonic zero modes removed from the strict-hard diagrams reappear once, with screening, in the reduced theory.

B. Free and ring-resummed terms

The free term is most transparent species by species because this exposes the scalar contact. For one real massless scalar with curvature coupling ξ, the separated operator bubble gives $\kappa_{s,\mathrm{corr}}/\Theta^2 = -1/72$; the label corr denotes the operator-correlator part before variational completion. The curvature contact adds $\xi/12$, giving $\kappa_s/\Theta^2 = -(1-6\xi)/72$. One two-component Weyl fermion gives $1/144$, and one massless gauge vector, including gauge-fixing and ghost completion, gives $1/18$.

The symmetric-phase Standard Model contains four real Higgs components with common coupling ξ_H, 45 Weyl fermions, and 12 gauge vectors. Summing those completed species contributions and applying the normalization in Eq. (49) gives

$$\frac{A_{2,E}^{(0)}}{\Theta^2} = \frac{133 + 48\xi_H}{576}. \tag{51}$$

At minimal Higgs curvature coupling, $\xi_H = 0$, Eq. (51) gives $133/576 = 0.230902777778$. The integer 133 is a count appropriate to the static completed Hessian. It is not a count of resolved graviton-like spectral polarizations, which would be a different response moment.

A bosonic zero mode is nonanalytic before resummation and cannot be expanded ordinarily about zero mass. For one three-dimensional scalar screening mass m, resummation changes the completed slope by $\delta\kappa_{s,0} = \Theta m(1 - 6\xi)/(24\pi)$, where δ denotes the screened result minus its strict massless expansion. Fermions have no zero Matsubara mode. Let m_{D3}, m_{D2}, and m_{DY} be the color-electric SU(3), weak-electric SU(2), and hypercharge screening masses, respectively, and let m_H be the Higgs screening mass. Their summed first-order correction is

$$\frac{\delta A_{2,E}^{(g)}}{\Theta^2} = \frac{8m_{D3} + 3m_{D2} + m_{DY} + 4m_H(1 - 6\xi_H)}{96\pi\Theta}. \tag{52}$$

This term is formally first order in a generic coupling because every screening mass is proportional to $g\Theta$ at leading order. Define $A_{2,E}^{(0+g)} \equiv A_{2,E}^{(0)} + \delta A_{2,E}^{(g)}$. At the benchmark point, the free and screened contributions combine to

$$\frac{A_{2,E}^{(0+g)}}{\Theta^2} = 0.261940621924 + 0.060106452558\,\xi_H. \tag{53}$$

For $\xi_H = 0$, the ring term raises the free value by 13.44%. The quoted benchmark is a dimensionless snapshot at the stated couplings and scales rather than a complete running trajectory, so it does not by itself define a scale-variation band.

C. Strict-hard two-loop sectors

The strict-hard term contains loop momenta of order $\pi\Theta$. Bosonic zero modes are excluded from this region and reserved for the three-dimensional theory, so ``hard'' describes a matching region rather than a new physical contribution. We start from the complete two-loop 12,12 operator correlator of Ref. [16]. Its five gauge-independent classes are Higgs self-coupling, gauge-gauge, Yukawa, scalar-gauge, and fermion-gauge.

For each class $x \in \{\lambda, gg, y, sg, fg\}$, let H_x denote the dimensionless second derivative of the corresponding hard master combination with respect to k, divided by Θ^2, and let C_x denote its Standard-Model coupling and multiplicity weight. The source-normalized contribution is then $\delta A_x/\Theta^2 = C_x H_x/4$. Dimensional regularization is performed in $D = 4 - 2\epsilon$ spacetime dimensions and $d = 3 - 2\epsilon$ spatial dimensions, where ϵ is the regulator taken to zero after renormalization.

Let $P = (P_0, \mathbf{p})$ and $Q = (Q_0, \mathbf{q})$ be Euclidean thermal four-momenta, with bosonic or fermionic Matsubara frequencies as assigned by the sector, and let $K = (0,0,0,k)$ be the external static four-momentum. Each thermal sum-integral contains a Matsubara sum multiplied by Θ and a spatial integral with measure $d^d\mathbf{p}/(2\pi)^d$; the double summation sign below contains one such factor for P and one for Q. The integers a, b, c, r, s are

propagator powers, and f, g, h are frequency powers. Before the master is used, define the homogeneity index $y \equiv a + b + c + r + s - f - g - h - 2$. With this notation, the static expansion reduces masters of the form

$$I^{fgh}_{abcrs} = \sum_{P,Q} \frac{P_0^f Q_0^g (Q-P)_0^h \left(K^2\right)^y}{\left(P^2\right)^a \left(Q^2\right)^b [(Q-P)^2]^c [(K-P)^2]^r [(K-Q)^2]^s}. \tag{54}$$

Terms with $y \geq 2$ start at k^4; a $y = 1$ term is evaluated at $K = 0$ after the explicit factor of k^2 is extracted; and a $y = 0$ term survives only if an external propagator remains to be differentiated. Tensor numerators are reduced by rotational averaging in exact d dimensions before the Laurent expansion in ϵ. Thermal integration-by-parts identities and frequency relabelings then reduce the surviving terms to products of one-loop tadpoles. This order of operations is part of the calculation, because an evanescent factor proportional to ϵ can multiply a pole and leave a finite remainder.

For the gauge-gauge class, define the bosonic tadpole $\mathcal{I}_n \equiv \sum_P \left(P^2\right)^{-n}$ and let $\mathcal{C}_{gg}(d)$ be the rational coefficient left by tensor reduction. Integration by parts reduces the complete slope to $\mathcal{C}_{gg}(d)\mathcal{I}_1\mathcal{I}_2$. The two factors needed to expose the finite evanescent remainder are

$$\begin{aligned} \mathcal{C}_{gg}(d) &= \frac{(d-8)(d-3)(d-2)(d-1)^3(d+1)}{12d(d-5)}, \\ \mathcal{C}_{gg}(3-2\epsilon) &= -\frac{40}{9}\epsilon + O\left(\epsilon^2\right), \\ \mathcal{I}_1 &= \frac{\Theta^2}{12} + O(\epsilon), \qquad \mathcal{I}_2 = \frac{1}{16\pi^2\epsilon} + O(1). \end{aligned} \tag{55}$$

These factors combine so that the evanescent coefficient leaves $H_{gg} = -5/\left(108\pi^2\right)$. Setting $d = 3$ prematurely would erase this term. The result is a useful audit of the reduction because it arises entirely from the order in which tensor averaging and pole expansion are performed.

The fermion-gauge class is the largest independent check. Its 97 printed structures first reduce by static power counting and frequency relabeling; thermal integration by parts then collapses the survivors to 19 products of one-loop tadpoles. Both its double and simple poles cancel within the class. The finite slope is

$$H_{fg} = -\frac{7}{432\pi^2} = -0.001641778438649. \tag{56}$$

The remaining three classes provide complementary checks. The Higgs self-coupling and scalar-gauge classes reduce to bosonic tadpole products but retain simple poles that must be matched. The Yukawa class mixes bosonic and fermionic Matsubara assignments; after frequency mapping and integration by parts, its candidate structures also reduce to products of one-loop sums. Let γ_E be the Euler–Mascheroni constant, and let $\zeta'(-1)$ denote the derivative of the Riemann zeta function at -1. Before minimal subtraction, the three Laurent endpoints are

$$\begin{aligned} H_\lambda &= \frac{1}{12\pi^2\epsilon} + \frac{-48\zeta'(-1) + 8\ln[\bar{\mu}/(4\pi\Theta)] - 3 + 4\gamma_E}{24\pi^2} + O(\epsilon), \\ H_y &= \frac{1}{72\pi^2\epsilon} + \frac{4\ln[\bar{\mu}/(2\pi\Theta)] - 24\zeta'(-1) - 6\ln 2 - 1 + 2\gamma_E}{72\pi^2} + O(\epsilon), \\ H_{sg} &= \frac{11}{288\pi^2\epsilon} + \frac{-528\zeta'(-1) + 88\ln[\bar{\mu}/(4\pi\Theta)] - 49 + 44\gamma_E}{576\pi^2} + O(\epsilon). \end{aligned} \tag{57}$$

The Standard-Model weights are not independent fit parameters. In the stated top-Yukawa truncation they are $C_\lambda = \lambda/4, C_{gg} = \left(3g_2^2 + 12g_3^2\right)/4, C_y = 3y_t^2/4, C_{sg} = \left(g_Y^2 + 3g_2^2\right)/4$, and $C_{fg} = \left(5g_Y^2 + 9g_2^2 + 24g_3^2\right)/4$. At the benchmark these evaluate to -0.008396725, 0.899790408, 0.121181896, 0.246368054, and 2.274022931, respectively. Table 2 reports each minimally subtracted slope, its weight, and the resulting source-normalized contribution. The analytic reductions establish the structure; the decimal values are audit targets rather than substitutes for the derivation.

Table 2. Strict-hard operator contributions at $\bar{\mu} = 2\pi\Theta$.

Sector x	$H_x^{\overline{\mathrm{MS}}}$	C_x	$\delta A_x/\Theta^2 = C_x H_x/4$
Higgs self-coupling	+0.007193382340	-0.008396725	-0.000015100213339
Gauge-gauge	-0.004690795539	+0.899790408	-0.001055183207970
Yukawa	-0.000048330633	+0.121181896	-0.000001464199449
Scalar-gauge	+0.000482489584	+0.246368054	+0.000029717504960
Fermion-gauge	-0.001641778439	+2.274022931	-0.000933360454277
Strict-hard subtotal			**-0.001975390570075**

Let $A^{g^2}_{\mathrm{hard,corr}}$ be the sum of the five strict-hard operator terms. The gauge-gauge and fermion-gauge classes are finite; the remaining three retain the pole and explicit scale dependence required for matching:

$$\left.\frac{A^{(g^2)}_{\mathrm{hard,corr}}}{\Theta^2}\right|_{1/\epsilon} = \frac{0.000263263990495}{\epsilon}. \tag{58}$$

Define the explicit hard logarithmic derivative by $D_{\mathrm{hard,corr}} \equiv \partial\left(A^{(g^2)}_{\mathrm{hard,corr}}/\Theta^2\right) / \partial \ln\bar{\mu}$ with all running couplings held fixed during the partial derivative. Its value is $D_{\mathrm{hard,corr}} = 0.001053055961979$. The identity $D_{\mathrm{hard,corr}} = 4P_{\mathrm{hard,corr}}$, where $P_{\mathrm{hard,corr}}$ is the coefficient of $1/\epsilon$ in Eq. (58), checks the normalization of the logarithms. The residual pole and derivative are retained as matching data.

D. Higgs contact, electric theory, and scale closure

The strict-hard operator correlator is not yet the completed stress-response Hessian. Its pole and scale derivative must be combined with the second variation of the nonminimal Higgs operator and with the screened zero modes. The Higgs variation contributes:

$$A_{H,\mathrm{contact}} = \frac{\xi_H}{2}\langle H^\dagger H\rangle. \tag{59}$$

At the benchmark point, define the dimensionless hard contact $\delta a_H \equiv \delta A^{(g^2)}_{H,\mathrm{contact}}/\Theta^2$, its pole coefficient $p_H \equiv 0.000780071962924$, and its finite coefficient $f_H \equiv 0.002835981753442$. The hard condensate correction [17] gives

$$\delta a_H = -\xi_H\left(\frac{p_H}{\epsilon} + f_H\right). \tag{60}$$

To state the required four-dimensional running, define $\beta_{\xi_H} \equiv d\xi_H/d\ln\bar{\mu}$ and use the top-only invariant $Y_2 = 3y_t^2$ fixed above. The curved-space Standard-Model result [15] is

$$16\pi^2\beta_{\xi_H} = \left(\xi_H - \frac{1}{6}\right)\left(12\lambda + 2Y_2 - \frac{3}{2}g_Y^2 - \frac{9}{2}g_2^2\right), \tag{61}$$

Define D_{required} as the explicit derivative of the completed second-order coefficient with respect to $\ln\bar{\mu}$, holding the running parameters fixed inside that partial derivative. Renormalization-group invariance then requires

$$D_{\text{required}} = -\frac{\beta_{\xi_H}}{12}. \tag{62}$$

Numerically, $D_{\text{required}} = -0.000080194657458 + 0.000481167944750\,\xi_H$. This is an independent target, not a coefficient fitted to make the subsequent ledger close.

The electrostatic effective theory contains the Higgs zero mode, the temporal weak adjoint A_0^a with $a \in \{1,2,3\}$, and the temporal hypercharge scalar B_0. Its potential block consists of the Higgs quartic graph and the Higgs-electric figure-eight graphs. At leading order the three-dimensional couplings are $\lambda_3 = \lambda\Theta$, $h_3 = g_2^2\Theta/4$, and $h'_3 = g_Y^2\Theta/4$. For N_X real electric scalars X_A of mass m_X coupled through $h_X H^\dagger H X_A X_A$, the two finite contributions are

$$\frac{\delta A_{\lambda_3}}{\Theta^2} = -\frac{\lambda}{32\pi^2}(1 - 6\xi_H), \qquad \frac{\delta A_{HX}}{\Theta^2} = -\frac{(h_X/\Theta)N_X}{32\pi^2}\left[\left(\frac{1}{6} - \xi_H\right)\frac{m_X}{m_H} + \frac{1}{6}\frac{m_H}{m_X}\right]. \tag{63}$$

The hypercharge block uses $(X, N_X, h_X, m_X) = (B_0, 1, h'_3, m_{DY})$, while the weak block uses $(A_0, 3, h_3, m_{D2})$. Their constant and ξ_H coefficients are, respectively, $(+0.000106345764, -0.000638074587)$, $(-0.000080242946, +0.000399966933)$, and $(-0.000275777025, +0.001397449055)$. Define the dimensionless sum $\delta a_{3D,\text{pot}} \equiv \delta A^{(g^2)}_{3D,\text{pot}}/\Theta^2$, its curvature-independent coefficient $q_0 \equiv -0.000249674206$, and its coefficient linear in ξ_H, $q_\xi \equiv 0.001159341402$. The summed potential contribution is

$$\delta a_{3D,\text{pot}} = q_0 + q_\xi \xi_H. \tag{64}$$

The same expression follows from inserting the soft one-loop mass shifts into the ring term. This equality is the double-counting check: next-to-leading screening masses and explicit three-dimensional potential graphs represent the same contribution and cannot both be included. Accordingly, the ring term uses leading screening masses and the second-order potential graphs are retained explicitly.

The pole ledger has two different parts. The reduced Higgs condensate supplies $+p_H\xi_H/\epsilon$, which cancels the hard contact pole in Eq. (60) directly. The operator poles are instead distributed among the hard region, the electric and magnetostatic regions, and the local-curvature counterterm. Let $P_E = -0.001079676881389$ be the electric operator pole coefficient, let $P_M = 0.000949664887569$ be the perturbative magnetostatic pole coefficient, and let P_R be the local-curvature counterterm coefficient. Ultraviolet finiteness requires

$$-p_H\xi_H + p_H\xi_H = 0, \qquad P_{\text{hard,corr}} + P_E + P_M + P_R = 0, \qquad P_R = -0.000133251996674. \tag{65}$$

The second equality defines the pole subtraction of the renormalized local operator; it is a consistency requirement, not an independently predicted finite number. Stating the ledger this way avoids the incorrect impression that the electric theory alone cancels Eq. (58).

For each reduced-theory block, D denotes the coefficient of its explicit logarithmic derivative with respect to μ_3. The subscripts $E3$, $E2$, and EY label color-electric, weak-electric, and hypercharge-electric operator terms, while $M3$ and $M2$ label the color and weak magnetostatic terms. Their values are $D_{E3}^{\mathrm{corr}} = -0.002999279576$, $D_{E2}^{\mathrm{corr}} = -0.001199069961$, $D_{EY}^{\mathrm{corr}} = -0.000120357988$, $D_{M3} = +0.002999279576$, and $D_{M2} = +0.000799379974$. In this pole convention, $P_M = (D_{M3} + D_{M2})/4$. Define $D_{\mathrm{soft,corr}}$ as their sum. It is

$$D_{\mathrm{soft,corr}} = -\frac{g_Y^2 + 3g_2^2}{192\pi^2}. \tag{66}$$

The reduced Higgs condensate contributes $+0.003120287851698\,\xi_H$ to the scale derivative. Together with the hard Higgs-contact derivative $-0.002639119906948\,\xi_H$, it yields $+0.000481167944750\,\xi_H$, exactly the coefficient in Eq. (62). Define the dimensionless matching coefficient $c_\xi \equiv \left(g_Y^2 + 3g_2^2\right)/\left[2(4\pi)^2\right] = 0.003120287851698$. The finite Higgs matching is

$$\frac{\delta A_{H,\mathrm{matched}}^{(g^2)}}{\Theta^2} = c_\xi \xi_H \left[\ln\frac{\bar{\mu}}{2m_H} + \frac{1}{4}\right]. \tag{67}$$

The massive electric scalar-gauge graphs also contain an absolute finite threshold. Let m denote the massive scalar screening mass in one charged block, and let Γ denote the Euler gamma function. In dimensional regularization, the combined exchange-plus-seagull curvature term is proportional to $\Gamma(3-d)(m^2)^{d-3}I(d)$. To specify the dimensionless Feynman-parameter integral, take $x \in [0,1]$, $u \in [0,\infty)$, and define $a_x \equiv x(1-x)$. Then

$$N_d(x,u) \equiv a_x(d+4a_x) + d(4a_x+1)u + (5d-32a_x-2)u^2 + 4(d-6)u^3,$$
$$I(d) \equiv -\frac{1}{24}\int_0^1 dx \int_0^\infty du\, \frac{N_d(x,u)}{[a_x+u]^{d/2+2}}. \tag{68}$$

Here d is the continued spatial dimension defined above, and a prime denotes differentiation with respect to d. Evaluation gives $I(3) = -\pi$ and $I'(3)/I(3) = 2\ln 2 - 5/9$. Defining $c_{sg}^{(m)}$ as the finite massive scalar-gauge threshold relative to the logarithm, one obtains

$$c_{sg}^{(m)} = -\frac{I'(3)}{2I(3)} = \frac{5}{18} - \ln 2 = -0.415369402782167. \tag{69}$$

For a charged-scalar block indexed by i, let m_i be its screening mass and D_i its logarithmic coefficient. That block contributes $D_i\left[\ln(\mu_3/m_i) + c_{sg}^{(m)}\right]$. The three coefficients required here are $D_{A_0^{(3)}} = -24g_3^2/\left(192\pi^2\right)$ for the color-electric adjoint, $D_{A_0^{(2)}} = -6g_2^2/\left(192\pi^2\right)$ for the weak-electric adjoint, and $D_{Hg} = -\left(g_Y^2 + 3g_2^2\right)/\left(192\pi^2\right)$ for the Higgs-gauge block. Their masses are m_{D3}, m_{D2}, and m_H, respectively. At $\mu_3 = g_2^2\Theta$, the three completed blocks are 0.004253519982, 0.001124469063, and 0.000291448990, which sum to 0.005669438036. This exposes both the unequal mass logarithms and the common threshold in Eq. (69).

A temporary magnetic regulator mass m_M provides a direct Ward-identity check on the separation. The current-exchange and covariant-kinetic seagull terms each contain a contribution proportional to $1/m_M$, but

their coefficients are equal and opposite in the completed metric variation. Their sum therefore has no powerlike magnetic term. The regulator is removed after this cancellation and does not enter any reported coefficient.

Finally, let C_R^{3D} denote the dimensionless local three-dimensional curvature matching coefficient and define $D_{\mathrm{soft}} \equiv D_{\mathrm{soft,corr}} = -0.000520047975283$. The constant part of the four-dimensional renormalization condition fixes, rather than predicts, the diagonal logarithmic coefficient:

$$D_{R,\mathrm{diag}} \equiv D_{\mathrm{required}}\big|_{\xi_H=0} - D_{\mathrm{hard,corr}} - D_{\mathrm{soft}} = -0.000613202644155. \tag{70}$$

It is the derivative obtained when $\bar{\mu}$ and μ_3 are varied together, not a third independent beta function. Renormalization-group equations determine the logarithms but leave an additive finite local-R convention. We define that constant to vanish at $\bar{\mu} = 2\pi\Theta$ in the thermal-subtracted $\overline{\mathrm{MS}}$ matching convention. With this definition, the two-scale bridge is

$$C_R^{3D} = D_{\mathrm{soft}} \ln\frac{\bar{\mu}}{\mu_3} + D_{R,\mathrm{diag}} \ln\frac{\bar{\mu}}{2\pi\Theta}, \tag{71}$$

At $\mu_3 = g_2^2\Theta$ this gives $C_R^{3D} = -0.001671619098126$. A different finite local-R convention would shift this coefficient and the quoted susceptibility by the same local amount, so numerical comparisons must use the convention just stated. At fixed $\bar{\mu}$, the μ_3 derivative of the electric, magnetostatic, and curvature terms vanishes; this is an independent reduced-theory closure. At fixed μ_3, the constant equality $0.001053055961979 - 0.000520047975283 - 0.000613202644155 = -0.000080194657459$ verifies the numerical assembly of the imposed four-dimensional matching condition, but it is not a second prediction. By contrast, the ξ_H coefficient closes against Eq. (62) after the hard and reduced condensate derivatives are evaluated.

Define the perturbatively matched part by $a_{\mathrm{pert}} \equiv \left(A_{2,E} - A_M^{\mathrm{NP}}\right)/\Theta^2$. With $p_0 \equiv 0.263713376086$ and $p_\xi \equiv 0.066624158194$, the endpoint before the nonperturbative magnetostatic boundary is

$$a_{\mathrm{pert}} = p_0 + p_\xi \xi_H. \tag{72}$$

Table 3 exposes the numerical assembly. Each row has a distinct origin, so agreement of only the final sum would be insufficient. The pole ledger, reduced-theory scale cancellation, four-dimensional running condition, and row-by-row arithmetic are tested separately. The subtotal is evaluated from the unrounded analytic expressions; the displayed row values are rounded audit summaries.

Table 3. Assembly of the matched coefficient.

Contribution	Constant part	Coefficient of ξ_H
Free plus ring	+0.261940621924	+0.060106452558
Strict-hard operator	-0.001975390570075	0
Hard Higgs contact	0	-0.002835981753442
Three-dimensional potential	-0.000249674206	+0.001159341402
Matched finite Higgs contact	0	+0.008194345988
Massive electric loop	+0.005669438036	0
Local-curvature bridge	-0.001671619098	0
Through electric/local matching	**+0.263713376086**	**+0.066624158194**

E. Magnetostatic remainder as a lattice observable

After the hard and electric scales are integrated out, the spatial SU(3) and SU(2) zero modes form confining pure three-dimensional Yang–Mills theories. Magnetostatic denotes this unscreened spatial gauge sector, not an externally applied magnetic field. No confining Abelian contribution occurs at this order.

Let $G = \mathrm{SU}(N_c)$ denote either non-Abelian group. Its adjoint Casimir and adjoint dimension are $C_A \equiv N_c$ and $d_G \equiv N_c^2 - 1$. Let g_G denote the corresponding four-dimensional gauge coupling. Define the three-dimensional gauge coupling by $g_{M,G}^2 \equiv g_G^2 \Theta$ and its intrinsic dimensionful scale by $\lambda_G \equiv C_A g_{M,G}^2$. Let $A_{M,G}^{\mathrm{NP}}$ be the contribution of group G to the magnetostatic remainder, let $D_{M,G}$ be the known coefficient of its logarithm, and let c_G^{NP} be the finite, scheme-defined nonperturbative boundary constant. Dimensional analysis and the ultraviolet logarithm then imply

$$\frac{A_{M,G}^{\mathrm{NP}}(\mu_3)}{\Theta^2} = D_{M,G}\left[\ln\frac{\mu_3}{\lambda_G} + c_G^{\mathrm{NP}}\right]. \tag{73}$$

For SU(3) and SU(2), respectively, $D_{M,3} = 0.002999279576$ and $D_{M,2} = 0.000799379974$. At the benchmark choice $\mu_3 = g_2^2\Theta$, define the known logarithmic offset $b_0 \equiv -0.003657169408$, and define the two weights $b_3 \equiv D_{M,3}$ and $b_2 \equiv D_{M,2}$. The two-group result is

$$\frac{A_M^{\mathrm{NP}}}{\Theta^2} = b_0 + b_3 c_3^{\mathrm{NP}} + b_2 c_2^{\mathrm{NP}}, \tag{74}$$

Combining Eqs. (72) and (74), define $a_0 \equiv 0.260056206678$, $a_\xi \equiv 0.066624158194$, $a_3 \equiv 0.002999279576$, and $a_2 \equiv 0.000799379974$. The central result can then be written compactly as

$$\frac{A_{2,E}}{\Theta^2} = a_0 + a_\xi \xi_H + a_3 c_3^{\mathrm{NP}} + a_2 c_2^{\mathrm{NP}}, \tag{75}$$

The c_G^{NP} are not fitting coefficients and are not adjusted to reproduce a gravitational measurement. They are local responses of pure Yang-Mills theory in the same $\overline{\mathrm{MS}}$ convention as the perturbative matching. The SU(3) and SU(2) weights carry 78.956% and 21.044% of the known magnetostatic logarithmic coefficient, respectively; this split does not assume that the two finite constants are equal.

To determine the constants, let g_{ij} be a weak Euclidean spatial metric with determinant $g \equiv \det(g_{ij})$, inverse g^{ij}, and shear component $g_{12} = g_{21} = h(z)$. Let F_{ij}^a be the Yang-Mills field strength, with spatial indices $i, j, k, l \in \{1,2,3\}$ and adjoint color index a; repeated spatial and color indices are summed. For one group, abbreviate $g_{M,G}$ to g_M. The continuum action is

$$S_G[g] = \frac{1}{4g_M^2} \int d^3x \sqrt{g}\, g^{ik} g^{jl} F_{ij}^a F_{kl}^a, \tag{76}$$

The first derivative with respect to h defines $T_{12} \equiv F_{13}^a F_{23}^a / g_M^2$. The second derivative gives the local Yang-Mills Lagrangian density $\mathcal{L}_{\mathrm{YM}} \equiv F_{ij}^a F_{ij}^a / (4g_M^2)$ on the flat background. Consequently, define the completed positive-correlator response by

$$\mathcal{G}_G(k) = \int d^3x\, e^{ikz} \langle T_{12}(x) T_{12}(0) \rangle_c - \langle \mathcal{L}_{\mathrm{YM}} \rangle. \tag{77}$$

The second term is the microscopic contact required by the Ward identity. Since it is momentum independent on the flat background, it cancels from momentum differences, but it is retained in the definition so that the lattice and continuum Hessians have the same normalization. For the shear metric used here, the geometric normalization is $\int d^3x \sqrt{g}\, R = -1/2 \int d^3x (\partial_z h)^2 + O(h^3)$ up to a periodic total derivative, where R is the three-dimensional Ricci scalar. Define $C_G^{\overline{\mathrm{MS}}}(\mu_3)$ as the renormalized coefficient of k^2 in the small-momentum expansion,

$$\mathcal{G}_G^{\overline{\mathrm{MS}}}(k; \mu_3) = \mathcal{G}_G(0) + C_G^{\overline{\mathrm{MS}}}(\mu_3) k^2 + O(k^4). \tag{78}$$

The perturbative ultraviolet result fixes the running of this slope, and dimensional reduction fixes its conversion to the four-dimensional susceptibility:

$$\frac{\partial C_G^{\overline{\mathrm{MS}}}}{\partial \ln \mu_3} = \frac{d_G \lambda_G}{96\pi^2}, \qquad \frac{A_{M,G}^{\mathrm{NP}}}{\Theta^2} = \frac{C_G^{\overline{\mathrm{MS}}}(\mu_3)}{2\Theta}. \tag{79}$$

The first equality reproduces $D_{M,G}$ after the second is differentiated, independently fixing the factor of two. Combining these relations with Eq. (73) and solving for the finite boundary constant gives

$$c_G^{\mathrm{NP}} = \frac{96\pi^2}{d_G \lambda_G} C_G^{\overline{\mathrm{MS}}}(\mu_3) - \ln \frac{\mu_3}{\lambda_G}. \tag{80}$$

On a flat cubic lattice, let a_L be the lattice spacing, let L be the linear physical extent, and let $\mathcal{G}_G^L(k; a_L, L)$ be the completed Hessian obtained from the same two source derivatives of the lattice action. An absolute lattice slope contains an additive local coefficient $C_R^L(a_L)$. It need not be computed separately. For a momentum k along the third lattice axis, define the improved lattice momentum $\widehat{k} \equiv (2/a_L)\sin(a_L k/2)$. For any two distinct momenta k_a and k_b—whose subscripts are labels, not powers of the spacing—define the secant

$$\mathcal{S}_G^L(k_a, k_b) = \frac{\mathcal{G}_G^L(k_a) - \mathcal{G}_G^L(k_b)}{\widehat{k}_a^2 - \widehat{k}_b^2}, \tag{81}$$

Choose k_{i1} and k_{i2} in an infrared window and k_{u1} and k_{u2} in an ultraviolet matching window on the same ensemble. Define the infrared-minus-ultraviolet difference

$$\Delta\mathcal{S}_G^L = \mathcal{S}_G^L(k_{i1}, k_{i2}) - \mathcal{S}_G^L(k_{u1}, k_{u2}). \tag{82}$$

Both $C_R^L(a_L)$ and the momentum-independent contact cancel algebraically because they are common to the two secants. Continuum perturbation theory supplies the ultraviolet anchor in the window $\lambda_G \ll k_u \ll a_L^{-1}$, where k_u denotes either ultraviolet momentum, and confinement supplies the analytic infrared anchor. Let $\mathcal{S}_{G,\mathrm{pert}}^{\overline{\mathrm{MS}}}$ denote the secant computed from the renormalized continuum ultraviolet response, and let $k_i \to 0$ mean that both members of the infrared pair approach zero at fixed ratio. The absolute coefficient is

$$C_G^{\overline{\mathrm{MS}}}(\mu_3) = \mathcal{S}_{G,\mathrm{pert}}^{\overline{\mathrm{MS}}}(k_{u1}, k_{u2}; \mu_3) + \lim_{k_i \to 0} \lim_{L \to \infty} \lim_{a_L \to 0} \Delta\mathcal{S}_G^L. \tag{83}$$

The order of limits is operational: at fixed physical momenta take the continuum limit $a_L \to 0$, then the infinite-volume limit $L \to \infty$, and only then send the infrared pair to zero. Reversing these limits can mix discretization or finite-volume effects with the curvature slope.

For the ultraviolet anchor, define $\ell(k) \equiv 8\ln(k^2/\mu_3^2)/(3\pi^2)$, $u_1 \equiv -1 + 20/(3\pi^2)$, and $u_2 \equiv 431/768 - 37/(9\pi^2)$. The ellipsis below denotes terms of higher order in λ_G/k. The three-loop ultraviolet response [18] is

$$\mathcal{G}_{G,\mathrm{pert}}^{\overline{\mathrm{MS}}}(k) = \frac{d_G}{512}\left\{k^3 + \lambda_G k^2[u_1 - \ell(k)] + \lambda_G^2 k u_2 + \cdots\right\}, \tag{84}$$

A constant ultraviolet contact is omitted because it drops out of the secant. The extraction is deliberately overconstrained: continuum, volume, infrared-momentum, and ultraviolet-window extrapolations must each form a stable plateau; variation of μ_3 must reproduce the known logarithm; and a second shear discretization or gradient-flow energy-momentum tensor [19–20] must give the same continuum result. Agreement among these routes tests both the normalization of the source derivative and the cancellation of the additive lattice counterterm.

Published glueball masses [21] do not supply $C_G^{\overline{\mathrm{MS}}}$. If $A_0(p^2)$ and $A_2(p^2)$ denote the scalar and tensor stress form factors at Euclidean momentum squared p^2, the commonly studied superconvergent combination $[A_0(p^2) + A_2(p^2)]/(p^2)^2$ cancels local terms proportional to p^2 [18]. A gapped spectrum validates the analytic infrared expansion but is intentionally insensitive to the local curvature coefficient isolated by Eq. (83). Equation (83) therefore defines a curvature-response observable that is not determined by the published glueball spectrum.

F. Closure tests

Equation (75) gives the renormalized static transverse-traceless susceptibility through $\mathcal{O}(g^2)$ for the symmetric high-temperature Standard Model in the stated top-Yukawa truncation. It combines the free response, ring-resummed zero modes, five strict-hard interaction sectors, the Higgs second-variation contact, the electrostatic effective theory, the massive scalar-gauge threshold, and the local-curvature bridge. In the thermal-subtracted $\overline{\mathrm{MS}}$ convention, these contributions define one matched coefficient rather than a sum of independently adjustable terms.

The closure tests establish the consistency of this coefficient across its effective theories. The hard Higgs-contact poles cancel against the reduced condensate, and the remaining operator poles are removed by the local-

curvature counterterm. The dependence on the three-dimensional factorization scale μ_3 cancels at fixed $\bar{\mu}$. The residual four-dimensional scale derivative equals the running required by the Higgs-curvature coupling ξ_H. The analytic massive scalar-gauge threshold, the sector-by-sector hard reductions, and the numerical assembly of Eq. (72) provide additional checks on the finite terms. The quantities P_R and $D_{R,\mathrm{diag}}$ retain their distinct roles as the required pole subtraction and the finite matching condition, while the additive finite local-curvature boundary is fixed by the declared renormalization convention.

The two coefficients c_3^{NP} and c_2^{NP} are the finite magnetostatic responses of pure three-dimensional SU(3) and SU(2) Yang-Mills theory. They are scheme-defined observables rather than fit parameters. Equation (83) expresses each coefficient as a continuum ultraviolet anchor plus an infrared-minus-ultraviolet lattice difference. The common additive curvature counterterm and the momentum-independent contact cancel in that difference. The ordered continuum, infinite-volume, and zero-momentum limits, together with the ultraviolet matching window and operator-normalization comparison, define the measurement completely.

The result applies to the symmetric phase and the stated top-Yukawa truncation. The numerical benchmark is a dimensionless snapshot at $\mu = 2\pi\Theta$, not a unique thermal epoch. Within that domain, every perturbative term through $\mathcal{O}(g^2)$ belongs to a hard, electric/Higgs, contact, magnetostatic, or local-curvature sector. The final expression depends only on the renormalized ξ_H, the two Yang–Mills response coefficients, and the stated finite local-curvature convention.

The completed matter stress Hessian therefore has a finite local k^2 coefficient after variational contacts, counterterms, and scale matching are included. Its perturbative and effective-theory content closes through second order, and all nonperturbative dependence is isolated in two explicitly defined lattice observables. The static matter-response normalization is thereby fixed independently of any measured gravitational value.

IV. CAUSAL FINITE-FREQUENCY AND FINITE-WAVE-NUMBER RESPONSE

A. Retarded kernel and the underdamped branch

The static TT coefficient fixes the zero-frequency slope but not the causal content of the same matter response. At finite frequency and wave number, isolated poles represent discrete resonances, branch cuts represent excitation continua, and adjacent Riemann sheets encode analytic continuation through those cuts. These structures determine whether the infrared compliance is accompanied by ordinary damping, additional modes, or a critical pole merger.

The static and finite-frequency results play distinct roles. Inverting the completed small-k kernel gives the $1/k^2$ long-range compliance, whereas the retarded calculation gives two excitation continua and one simple underdamped resonance on the two-particle-production sheet. A Lorentzian massless wave operator additionally requires the locally analytic two-helicity infrared branch stated in Sec. V; it is not inferred from the thermal resonance.

The long-range contribution follows directly by inverting the small-k static kernel of Sec. III. Let $D_{\mathrm{TT},E}(0,k)$ be the static Euclidean TT compliance at spatial wave-number magnitude k. Then

$$D_{TT,E}(0,k) = \frac{1}{A_{2,E}k^2 + O(k^4)} \qquad (k \to 0). \tag{85}$$

Thus the compliance contains the massless $1/k^2$ branch required for a long-range interaction before any finite-frequency resonance is discussed. The role of the causal calculation is instead to determine the additional microscopic dispersive and absorptive structure carried by the same stress Hessian.

A thermal equilibrium state selects a rest frame, so temporal frequency and spatial wave number become independent response coordinates. Let z be the complex retarded frequency, k the spatial wave-number magnitude,

T the equilibrium temperature, k_B Boltzmann's constant, c the locally measured limiting signal speed, and $\hbar$ the reduced Planck constant. It is useful to write

$$u=\frac{\hbar z}{k_B T}, \qquad y=\frac{\hbar c k}{k_B T}, \tag{86}$$

The real-time free-field structures for scalar, Weyl and gauge sectors follow the thermal spinning-field response of Ota, Zhu and Zhu and the finite-temperature graviton self-energy decomposition of Brandt and Frenkel [22–23]. Let ω denote a real positive angular frequency. The collisionless tensor response contains two physically distinct continua. The scattering or Landau cut occupies $0 < \omega < ck$, while the two-particle-production cut begins at $\omega > ck$ for massless constituents. These two cuts are not artifacts of a particular representation; they are the real-time manifestation of the distinct kinematic processes available to the thermal matter state.

This is also why the dynamical and static limits must be kept separate. The Landau continuum extends all the way to the origin along the real-frequency direction, so the response remembers the path by which (ω, k) approaches $(0,0)$. A static shear experiment and a collisionless low-frequency experiment therefore probe different limits of the same kernel. In general,

$$\lim_{\omega\to 0}\lim_{k\to 0}\Pi^R_{TT}(\omega,k) \neq \lim_{k\to 0}\lim_{\omega\to 0}\Pi^R_{TT}(\omega,k). \tag{87}$$

The Euclidean static stiffness of Sec. III is therefore a legitimate equilibrium projection, but it should not be confused with the collisionless retarded limit taken along another path. Both belong to the same matter response, but they answer different physical questions.

The general-kinematics thermal master functions of Jackson and the complete Standard-Model TT algebra of Ghiglieri, Jackson, Laine and Zhu make this construction explicit [16, 24]. The finite-ω, k kernel can be constructed without relying on unstable near-cut two-dimensional quadrature. After subtracting the local polynomial terms already assigned to the static response, let $R(z,k)$ denote the resulting dimensionless nonlocal thermal TT response factor and $R^I(z,k)$ its principal-sheet reconstruction. The notation Im means imaginary part, and $+i0$ means that the real axis is approached from the retarded upper side. For fixed real k, the principal-sheet response is obtained from the exact real-axis discontinuity by the subtracted spectral relation

$$R_{\mathrm{I}}(z,k)=\frac{2}{\pi}\int_0^\infty d\,\omega\,\frac{\omega\,\mathrm{Im}\,R(\omega+i0,k)}{\omega^2-z^2}. \tag{88}$$

The two-particle-production and Landau intervals are integrated separately, with scalar, Weyl, and gauge-vector contributions carrying the physical TT weights derived in Sec. II.B. At the benchmark wave number, the reconstruction agrees with direct imaginary-axis evaluation of the complete collisionless Standard-Model thermal kernel to better than one part in 10^9. This agreement distinguishes convergent sheet poles from numerical discretizations of a continuum.

A decaying resonance is found after the retarded boundary value is continued onto the adjacent sheet of the appropriate cut. Write $\mathrm{Disc}_P R$ for the discontinuity across the positive-frequency two-particle-production cut, use the label II, P for the adjacent two-particle-production sheet, and let s denote the local argument of the logarithmic factor whose branch cut is crossed. The label P denotes the two-particle-production cut throughout, and Log on sheet I is the principal complex logarithm. Then

$$R_{\mathrm{II},P}(z,k)=R_{\mathrm{I}}(z,k)+\mathrm{Disc}_P R(z,k), \qquad \mathrm{Log}_{\mathrm{II},P}s=\mathrm{Log}s-2\pi i, \tag{89}$$

where the logarithmic continuation is applied only to the cut actually crossed. The Landau continuation uses its own thermal discontinuity and does not inherit this vacuum-log shift. This sheet-by-sheet construction is essential: a global replacement of every logarithm by $\mathrm{Log}-2\pi i$ does not represent the retarded analytic structure correctly.

Let $F_{\mathrm{II},P}(u,y)$ be the dimensionless zero-forming inverse-response eigenvalue on that positive-frequency two-particle-production sheet, so a pole satisfies $F_{\mathrm{II},P}(u_P,y)=0$. At

$$y=0.758224684, \tag{90}$$

the physical positive-frequency two-particle-production sheet contains the stable pole

$$u_P=1.353089335-0.093439111\,i, \qquad \left|\partial_u F_{\mathrm{II},P}(u_P)\right|=5.39112. \tag{91}$$

The derivative is decisively nonzero. The pole is therefore simple. Here $\mathrm{Re}u_P$ and $\mathrm{Im}u_P$ denote the real and imaginary parts of the pole coordinate. Its damping-to-oscillation ratio is

$$\frac{-\mathrm{Im}\,u_P}{\mathrm{Re}\,u_P}=0.0691. \tag{92}$$

It is an ordinary underdamped resonance rather than an exceptional point. The distinction is not semantic: at an exceptional point both the inverse response and its first derivative vanish, whereas Eq. (91) shows that the known causal resonance branch on the two-particle-production sheet is far from that condition.

The same pole can be tracked continuously over real wave number. Representative collisionless values are

y	$u_P(y)$
0.4	$0.9600-0.0629\,i$
0.7582	$1.35309-0.09344\,i$
1.0	$1.58615-0.10564\,i$
2.0	$2.48635-0.11775\,i$
3.0	$3.38920-0.10557\,i$
4.0	$4.31703-0.08972\,i$
5.0	$5.26477-0.07568\,i$

(93)

and the magnitude of $\partial_u F$ grows rather than tends toward zero across the upper part of this range. Extending the branch much farther in y preserves the same qualitative behavior: the ordinary resonance remains underdamped, and the damping fraction decreases at sufficiently large momentum.

The Landau sector behaves differently. Finite-quadrature representations generate many apparent zeros immediately below the Landau interval, but their imaginary parts shrink in inverse proportion to the spectral resolution. They therefore collapse onto the branch cut rather than converge to isolated physical poles. The causal matter response contains a Landau continuum, not a hidden forest of stable long-lived modes. This numerical convergence test is important because near-cut discretization can otherwise mimic precisely the sort of extra pole one might incorrectly interpret as new gravitational physics.

Standard-Model interactions contribute thermal dispersion shifts, quasiparticle decay widths, and a finite Ward-completed vertex/self-energy deformation. Ward consistency requires the self-energy and gravitational vertex to be resummed together [25–27]. The first two effects translate an existing analytic structure and therefore preserve the multiplicity of a simple root. Let F_0 be the unbroadened inverse response and u_0 one of its simple pole coordinates. If Γ_{phys} is the physical decay rate, define the dimensionless width $\Gamma \equiv \hbar\Gamma_{\mathrm{phys}}/(k_B T)$. A common hard width gives

$$F_\Gamma(u,y)=F_0\left(u+i\hat{\Gamma},y\right) \quad \Rightarrow \quad u_\Gamma=u_0-i\hat{\Gamma}, \qquad F_\Gamma'(u_\Gamma)=F_0'(u_0). \tag{94}$$

A width moves the pole deeper into the lower half-plane but cannot make a simple root double. The same statement holds for a pure invariant thermal-mass translation as long as the shifted pole remains away from the origin.

A two-particle-production threshold also does not become an exceptional point merely because a pole reaches it. Near a two-particle branch point u_{th}, introduce the uniformizing coordinate

$$w=\sqrt{u\text{-}u_{\mathrm{th}}}. \tag{95}$$

Let $F(w)$ denote the inverse response re-expressed in the uniformizing coordinate w. It is locally analytic in w. A simple pole passing through $w = 0$ changes sheets while remaining a simple zero. An exceptional point requires the additional independent condition

$$F(0)=0, \qquad \partial_w F(0)=0. \tag{96}$$

This distinction is central to the interacting topology. Threshold crossing is generically a sheet transition, not a critical pole merger.

The quasiparticle-pole interpretation used in this high-momentum discussion follows Weldon, while the non-Abelian asymptotic-mass organization and its nonperturbative context are discussed by Moore and Schlusser [28–29]. The leading Standard-Model thermal masses make the nearest gauge thresholds explicit. Let $m_{\infty,a}$ denote the leading asymptotic thermal quasiparticle mass for field class a, and define its frequency scale by $\Omega_{\infty,a}=m_{\infty,a}c^2/\hbar$. Then

$$\omega_{\mathrm{th},a}=\sqrt{c^2k^2+4\Omega_{\infty,a}^2}. \tag{97}$$

At the benchmark wave number, the leading dressed thresholds are approximately $1.233752\,k_BT/\hbar$ for gluons, $1.225000\,k_BT/\hbar$ for the weak gauge bosons, and $1.187933\,k_BT/\hbar$ for hypercharge, while the real part of the causal resonance is $1.353089\,k_BT/\hbar$. Including the corresponding leading widths moves the nearest branch points into the lower half-plane. Their complex separations from the pole are approximately $0.120\,k_BT/\hbar$ for the gluon two-particle branch point and $0.134\,k_BT/\hbar$ for the weak two-particle branch point. The pole is close enough that the exact finite interaction correction matters for precision, but it is not coincident with either threshold.

The size of the deformation required to change the local topology can be estimated directly from the known simple root u_0. Let δF be a perturbation of the inverse response about that root and δu the induced pole shift. To first order,

$$\delta u=-\frac{\delta F(u_0,y)}{F_0{}'(u_0,y)}+O(\delta F^2). \tag{98}$$

Dragging the benchmark pole all the way to the nearest dressed gluon branch point requires a normalized change of the inverse response of approximately 0.65, and reaching the weak branch point requires approximately 0.72. Those are order-one deformations of the zero-forming function, not ordinary percent-level shifts. The complete two-loop Standard-Model TT algebra is gauge independent in its assembled sectors, and the benchmark kinematics lies in the hard timelike regime addressed by the general-kinematics thermal master functions, well outside the external soft/light-cone region where fixed-order thermal perturbation theory loses its ordinary power counting. No known soft, collinear, threshold, polarization-mixing or kinetic mechanism supplies the anomalous enhancement needed to change the local root count.

The helicity structure supplies one further stability check. In an isotropic parity-invariant state, rotations about the propagation axis keep helicities +2 and -2 distinct while parity keeps their eigenvalues equal. The physical TT block therefore remains proportional to the identity in the two-helicity subspace. This degeneracy is semisimple: the

two polarization eigenvectors remain independent, so ordinary Standard-Model interactions cannot turn polarization degeneracy itself into a non-Hermitian exceptional point.

The resulting causal picture is consequently simple and physically interpretable:

massless long-range branch
+one underdamped causal TT resonance
on the two-particle-production sheet
+Landau/scattering and two-particle-production continua. (99)

The controlled interaction deformation adds ordinary dispersion and damping while leaving that topology intact. The nonzero derivative at the pole and the interaction bounds fix the branch classification: higher-order corrections refine its coordinate, reactive dispersion scale and damping rate without changing the established simple underdamped topology in the absence of a new singular mechanism.

The hierarchy is therefore explicit. Equation (85) fixes the static long-range compliance, while the finite-frequency pole measures microscopic coherence and damping of the matter state. The pole need not become critically damped for the static interaction to exist: ordinary weakly coupled Standard-Model matter realizes a simple underdamped resonance. The causal massless propagation used below is the separate, testable infrared branch condition stated in Sec. V.

B. Finite wave number and the analytic benchmark

Three analytically distinct structures remain: the ordinary retarded resonance of weakly coupled matter, an exact double zero of a simplified analytic kernel, and a critical self-loading condition that would belong to a different physical boundary. The first is a causal pole, the second a mathematical benchmark, and the third an independent constitutive condition.

The principal sheet is the default logarithmic branch before a physical cut is crossed; the retarded sheet of a decaying resonance is reached by continuing through the appropriate cut. The general-momentum Standard-Model tensor organization and spectral masters follow Refs. [16, 24], and the Ward-complete interaction consistency follows Ref. [27].

A Kubo–Martin–Schwinger (KMS) equilibrium state selects a rest frame, so temporal and spatial response coordinates are independent. Let z be complex frequency, $k \equiv |\mathbf{k}|$ the real spatial wave-number magnitude, T the equilibrium temperature, k_B Boltzmann's constant, c the locally measured limiting signal speed, and $\hbar$ the reduced Planck constant. Define $u \equiv \hbar z/(k_B T)$ and $y \equiv \hbar c k/(k_B T)$. On the positive-frequency two-particle-production sheet, write the pole in terms of a real reactive coordinate $E_P(y)$ and positive dimensionless damping coordinate $\gamma_P(y)$ as

$$u_P(y) = \mathcal{E}_P(y) - i\,\gamma_P(y), \tag{100}$$

the real part $\mathcal{E}_P$ describes reactive dispersion and $\gamma_P > 0$ describes decay. This decomposition is preferable, once interactions are present, to treating the entire quantity $u_P^2 - y^2$ as one fixed complex "mass." Thermal forward scattering shifts the reactive dispersion while collisions supply an additional width, and those two effects need not share the same large-y asymptote.

The collisionless calculation nevertheless provides a useful guide to the finite-wave-number structure. At small real wave number the tracked resonance branch is gapless and behaves approximately as

$$u_P^2(y) \simeq 2.28\,y \qquad (y \ll 1), \tag{101}$$

whereas at large real wave number the collisionless branch approaches

$$u_P^2(y) - y^2 \to 2.931 - 0.480\,i. \tag{102}$$

Equation (102) is not promoted to a permanent complex particle mass. It is the asymptotic form of the collisionless thermal branch. In the interacting theory the cleaner observables are the reactive dispersion scale obtained from $\mathcal{E}_P^2$-y^2 and the damping rate γ_P separately. Ordinary hard Standard-Model collisions dominate the collisionless $1/y$ decay width beyond several thermal momenta while the fractional damping continues to decrease, so the large-y continuation remains naturally underdamped rather than evolving toward a critical pole merger.

The static finite-wave-number response provides an independent projection. The scalar, Weyl and vector expressions can all be evaluated at zero Matsubara frequency while retaining nonzero spatial momentum, and their small-y slopes independently reproduce the Kubo coefficients of Sec. III. This is a strong normalization check because the finite-y functions are not obtained by fitting the zero-momentum answer. Let $F_{\mathrm{stat}}(y)$ denote the dimensionless static transverse-traceless inverse-response factor in this projection. When the leading resummed soft contribution is combined with the free thermal functions, F_{stat} develops two positive roots,

$$F_{\mathrm{stat}}(y_i)=0, \qquad F_{\mathrm{stat}}{}'(y_i)\neq 0, \qquad i=1,2. \tag{103}$$

Neither root is a spatial critical double zero. More importantly, when the same free thermal model is recomputed without the soft ring contribution, the zeros disappear and are replaced by a shallow positive minimum. The finite-y roots are therefore useful diagnostics of thermal spatial dispersion rather than fundamental correlation lengths. They show directly that a thermal rest frame breaks the simple temporal/spatial coincidence of the vacuum skeleton; they do not authorize the introduction of a new spatial scale by hand.

The leading one-log model is exactly soluble. Let $\mathrm{Log}x$ denote the principal complex logarithm, and let the Lambert-W function be the inverse of $w\mapsto we^w$. Let x be the dimensionless invariant coordinate and $A>0$ its dimensionless matter-response coefficient. The zero-forming factor is

$$F_{\mathrm{an}}(x;\mathcal{A})=1+\frac{x\mathrm{Log}x}{\mathcal{A}}. \tag{104}$$

Its nonzero roots satisfy

$$x\mathrm{Log}x=-\mathcal{A}, \tag{105}$$

and are generated by the two real Lambert-W branches W_0 and W_{-1}, defined by $W(q)e^{W(q)}=q$,

$$x_{\pm}=\exp\left[W_{0,-1}(-\mathcal{A})\right]. \tag{106}$$

The two real branches meet at the critical value

$$\mathcal{A}_c=\frac{1}{e} \tag{107}$$

At this value, the two principal-sheet roots merge at

$$x_c=\frac{1}{e}, \qquad F_{\mathrm{an}}(x_c)=0, \qquad F_{\mathrm{an}}{}'(x_c)=0, \qquad F_{\mathrm{an}}{}''(x_c)=e^2\neq 0. \tag{108}$$

Equation (108) is an exact nondegenerate double zero of the leading analytic representation. The assembled leading interaction correction alters the coefficient that maps the Standard-Model matter content into $\mathcal{A}$ but does not destroy this Lambert-W branch topology, and the leading thermal statistical memory likewise leaves a well-defined principal-sheet benchmark. At the central thermal state the corresponding dimensionless root magnitude is 3.507691515625.

The causal meaning is nevertheless different. A decaying physical resonance is obtained by continuing the retarded response through the appropriate cut into the lower half-plane. As Sec. IV.A emphasized, that continuation must be performed cut by cut. The derivative condition satisfied by Eq. (108) on the principal logarithmic sheet is changed by the second-sheet discontinuity. The leading retarded-sheet calculation therefore does not identify the Lambert merger with the physical resonance on the two-particle-production sheet of Eq. (91). The principal-sheet double zero remains valuable as an analytic benchmark against which interaction and memory corrections can be organized; it is not relabeled as a critical-damping pole merely because both calculations contain a double-root condition.

This separation also clarifies the role of complex spatial continuation. Let $m_\infty^2 \equiv 2.931 - 0.480\,i$ denote the dimensionless collisionless asymptotic constant in Eq. (102), and let m_∞ denote a chosen square root of m_∞^2, with the branch selected consistently for the reduced dispersion. If one approximates the high-y collisionless branch by

$$u_\pm(y) \simeq \pm\sqrt{y^2+m_\infty^2}, \tag{109}$$

the reduced dispersion has square-root branch points at formally complex wave number $y=\pm i m_\infty$. That observation is useful as a guide to a spatial coherence problem, but it is not an established exceptional point of the complete thermal kernel. Continuing the exact thermal integrals to complex y rotates internal integration contours through the poles of the Bose and Fermi distributions, so the correct continuation requires contour deformation and the associated thermal residues. Spatial screening, temporal pole position and causal attenuation are therefore distinct response observables in a thermal state.

The response structure therefore contains a static $1/k^2$ compliance, Landau and two-particle-production continua, and one simple underdamped resonance on the adjacent production sheet. The principal-sheet Lambert-W double zero remains an analytic benchmark and is not the causal resonance; the Lorentzian massless branch condition is stated separately in Sec. V.

V. MASSLESS COMPLIANCE AND INFRARED CONSTITUTIVE INVERSION

Inverting the completed TT stiffness converts a response of stress to an imposed source into the source-conjugate response generated by a specified stress. This algebraic reversal produces the massless pole without adding a microscopic gravitational field.

Let $A_{2,E,\mathrm{tot}}[S]$ be the full positive TT stiffness of state S, including its reference-state intercept. Let $p = (p_0, \mathbf{p})$ be Euclidean four-momentum with $p^2 = p_0^2 + |\mathbf{p}|^2$; the static shear has $p_0 = 0$ and $|\mathbf{p}| = k$. Let $h_{\mu\nu}^{\mathrm{TT}}(p)$ be the source-conjugate operational tensor restricted to the TT subspace, $\tau_{\mu\nu}^{\mathrm{TT}}(p)$ its conjugate stress perturbation, and $K_{\mathrm{TT},E}(p)$ the subtracted Euclidean constitutive kernel. Their low-momentum relation is

$$\begin{aligned} \tau_{\mu\nu}^{TT}(p) &= \mathcal{K}_{TT,E}(p) h_{\mu\nu}^{TT}(p), \\ \mathcal{K}_{TT,E}(p) &= A_{2,E}^{tot} p^2 + \mathcal{O}(p^4). \end{aligned} \tag{110}$$

The TT compliance $D_{\mathrm{TT},E}(p)$ is the inverse of $K_{\mathrm{TT},E}(p)$ on the physical TT subspace:

$$\mathcal{D}_{TT,E}(p) \equiv \mathcal{K}_{TT,E}^{-1}(p) = \frac{1}{A_{2,E}^{tot} p^2} + \mathcal{O}(p^0). \tag{111}$$

Factoring $K_{\mathrm{TT},E} = A_{2,E,\mathrm{tot}} p^2 [1 + \mathcal{O}(p^2)]$ makes the inversion explicit: its leading inverse is $1/\left(A_{2,E,\mathrm{tot}} p^2\right)$, while $\mathcal{O}(p^0)$ collects terms finite at $p \to 0$. The denominator therefore vanishes at $p^2 = 0$ without a mass term. This massless pole is the Fourier-space origin of a $1/r$ static Green function; it is a property of the inverted matter

response, not an independently postulated microscopic metric. With $\int_p \equiv \int d^4p/(2\pi)^4$, the corresponding dimensionless quadratic effective action is

$$\Gamma^{(2)}_{OQG,E} = \frac{1}{2}\int_p h^{TT}_{\mu\nu}(-p) A^{tot}_{2,E} p^2 h^{\mu\nu}_{TT}(p). \tag{112}$$

The TT projector is implicit because both tensors in Eq. (112) already lie in that subspace. The action fixes the two helicity states and their residue. Trace variables, gauge redundancy, and nonlinear tensor representation are independent of this projected result.

The equilibrium calculation fixes the zero-frequency spatial residue but does not determine the temporal residue of a Lorentzian wave equation. The causal continuation used here therefore selects a locally analytic, massless two-helicity infrared branch on which the angular frequency ω and spatial wave-number magnitude k obey $\omega^2 = c^2k^2$, with c the locally measured limiting signal speed. This branch condition is independently testable and is not inferred from the static slope. Once it is imposed, analytic continuation turns the Euclidean derivative block into the Lorentzian massless wave operator with residue $A_{2,E,\mathrm{tot}}$; no microscopic gravitational field or additional fitted coefficient is introduced.

VI. FIXED-MEASURE WTDIFF AND EINSTEIN-EQUIVALENT DYNAMICS

A. Fixed-measure WTDiff completion

To represent the selected TT branch for conserved sources, choose a local scalar-free completion. At two derivatives, stability and ghost freedom require at least transverse diffeomorphism symmetry (TDiff), whose vector parameter is divergence free [30–31]. Generic TDiff retains a scalar; enhancement to full diffeomorphisms (Diff) or to TDiff plus local Weyl symmetry (WTDiff) removes it. Diff and WTDiff give the same on-shell TT exchange between conserved sources, while organizing the trace and determinant differently [30, 32–35]. Table 4 compares the admissible representations.

Table 4. Local infrared representations of the calculated transverse-traceless block.

Completion	Physical TT block	Additional content	Role in the response representation
Generic TDiff	Same p^2 helicity-two block	Generally contains a propagating scalar	Excluded by the no-scalar condition
Full Diff	Same p^2 helicity-two block	Uses full diffeomorphism redundancy	Viable and on-shell equivalent in the calculated sector
WTDiff	Same p^2 helicity-two block	Uses Weyl redundancy and transverse diffeomorphisms with a fixed measure	Chosen to implement the OQG fixed-measure condition directly
Massive or higher-derivative tensor	Changes the pole or adds modes	Introduces further scales, poles, or degrees of freedom	Outside the local massless two-derivative response

WTDiff realizes the two properties already fixed by the matter calculation: the Hessian is evaluated in a traceless shear channel, and the volume element is not a dynamical response coordinate. It adds no scalar. Full Diff remains an on-shell-equivalent representation of the calculated TT sector, so WTDiff is an admissible fixed-measure completion rather than a uniqueness claim.

To define the fixed measure, let $\varpi(x)$ be a nondynamical positive scalar density with the same density weight as $\sqrt{-q}$, let $q_{\mu\nu}$ be an auxiliary Lorentzian symmetric tensor, and write $q \equiv \det q_{\mu\nu}$. Let $\hat{g}_{\mu\nu}$ denote the composite metric and $\hat{g} \equiv \det \hat{g}_{\mu\nu}$ its determinant. In four dimensions define

$$\hat{g}_{\mu\nu} = \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2} q_{\mu\nu}, \qquad \sqrt{-\hat{g}} = \varpi. \tag{113}$$

Let $\sigma(x)$ denote the scalar parameter of a local Weyl rescaling. The composite metric $\hat{g}_{\mu\nu}$ is invariant under $q_{\mu\nu} \to e^{2\sigma(x)} q_{\mu\nu}$. The density ϖ fixes only the integration measure; it supplies neither curvature nor a causal cone. Around a flat background compatible with the chosen measure, write $q_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ is the Minkowski metric and $h \equiv \eta^{\rho\sigma} h_{\rho\sigma}$. The linear Weyl-invariant perturbation is

$$\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu} h, \qquad h \equiv \eta^{\rho\sigma} h_{\rho\sigma}. \tag{114}$$

Let ξ_μ be a vector gauge parameter constrained by $\partial_\mu \xi^\mu = 0$, where ∂_μ denotes the coordinate derivative. Together with the Weyl parameter σ, its linear gauge transformation is

$$\delta h_{\mu\nu} = 2\,\partial_{(\mu}\xi_{\nu)} + 2\sigma\eta_{\mu\nu}, \qquad \partial_\mu \xi^\mu = 0. \tag{115}$$

Parentheses around indices denote symmetrization with unit weight. The transverse vector supplies three independent gauge functions and the Weyl transformation supplies the fourth, leaving the two helicity-two degrees of freedom.

For coefficient matching, let X_E^μ denote Euclidean coordinates with dimensions of inverse energy. Let C_W be the positive energy-squared coefficient multiplying the dimensionless quadratic WTDiff action. With Euclidean signs chosen so that the TT kernel is positive,

$$\Gamma^{(2)}_{W,E} = C_W \int d^4X_E \left[\frac{1}{4}\partial_\lambda \hat{h}_{\mu\nu}\,\partial_\lambda \hat{h}_{\mu\nu} - \frac{1}{2}\partial_\mu \hat{h}_{\mu\nu}\,\partial_\rho \hat{h}_{\rho\nu}\right]. \tag{116}$$

On the TT subspace, $\hat{h}_{\mu\nu} = h^{TT}_{\mu\nu}$ and $\partial_\mu h^{TT}_{\mu\nu} = 0$, so Eq. (116) reduces to

$$\Gamma^{(2)}_{W,E}\Big|_{TT} = \frac{C_W}{4} \int d^4X_E\; \partial_\lambda h^{TT}_{\mu\nu}\,\partial_\lambda h^{TT}_{\mu\nu}. \tag{117}$$

Apply the real shear source of Sec. III to Eqs. (112) and (117). Because a symmetric tensor contains both h_{xy} and h_{yx}, the contraction gives

$$\partial_\lambda h^{TT}_{\mu\nu}\,\partial_\lambda h^{TT}_{\mu\nu} = 2(\partial_z h_{xy})^2. \tag{118}$$

Integration of the two Fourier modes over V_4 gives

$$\int d^4X_E (\partial_z h_{xy})^2 = 2V_4 k^2 h_k h_{-k}. \tag{119}$$

Equations (117)–(119) therefore yield

$$\Gamma^{(2)}_{W,E} = C_W V_4 k^2 h_k h_{-k}, \tag{120}$$

whereas Eq. (112), with the same two Fourier modes and the same symmetric tensor contraction, yields

$$\Gamma^{(2)}_{OQG,E} = 2A^{tot}_{2,E} V_4 k^2 h_k h_{-k}. \tag{121}$$

Matching the identical TT inverse-propagator blocks fixes

$$C_W = 2A^{tot}_{2,E}. \tag{122}$$

The factor of two is fixed by the real symmetric shear source: $A_{2,E}$ is normalized to one off-diagonal source pair, whereas C_W multiplies the full symmetric-tensor contraction. For states S and S_0, define $\Delta C_W \equiv C_W[S] - C_W[S_0]$. The thermal-minus-reference result of Sec. III and Eq. (122) give

$$\frac{\Delta C_W}{\Theta^2} = 2\big(a_0 + a_\xi \xi_H + a_3 c_3^{NP} + a_2 c_2^{NP}\big). \tag{123}$$

The four coefficients in Eq. (123) are therefore 0.520112413356, 0.133248316388, 0.005998559152, and 0.001598759948, respectively. They are the curvature-action normalization of the same calculated state difference, not a second fit or an additional microscopic contribution.

B. Dimensional normalization and force invariant

Let x^μ denote coordinates measured in length, with $x^0 = ct$ and t the Lorentzian time, and let X^μ denote the corresponding inverse-energy coordinates. The global conversion is

$$X^\mu = \frac{x^\mu}{\hbar c}. \tag{124}$$

With the same dimensionless metric field expressed as a function of the rescaled coordinates, $d^4x = (\hbar c)^4 d^4X$ and $R_x = (\hbar c)^{-2} R_X$. Here R_x and R_X are the Ricci scalars formed with derivatives with respect to x^μ and X^μ, respectively. In a fixed-measure chart this gives

$$d^4x\, \varpi R_x = (\hbar c)^2 d^4X\, \varpi R_X. \tag{125}$$

Let G denote Newton's coupling. The Einstein-normalized fixed-measure WTDiff action in length coordinates is

$$S_W = \frac{c^3}{16\pi G} \int d^4x\, \varpi R_x. \tag{126}$$

Dividing by $\hbar$ and applying Eq. (125) gives the dimensionless quantum action

$$\frac{S_W}{\hbar} = \frac{\hbar c^5}{16\pi G} \int d^4X\, \varpi R_X \equiv C_W \int d^4X\, \varpi R_X. \tag{127}$$

Combining Eqs. (122) and (127) yields

$$A^{tot}_{2,E} = \frac{\hbar c^5}{32\pi G}. \tag{128}$$

The coefficient multiplying stress-energy in the Einstein equation is therefore

$$\frac{G}{c^4} = \frac{\hbar c}{32\pi A_{2,E}^{tot}}. \tag{129}$$

Both sides of Eq. (129) have units of inverse force. Its reciprocal is

$$\frac{c^4}{G} = \frac{32\pi A_{2,E}^{tot}}{\hbar c}. \tag{130}$$

The left side of Eq. (130) is commonly called the Planck force or superforce. The right side shows the engineering meaning of $A_{2,E}^{tot}$: after division by the action-to-length factor $\hbar c$, the TT stiffness equals the invariant force scale apart from the exact normalization 32π.

The field-theory calculation fixes state differences without altering the invariant relation between stiffness and force. For a state S and reference state S_0, write $A_{2,E,\Delta}[S; S_0]$ for the calculated thermal-minus-reference stiffness. Then

$$\begin{aligned} C_W(\mathcal{S}) - C_W(\mathcal{S}_0) &= 2A_{2,E}^{\Delta}(\mathcal{S}; \mathcal{S}_0), \\ C_W(\mathcal{S}) &= C_W(\mathcal{S}_0) + 2A_{2,E}^{\Delta}(\mathcal{S}; \mathcal{S}_0). \end{aligned} \tag{131}$$

The Hessian fixes the difference in Eq. (131). The total coefficient, and therefore a numerical value of the force scale, additionally requires the reference-state value $C_W[S_0]$. This is a declared physical boundary datum rather than an adjustable gravitational fit.

C. Einstein-equivalent nonlinear equations

The response calculation fixes the quadratic TT residue but does not select a unique nonlinear action. Within the local, scalar-free, two-derivative class identified in Sec. VI.A, the minimal fixed-measure representative extends the composite fixed-measure metric defined in Sec. VI.A to WTDiff form. Let Ψ denote the collection of matter fields, let $S_m[\hat{g}, \Psi]$ be their macroscopic action, and let $R[\hat{g}]$ be the Ricci scalar of $\hat{g}_{\mu\nu}$. The resulting action is

$$\begin{aligned} S[\hat{g}, \Psi] &= \frac{c^3}{16\pi G}\int d^4x\, \varpi R[\hat{g}] + S_m[\hat{g}, \Psi], \\ \sqrt{-\hat{g}} &= \varpi. \end{aligned} \tag{132}$$

Define the symmetric macroscopic stress tensor by $\delta S_m = -\frac{1}{2c}\int d^4x\, \varpi T_{\mu\nu}\delta\hat{g}^{\mu\nu}$, as appropriate when $x^0 = ct$, and define its trace by $T \equiv \hat{g}^{\mu\nu}T_{\mu\nu}$. Let $R_{\mu\nu}$ and R denote the Ricci tensor and scalar of $\hat{g}_{\mu\nu}$. Because the fixed determinant permits only traceless metric variations, Eq. (132) gives

$$R_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}R = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}T\right). \tag{133}$$

Let $\hat{\nabla}$ denote the Levi-Civita covariant derivative of $\hat{g}_{\mu\nu}$. For covariantly conserved macroscopic matter,

$$\hat{\nabla}^{\mu}T_{\mu\nu} = 0. \tag{134}$$

Taking the divergence of Eq. (133) and using the contracted Bianchi identity $\widehat{\nabla}^{\mu}R_{\mu\nu} = 1/2\,\widehat{\nabla}_{\nu}R$ gives

$$\widehat{\nabla}_{\nu}\left(R + \frac{8\pi G}{c^4}T\right) = 0. \tag{135}$$

On each connected spacetime region, define Λ as one quarter of this integration constant. Then

$$R + \frac{8\pi G}{c^4}T = 4\Lambda, \tag{136}$$

Here Λ is a cosmological integration datum. Substitution into Eq. (133) gives

$$R_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}R + \Lambda\hat{g}_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{137}$$

Equation (137) is the Einstein equation with the source normalization fixed by Eqs. (122) and (129). The cosmological term is not a coupling in the fixed-measure action; it enters when stress conservation and the Bianchi identity restore the trace. Under a homogeneous change of energy origin by a constant density ρ_0, the macroscopic stress transforms as:

$$T_{\mu\nu} \to T_{\mu\nu} - \rho_0\hat{g}_{\mu\nu} \tag{138}$$

This homogeneous shift cancels from the trace-free local source and only relabels the integration datum. The fixed-measure variation yields the trace-free equation, and stress conservation with the Bianchi identity restores its Einstein representative. The calculated matter stiffness remains the origin of the common source coefficient throughout the completion.

VII. VACUUM-SHIFT BLINDNESS AT FULL RESPONSE LEVEL

A homogeneous identity displacement can affect gravity only if it survives differentiation of the normalized matter functional. Fixing the measure at the source level removes that direction before any derivative is taken, allowing the claim to be tested for the complete Hessian and every higher response vertex.

A. Matter-first source definition

Let $q_{\mu\nu}$ be an unconstrained, nonsingular Lorentzian source of fixed signature, let $q^{\mu\nu}$ denote its matrix inverse, let $q \equiv \det q_{\mu\nu}$, and let ϖ be a fixed positive scalar density. In four dimensions, define the composite source metric $\hat{g}_{\mu\nu}$ by

$$\hat{g}_{\mu\nu}(q,\varpi) = \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2} q_{\mu\nu}, \qquad \sqrt{-\hat{g}} = \varpi. \tag{139}$$

The determinant identity follows algebraically throughout this fixed-signature source domain. The density ϖ specifies the integration measure; it does not prescribe lengths, null cones, curvature, or a pre-existing gravitational field. Those relations enter only through the response carried by the unimodular tensor $\hat{g}_{\mu\nu}$. This ordering implements the OQG statement that matter supplies the physical response and geometry records it.

The complete renormalized Standard Model action is coupled to the source only through the composite metric,

$$S_{\mathrm{SM}}[\Phi, q; \varpi] = \int_{\Omega} d^4x\, \varpi\, \mathcal{L}_{\mathrm{SM}}\big(\Phi, \hat{g}(q,\varpi); g_i(\mu)\big). \tag{140}$$

Here Φ denotes all Standard Model fields, $g_i(\mu)$ denotes the renormalized nonidentity couplings at scale μ, and Ω is the fixed integration domain. The matter action may contain interactions, masses, gauge fixing, ghosts, improvement terms, and the counterterms needed to define the stress response. Equation (140) imposes only one restriction relevant to the theorem: the source variation is performed at fixed ϖ rather than through a variable determinant.

The transformation to be tested is a homogeneous change of the energy origin,

$$\mathcal{L}_{\mathrm{SM}} \to \mathcal{L}_{\mathrm{SM}} - \rho_0, \qquad S_{\mathrm{SM}} \to S_{\mathrm{SM}} - \rho_0 \mathcal{V}_{\varpi}, \qquad \mathcal{V}_{\varpi} \equiv \int_{\Omega} d^4\, x\, \varpi. \tag{141}$$

The constant ρ_0 multiplies the identity operator and is independent of spacetime, fields, state labels, and sources. The physical preparation, nonidentity couplings, domain, measure, topology, and boundary conditions are held fixed. A change in any of those data is a different physical comparison and is not represented by Eq. (141).

Fixing the measure before source variation is essential. If one first constructs an ordinary diffeomorphism-invariant functional of an unrestricted metric and only afterward imposes a determinant condition, the vacuum term has already acquired a metric derivative. Equation (140) instead defines a different off-shell source map. Its Weyl redundancy removes the trace direction where the response functional is defined, rather than imposing a gauge choice on a completed Einstein theory [33, 35–37].

The same restriction is visible in the first variation. Parentheses around indices denote unit-weight symmetrization. From Eq. (139), a general source displacement obeys

$$\delta \hat{g}_{\alpha\beta} = \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2} \left(\delta q_{\alpha\beta} - \frac{1}{4} q_{\alpha\beta} q^{\mu\nu} \delta q_{\mu\nu}\right). \tag{142}$$

The expression in parentheses is trace free with respect to $q_{\mu\nu}$. Consequently, $q_{\mu\nu}$ couples to the projected stress response rather than to an independent determinant mode. This projection will reappear at macroscopic order as the trace-free WTDiff source.

B. Full response-level theorem

The simplest part of the proof is already visible at the level of a normalized state. For a static regulated system, let $V_{\varpi}^{(3)}$ be its fixed spatial volume. Equation (141) changes the Hamiltonian by

$$H' = H + \rho_0 V_{\varpi}^{(3)} \mathbf{1}. \tag{143}$$

Every energy eigenvalue receives the same displacement, while eigenvectors, energy differences, transition frequencies, and matrix elements remain unchanged. For a thermal preparation, $Z_\beta \equiv \mathrm{Tr} e^{-\beta H}$, $\hat{\rho}_\beta \equiv e^{-\beta H}/Z_\beta$, and $\beta = (k_{\mathrm{B}} T)^{-1}$, where k_{B} is Boltzmann's constant and T is the temperature.

$$Z'_\beta = e^{-\beta \rho_0 V_{\varpi}^{(3)}} Z_\beta, \qquad \hat{\rho}'_\beta = \frac{e^{-\beta H'}}{Z'_\beta} = \frac{e^{-\beta H}}{Z_\beta} = \hat{\rho}_\beta. \tag{144}$$

A normalized pure state acquires only a common phase. Thus the offset changes an unnormalized partition function or vacuum amplitude, but no normalized expectation value. This cancellation is necessary but not sufficient for gravity, because gravitational response is obtained by differentiating with respect to a source. The decisive step is to establish the cancellation before those derivatives are taken.

Let q_+ and q_- denote independent sources on the forward and backward branches of a closed-time-path (CTP) contour, let $U[q]$ be the corresponding Standard Model time-evolution operator, and let $\hat{\rho}_i$ be a normalized initial

density operator. With the same fixed measure and domain on both branches, define the normalized Schwinger-Keldysh generating functional by [38]

$$Z_{\mathrm{CTP}}[q_+, q_-] \equiv \mathrm{Tr}\big(U[q_+]\hat{\rho}_i U^\dagger[q_-]\big), \qquad Z_{\mathrm{CTP}}[q, q] = 1. \tag{145}$$

Equivalently, the path-integral representation contains the initial kernel $\langle \Phi_+^i | \hat{\rho}_i | \Phi_-^i \rangle$, integrates over a common final configuration satisfying $\Phi_+(t_f) = \Phi_-(t_f)$, and uses the same fixed measure and domain on both branches. Under Eq. (141), $U[q_+]$ and $U^\dagger[q_-]$ acquire opposite phases. Neither phase depends on q_+ or q_-, and both branches contain the same fixed four-volume, so

$$Z'_{\mathrm{CTP}}[q_+, q_-] = \exp\left[-\frac{i\rho_0}{\hbar}(\mathcal{V}_\varpi - \mathcal{V}_\varpi)\right] Z_{\mathrm{CTP}}[q_+, q_-] = Z_{\mathrm{CTP}}[q_+, q_-]. \tag{146}$$

This identity is exact for independent branch sources. It is not obtained by setting $q_+ = q_-$, truncating a loop expansion, or discarding disconnected diagrams after differentiation. The initial density operator is held fixed; for the thermal example its invariance follows from Eq. (144). No hidden normalization factor survives at the initial end of the contour.

Define the connected CTP generating functional by

$$W_{\mathrm{CTP}}[q_+, q_-] = -i\hbar \ln Z_{\mathrm{CTP}}[q_+, q_-]. \tag{147}$$

Equation (146) gives $W'_{\mathrm{CTP}} = W_{\mathrm{CTP}}$. Therefore, for every integer $n \geq 1$, every choice of branch labels $a_j \in \{+, -\}$, and every set of spacetime points,

$$\frac{\delta^n W'_{\mathrm{CTP}}}{\delta q_{a_1,\mu_1\nu_1}(x_1) \cdots \delta q_{a_n,\mu_n\nu_n}(x_n)} = \frac{\delta^n W_{\mathrm{CTP}}}{\delta q_{a_1,\mu_1\nu_1}(x_1) \cdots \delta q_{a_n,\mu_n\nu_n}(x_n)}. \tag{148}$$

Theorem 1. Let the renormalized Standard Model be coupled to an unconstrained symmetric source only through the fixed-measure composite metric in Eq. (139). Hold the normalized physical preparation, nonidentity couplings, integration domain, measure, topology, and boundary conditions fixed. Then any spacetime-independent identity-operator shift of the form in Eq. (141) leaves the normalized CTP generating functional invariant. Consequently, the projected mean stress, every retarded and fluctuation kernel, the full contact-completed Hessian, and every higher response vertex are invariant. The theorem holds before perturbative or derivative expansion and hence to all orders in Standard Model interactions in a renormalization scheme that preserves the fixed-measure source definition.

It is useful to contrast Eq. (146) with an ordinary metric source. A conventional vacuum term contributes to the doubled action as

$$\Delta S_{\mathrm{Diff}}[g_+, g_-] = -\rho_0\left[\int d^4x\sqrt{-g_+} - \int d^4x\sqrt{-g_-}\right]. \tag{149}$$

This expression vanishes at coincident sources, but its derivative with respect to the difference source does not. Define $g_r = (g_+ + g_-)/2$ and $g_a = g_+ - g_-$. Differentiating at fixed g_r and then taking the physical limit $g_a = 0$, for which $g_+ = g_- = g$, gives

$$\left.\frac{\delta \Delta S_{\mathrm{Diff}}}{\delta g_{a,\mu\nu}(x)}\right|_{g_a=0} = -\frac{1}{2}\rho_0\sqrt{-g}\, g^{\mu\nu} \neq 0. \tag{150}$$

The doubled contour by itself therefore does not solve the problem. A common phase cancels at the value of the functional, while the variable determinant retains a nonzero response. OQG's fixed measure removes that derivative

before the contour identity is used. The two ingredients—normalized doubling and a source-level fixed measure—are jointly responsible for the full theorem.

C. Complete Hessian and shear-channel audit

The second derivative of a matter generating functional contains more than an operator commutator or a separated-point stress correlator. Because the stress operator itself depends on the source, a second variation also produces local seagull terms, improvement terms, gauge-fixing contributions, and the contacts required by Ward identities. A proof based only on the fact that the identity operator commutes with everything would establish invariance of connected separated-point correlators, but it would leave the local part of the Hessian untested [39–40].

For branch labels $a, b \in \{+, -\}$, write the complete source Hessian schematically as

$$\mathcal{H}_{ab}^{\mu\nu,\rho\sigma}(x,y) \equiv \frac{\delta^2 W_{\mathrm{CTP}}}{\delta q_{a,\mu\nu}(x)\delta q_{b,\rho\sigma}(y)} = \mathcal{H}_{ab,\mathrm{conn}}^{\mu\nu,\rho\sigma}(x,y) + \mathcal{H}_{ab,\mathrm{contact}}^{\mu\nu,\rho\sigma}(x,y). \tag{151}$$

Equation (148) proves the invariance of the entire left-hand side, which automatically includes both terms on the right. More directly, the shifted action in Eq. (141) is independent of q_+ and q_-. Its first variation is zero and its second variation is zero, so it generates neither a mean projected stress nor a local seagull. This is stronger than cancelling disconnected vacuum bubbles after they have been calculated.

The shear source of Sec. III provides a component-level check. Work in a local Cartesian Euclidean background with unit diagonal entries and fixed density $\varpi = 1$. Introduce an off-diagonal perturbation in the x-y block of the ordinary source matrix, with z the direction of variation and k its conjugate wave number:

$$q_{xy} = q_{yx} = h(z), \qquad \det q = 1 - h^2, \qquad \sqrt{\det q} = 1 - \frac{1}{2}h^2 + \mathcal{O}(h^4). \tag{152}$$

If a vacuum term is coupled through the ordinary determinant, Eq. (152) produces a quadratic local contribution proportional to $\rho_0 h^2$. In momentum space that contribution is independent of the wave number. It can alter the algebraic k^0 contact but cannot alter the k^2 derivative slope. With the exact fixed-measure source, however,

$$\hat{g}_{\mu\nu} = (\det q)^{-1/4} q_{\mu\nu}, \qquad (\det q)^{-1/4} = 1 + \frac{1}{4}h^2 + \mathcal{O}(h^4), \qquad \sqrt{\det \hat{g}} = 1. \tag{153}$$

The compensating diagonal terms in Eq. (153) remove even the k^0 vacuum contact. Thus the general functional proof and the explicit shear algebra agree.

Let $Z_E[h]$ be the Standard Model Euclidean partition function and $W_E[h] \equiv -\ln Z_E[h]$ its connected equilibrium generating functional. Define $G_E^{xy,xy}$ as the full second shear derivative of W_E, including the local source variations and contact terms described above. Assuming the analytic small-momentum expansion in this channel and prescription, the response is

$$G_E^{xy,xy}(0,k) = G_E^{xy,xy}(0,0) + \frac{\kappa}{2}k^2 + \mathcal{O}(k^4), \qquad A_{2,E} \equiv \frac{\kappa}{4}. \tag{154}$$

The static observable of Sec. III subtracts the equilibrium block and retains the positive derivative stiffness. Even with the ordinary diagnostic source, a homogeneous vacuum shift could enter only $G_E^{xy,xy}(0,0)$ and would disappear from the subtracted k^2 slope. With the exact composite source it enters neither term. In both descriptions,

$$\frac{\partial A_{2,E}}{\partial \rho_0} = \frac{1}{4}\frac{\partial}{\partial \rho_0}\left.\frac{\partial^2 G_E^{xy,xy}(0,k)}{\partial k^2}\right|_{k=0} = 0. \tag{155}$$

The derivative stiffness is already blind to a momentum-independent k^0 contact under the ordinary diagnostic shear source. The fixed-measure source gives the stronger statement: the mean, the k^0 contact, the derivative stiffness, and every higher response vertex are invariant before any momentum subtraction.

D. Invariance of the coefficient chain

Let p denote Euclidean four-momentum. After the background and zero mode have been treated, the transverse-traceless Euclidean inverse response at nonzero p and its inverse on the physical TT subspace are defined at low momentum by

$$\mathcal{K}_{TT,E}(p) = A_{2,E}^{\mathrm{tot}} p^2 + \mathcal{O}(p^4), \qquad \mathcal{D}_{TT,E}(p) = \mathcal{K}_{TT,E}^{-1}(p) = \frac{1}{A_{2,E}^{\mathrm{tot}} p^2} + \mathcal{O}(p^0). \tag{156}$$

Here $A_{2,E}^{\mathrm{tot}}$ is the total derivative stiffness of the chosen reference state plus any calculated state increment. The superscript is important. The detailed Standard Model calculation determines a thermal-minus-reference increment; it does not determine the state-independent intercept. The vacuum-shift theorem removes dependence on the arbitrary identity coefficient from both the increment and the intercept, but it does not supply the intercept's numerical value.

The coefficient normalization follows from the same shear mode. Let $X^\mu = x^\mu/(\hbar c)$ be inverse-energy coordinates, $z \equiv X^z$, k their energy-dimension conjugate wave number, and $V_4 \equiv \int d^4X$. Let C_W multiply the positive dimensionless Euclidean WTDiff action. For $h_{xy} = h_{yx} = h_k e^{ikz} + h_{-k} e^{-ikz}$ with $h_{-k} = h_k^*$, the symmetric contraction counts both off-diagonal components. Matching the two quadratic forms and restoring the fixed-measure Einstein action gives

$$\begin{aligned} \Gamma_{W,E}^{(2)}[h^{TT}] &= \frac{C_W}{4} \int d^4X \; \partial_\lambda h_{\mu\nu}^{TT} \, \partial_\lambda h_{TT}^{\mu\nu} = C_W V_4 k^2 h_k h_{-k}, \\ \Gamma_{\mathrm{OQG},E}^{(2)} &= 2A_{2,E}^{\mathrm{tot}} V_4 k^2 h_k h_{-k}, \qquad C_W = 2A_{2,E}^{\mathrm{tot}} = \frac{\hbar c^5}{16\pi G}, \\ A_{2,E}^{\mathrm{tot}} &= \frac{\hbar c^5}{32\pi G}, \qquad \frac{G}{c^4} = \frac{\hbar c}{32\pi A_{2,E}^{\mathrm{tot}}}. \end{aligned} \tag{157}$$

No natural-unit convention is used. The quantity c is the locally realized causal signal speed of the matter response. An identity shift changes no normalized state, transition threshold, pole position, or retarded kernel; it therefore changes neither c nor the derivative coefficient. Equations (148), (155), and (157) give the response-chain identity

$$\frac{\partial A_{2,E}^{\mathrm{tot}}}{\partial \rho_0} = \frac{\partial \mathcal{K}_{TT,E}}{\partial \rho_0} = \frac{\partial \mathcal{D}_{TT,E}}{\partial \rho_0} = \frac{\partial C_W}{\partial \rho_0} = \frac{\partial}{\partial \rho_0}\left(\frac{G}{c^4}\right) = 0. \tag{158}$$

Equation (158) is the OQG-specific closure of the vacuum-shift argument. It begins at the normalized Standard Model functional, includes the local completion of the stress Hessian, and ends at the invariant coefficient multiplying the macroscopic source. There is no stage at which a cutoff-sized homogeneous offset is converted into an enormous gravitational response.

The theorem does not make all matter-dependent corrections to gravitational response vanish. Curvature susceptibilities, nonlocal polarization, thermal corrections, density changes, and other nonidentity operators can alter $A_{2,E}^{\mathrm{tot}}$ and hence the effective response. Those are measurable changes in state or dynamics. Equation (158) removes only the arbitrary common zero.

E. Renormalization and anomalies

Radiative stability can be stated without assigning a cutoff value to vacuum energy. Let $\rho_{\mathrm{vac}}(\mu)$ be the renormalized coefficient of the identity operator, $f_1(\mu)$ the coefficient of the two-derivative curvature scalar R, $\{R_i^2\}$ a basis of local quadratic and higher curvature invariants with coefficients $\alpha_i(\mu)$, and $\Gamma_{\mathrm{nonlocal}}$ the nonlocal remainder. In a symmetry-preserving scheme, the matter effective action has the form

$$\Gamma_{\mathrm{SM}}[\hat{g};\mu] = \int d^4x\, \varpi \left[-\rho_{\mathrm{vac}}(\mu) + f_1(\mu) R[\hat{g}] + \sum_i \alpha_i(\mu) \mathcal{R}_i^2 + \cdots \right] + \Gamma_{\mathrm{nonlocal}}[\hat{g};\mu]. \tag{159}$$

Because ϖ is fixed, the identity term contains no source dependence. Regulator, subtraction, or loop-order changes may alter $\rho_{\mathrm{vac}}(\mu)$, but no source derivative detects that change. By contrast, the curvature and nonlocal sectors depend on $g_{\mu\nu}$ and may renormalize the derivative stiffness or generate higher response.

This separation is visible in heat-kernel language. The leading volume coefficient contributes only to the fixed four-volume, whereas subsequent coefficients contain curvature and field strengths. Carballo-Rubio showed that the absence of longitudinal diffeomorphisms prevents the volume term from becoming a dynamical source in a fixed-measure theory [36]. One-loop calculations in unimodular gravity likewise find that the cosmological sector is not renormalized as an ordinary metric volume term, while the remaining effective action retains quantum corrections [41–42]. The result is radiative insulation, not the absence of quantum response.

The renormalization-group statement follows directly. Differentiating Eq. (159) with respect to the source removes $\rho_{\mathrm{vac}}(\mu)$ before any scale dependence is considered. The source-dependent response still obeys its Callan-Symanzik equation, including beta functions, anomalous dimensions, operator mixing, and permitted local anomaly or contact terms [43]. The identity coefficient can run in a chosen bookkeeping scheme without entering that equation. Thus two symmetry-preserving schemes that differ only by a homogeneous c-number produce identical normalized response functionals.

A trace anomaly does not reverse the conclusion. Running nonidentity couplings and curvature invariants can produce a nonzero trace response, and those contributions must be retained in a complete calculation. They do not make the fixed integral $\int d^4x\, \varpi$ depend on $q_{\mu\nu}$. Some formulations describe the Weyl redundancy of unimodular gravity as nonanomalous when the matter fields and functional measure are defined compatibly [44], but the present theorem does not require the stronger assertion that every trace anomaly vanishes. It requires only that an allowed anomaly be carried by source-dependent operators rather than by the identity coefficient.

The all-orders identity applies to the stated matter-first theory because Eq. (146) precedes every Standard-Model loop expansion. A theory containing an additional independently quantized fundamental graviton sector would have a different source and functional measure and is not included in the theorem.

F. Einstein-equivalent trace-free limit

The same vacuum-shift blindness appears in the nonlinear infrared representation. Let $R_{\mu\nu}$ be the Ricci tensor associated with the composite metric. Define $R \equiv \hat{g}^{\mu\nu} R_{\mu\nu}$, and let $\hat{\nabla}$ be the Levi-Civita derivative of $\hat{g}_{\mu\nu}$. Write $T \equiv \hat{g}^{\mu\nu} T_{\mu\nu}$. The full-metric tensor $T_{\mu\nu}$ is a representative whose trace-free projection is the actual fixed-measure source. With $\kappa_G \equiv 8\pi G/c^4$, variation of the fixed-measure WTDiff action gives

$$R_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu} R = \kappa_G \left(T_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu} T \right), \qquad \kappa_G \equiv \frac{8\pi G}{c^4}. \tag{160}$$

Under the homogeneous matter shift, the stress tensor and its trace transform as

$$T'_{\mu\nu} = T_{\mu\nu} - \rho_0 \hat{g}_{\mu\nu}, \qquad T' = T - 4\rho_0, \qquad T'_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}T' = T_{\mu\nu} - \frac{1}{4}\hat{g}_{\mu\nu}T. \tag{161}$$

The local field equation is therefore exactly invariant. This is not a cancellation between two large quantities. The identity direction is absent from the source space of Eq. (160).

For a conserved total stress tensor, the contracted Bianchi identity gives

$$\hat{\nabla}^{\mu}T_{\mu\nu} = 0 \quad \Rightarrow \quad \hat{\nabla}_{\nu}(R + \kappa_G T) = 0 \quad \Rightarrow \quad R + \kappa_G T = 4\Lambda, \tag{162}$$

where Λ is constant on each connected solution branch. Substitution into Eq. (160) yields the Einstein-form representative

$$R_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}R + \Lambda\hat{g}_{\mu\nu} = \kappa_G T_{\mu\nu}. \tag{163}$$

Equations (160)-(163) reproduce the standard local Einstein dynamics for conserved sources, but they assign a different status to the cosmological term [32–33, 35–37]. It is an integration datum rather than the coefficient of a local matter vacuum operator. If the representative stress tensor is shifted while the physical solution is held fixed, Eq. (162) requires

$$\Lambda' = \Lambda - \kappa_G \rho_0. \tag{164}$$

The separate symbols $T_{\mu\nu}$ and Λ therefore depend on the chosen zero of matter energy, while their local physical content does not. The invariant Einstein representation is the equivalence class of pairs

$$\left[\left(T_{\mu\nu}, \Lambda\right)\right] \equiv \left\{\left(T_{\mu\nu} - \rho\hat{g}_{\mu\nu},\ \Lambda - \kappa_G\rho\right) : \rho \in \mathbb{R},\ \partial_{\alpha}\rho = 0\right\}. \tag{165}$$

To name the invariant branch explicitly, write $T_{\mu\nu} = T^{(0)}_{\mu\nu} - \rho_{\mathrm{id}}\hat{g}_{\mu\nu}$, where $T^{(0)}_{\mu\nu}$ contains no homogeneous identity term, and define $\Lambda_{\mathrm{inv}} \equiv \Lambda + \kappa_G\rho_{\mathrm{id}}$. This quantity is constant across the class in Eq. (165). The identity-operator part of Standard Model loop corrections cannot force a retuning of the local gravitational equation, because it changes only the representative of that class. The theorem does not select Λ_{inv}. The zero branch means $\Lambda_{\mathrm{inv}} = 0$, equivalently $\Lambda = 0$ only in an identity-free representative.

G. Functional derivative conventions

The unconstrained source $q_{\mu\nu}$ carries ten formal components, but Eq. (139) makes one direction redundant. Parentheses on tensor indices denote unit-weight symmetrization, $A_{(\mu\nu)} \equiv \left(A_{\mu\nu} + A_{\nu\mu}\right)/2$. Define

$$\Omega_q^2 \equiv \left(\frac{\varpi}{\sqrt{-q}}\right)^{1/2}, \qquad \mathcal{P}_{\alpha\beta}^{\ \ \mu\nu} \equiv \delta^{\mu}_{(\alpha}\delta^{\nu}_{\beta)} - \frac{1}{4}q_{\alpha\beta}q^{\mu\nu}, \qquad \delta\hat{g}_{\alpha\beta} = \Omega_q^2\mathcal{P}_{\alpha\beta}^{\ \ \mu\nu}\delta q_{\mu\nu}. \tag{166}$$

The projector satisfies $q^{\alpha\beta}\mathcal{P}_{\alpha\beta}^{\ \ \mu\nu} = 0$. If the stress convention is fixed by $\delta S_{\mathrm{SM}} = 1/2\int d^4x\,\varpi T^{\alpha\beta}\delta\hat{g}_{\alpha\beta}$, the source conjugate is

$$\tau^{\mu\nu} \equiv \frac{2}{\varpi}\frac{\delta S_{\mathrm{SM}}}{\delta q_{\mu\nu}} = \Omega_q^2\mathcal{P}_{\alpha\beta}^{\ \ \mu\nu}T^{\alpha\beta}, \qquad q_{\mu\nu}\tau^{\mu\nu} = 0. \tag{167}$$

For CTP calculations, introduce average and difference sources $q_r = (q_+ + q_-)/2$ and $q_a = q_+ - q_-$. The physical limit is $q_a = 0$. Mean response and retarded response are obtained from derivatives with respect to the difference source followed by derivatives with respect to the average source. A representative convention is

$$\langle \tau^{\mu\nu}(x) \rangle = \frac{2}{\varpi(x)} \frac{\delta W_{\mathrm{CTP}}}{\delta q_{a,\mu\nu}(x)}\bigg|_{q_a=0}, \qquad G_R \sim \frac{\delta^2 W_{\mathrm{CTP}}}{\delta q_a \, \delta q_r}\bigg|_{q_a=0}. \tag{168}$$

Overall factors in the second expression depend on the source normalization and tensor symmetrization, but the vacuum-shift result does not. Since $W'_{\mathrm{CTP}} = W_{\mathrm{CTP}}$ as a functional of both independent branch sources, every derivative in any consistent r/a convention agrees exactly. Ward identities derived from the same functional are unchanged for the same reason.

H. Shear-source algebra

For the Euclidean off-diagonal shear used in Eq. (152), the nontrivial block and its determinant are

$$q_{ij} = \begin{pmatrix} 1 & h \\ h & 1 \end{pmatrix}, \qquad \det q_{ij} = 1 - h^2, \qquad (\det q)^{-1/4} = 1 + \frac{1}{4}h^2 + \mathcal{O}(h^4). \tag{169}$$

Multiplication by the compensating factor gives, through quadratic order,

$$\hat{g}_{xx} = \hat{g}_{yy} = 1 + \frac{1}{4}h^2 + \mathcal{O}(h^4), \qquad \hat{g}_{xy} = h + \mathcal{O}(h^3), \qquad \sqrt{\det \hat{g}} = 1. \tag{170}$$

For an ordinary variable-volume Euclidean functional, a constant density contributes

$$\Delta W_E^{\mathrm{ordinary}} = \rho_0 \int d^4x \sqrt{\det q} = \rho_0 \int d^4x \left(1 - \frac{1}{2}h^2 + \mathcal{O}(h^4)\right). \tag{171}$$

Its quadratic part is local. If h_k denotes the Fourier amplitude of $h(z)$, the transform produces a term proportional to $h_k h_{-k}$ with no factor of k^2. For the fixed-measure source, the corresponding functional is

$$\Delta W_E^{\mathrm{fixed}} = \rho_0 \int d^4x \, \varpi, \tag{172}$$

which contains no h dependence at any order. Equations (171) and (172) separate two useful statements: the derivative stiffness in Eq. (155) is blind even under the ordinary shear diagnostic because the residual term is k^0, while the exact OQG source removes the residual contact altogether.

I. Renormalization ledger

After the Standard Model fields have been integrated out, denote the renormalized effective action by $\Gamma_R \equiv \Gamma_{\mathrm{SM}}$. Let $\mathcal{O}_0 = \mathbf{1}$ be the identity and let $\mathcal{O}_A[\hat{g}]$ denote the local nonidentity operators admitted by the renormalized theory. The coefficient $c_0(\mu)$ multiplies the identity sector, the $c_A(\mu)$ are renormalized operator coefficients, and $\Gamma_{\mathrm{state}}[\hat{g}]$ collects the remaining state-dependent and nonlocal terms. The action can be organized as

$$\Gamma_{\mathrm{SM}} = c_0(\mu) \int d^4x \, \varpi + \sum_A c_A(\mu) \, \mathcal{I}_A[\hat{g}] + \Gamma_{\mathrm{state}}[\hat{g}], \qquad \mathcal{I}_A[\hat{g}] \equiv \int d^4x \, \varpi \, \mathcal{O}_A. \tag{173}$$

For every $n \geq 1$, the identity sector satisfies

$$\frac{\delta^n}{\delta q_{\mu_1\nu_1}(x_1) \cdots \delta q_{\mu_n\nu_n}(x_n)} \left[c_0(\mu) \int d^4x \, \varpi\right] = 0, \qquad n \geq 1. \tag{174}$$

The derivatives of $\mathcal{I}_A$ and Γ_{state} need not vanish. Vacuum polarization, anomalies, curvature counterterms, boundary operators, and state susceptibilities belong to this second class. A renormalization prescription preserves the theorem if counterterm mixing respects the fixed-measure source symmetry, so that the identity sector remains proportional to the source-independent fixed volume.

The source-dependent renormalization-group equation is therefore written in its complete form as

$$\begin{aligned} \mathcal{D}_{\text{CS}}\Gamma_R^{(n)} &= \mathcal{A}_R^{(n)}, \\ \Gamma_R^{(n)} &\equiv \frac{\delta^n \Gamma_R}{\delta q_{\mu_1\nu_1}(x_1)\cdots\delta q_{\mu_n\nu_n}(x_n)}, \qquad n \geq 1. \end{aligned} \tag{175}$$

Here $\mathcal{D}_{\text{CS}}$ is the full Callan-Symanzik operator, including $\mu\,\partial_\mu$, all beta functions, anomalous dimensions, and operator-source mixing, while $\mathcal{A}_R^{(n)}$ denotes allowed local anomaly or contact terms [43]. Source-dependent vertices obey their appropriate homogeneous or anomalous equations after contact completion. The identity coefficient contributes to neither side after one source derivative because its fixed-volume functional is $q_{\mu\nu}$ independent. Thus c_0 may be scheme dependent while the OQG response remains scheme independent.

J. Physical scope

Vacuum-shift blindness removes one redundant direction in operator space, not the physical quantum vacuum. A Casimir force compares configurations with different boundaries. A Lamb shift changes an energy difference. Vacuum polarization changes propagators and couplings. Connected stress fluctuations produce a noise kernel, while curvature-dependent terms change the derivative response. None of these effects is a common identity displacement, and all survive Eqs. (146) and (148). This distinction is consistent with the operational treatment of vacuum phenomena throughout quantum electrodynamics [45].

Choose a projected component $\tau(x)$ of the tensor $\tau^{\mu\nu}$ defined in Sec. VII.G, and set $\delta\tau \equiv \tau - \langle\tau\rangle$. With θ the Heaviside function and with no additional factor of $\hbar$ absorbed into the noise definition, the retained retarded kernel G_R and symmetrized noise kernel N are

$$G_R(x,x') = -\frac{i}{\hbar}\theta(t-t')\langle[\tau(x),\tau(x')]\rangle, \qquad N(x,x') = \frac{1}{2}\langle\{\delta\tau(x),\delta\tau(x')\}\rangle. \tag{176}$$

Adding a multiple of the identity changes neither the commutator nor the centered operator $\delta\tau$. More generally, Eq. (148) protects the contact-completed versions obtained by source differentiation. Spectral densities, damping widths, thresholds, and causal poles therefore remain physical. The theorem prevents a disconnected constant from masquerading as response; it does not make the vacuum empty.

Phase transitions require similar care. A common displacement of an entire effective potential, $V(\phi) \to V(\phi) + C$, leaves its derivatives, extrema, tunneling-exponent differences, and normalized transition probabilities unchanged. That displacement is covered by the theorem; a transition between physically distinct configurations is not. Let ϕ be a canonical scalar field and define $(\partial\phi)^2 \equiv \hat{g}^{\alpha\beta}\,\partial_\alpha\phi\;\partial_\beta\phi$. Its trace-free source contains

$$T_{\mu\nu}^{(\phi)} - \frac{1}{4}\hat{g}_{\mu\nu}T^{(\phi)} = \partial_\mu\phi\;\partial_\nu\phi - \frac{1}{4}\hat{g}_{\mu\nu}(\partial\phi)^2. \tag{177}$$

The metric-proportional potential cancels locally, while gradients, kinetic energy, defects, radiation, latent-heat products, and the transition history remain. A common shift cannot remove the energy difference between distinct endpoints. Equation (164) applies only to one common redefinition of the energy origin; it does not imply that every physical phase-transition contribution is globally sequestered [46].

Boundaries and global data provide another controlled limit. Equation (146) assumes that both contour branches have the same fixed domain, measure, topology, and compatible boundary conditions. Varying a boundary embedding, changing total four-volume, or comparing inequivalent topologies can produce boundary work or a different integration datum. Such effects are physical comparisons; they do not restore sensitivity to an arbitrary bulk zero within one fixed problem.

K. Consequence for the response chain

The fixed-measure and unimodular literature establishes vacuum-shift blindness at the macroscopic level [33, 35–37]. Here the same result is closed through the microscopic response chain: the normalized Standard-Model functional, its complete stress Hessian, the TT stiffness, and the Einstein-equivalent source coefficient all lie in the same invariant class.

Because the CTP identity holds for independent branch sources before differentiation, it covers projected mean stress, separated-point correlators, seagulls, Ward contacts, and higher vertices at once. The shear audit shows that an ordinary variable-volume diagnostic retains only a k^0 contact, whereas the exact fixed-measure source removes that contact as well. Equations (156)–(158) then carry the invariance through $A_{2,E,\mathrm{tot}}$, $K_{\mathrm{TT},E}$, $D_{\mathrm{TT},E}$, C_W, and G/c^4.

The radiative local-source vacuum catastrophe is therefore absent at its premise. A freely adjustable identity coefficient is not a measurable source coordinate of the normalized matter response, so no ultraviolet estimate of ρ_0 enters the local constitutive coefficient, no bare term is tuned against loop corrections, and no cancellation between large local sources is required.

Within the stated domain, a homogeneous shift of the Standard-Model Lagrangian changes no normalized state, complete stress-response vertex, TT stiffness, causal compliance, or invariant Einstein source coefficient. In the Einstein-equivalent representation it merely relabels $(T_{\mu\nu}, \Lambda)$ while leaving Λ_{inv} unchanged. The quantum vacuum retains its measurable polarization, boundaries, condensate differences, anomalies, transitions, and fluctuations.

VIII. CROSS-STATE CONTINUITY AND UNIVERSALITY

State dependence has two physically different meanings. Replacing an entire homogeneous KMS state changes the asymptotic susceptibility, whereas changing a compact body inside one ambient state changes only a localized part of the common response operator.

The distinction resolves the apparent tension between a temperature-dependent homogeneous stiffness and the observed universality of gravity between finite bodies. Threshold matching carries the first case between effective descriptions; an inhomogeneous Green function governs the second and returns to one massless operator in the far field.

For a family of homogeneous high-temperature equilibrium branches, let T be the KMS temperature, k_B Boltzmann's constant, and a_{TT} the dimensionless state-dependent TT coefficient. The stiffness may then be written

$$A_{2,E}(T)=a_T(T)(k_BT)^2, \tag{178}$$

where $a_T(T)$ is the dimensionless state-dependent stiffness coefficient, T is the KMS temperature of the homogeneous equilibrium state, and k_B is Boltzmann's constant.

Comparing two homogeneous branches may legitimately change the asymptotic compliance because the entire state defining the response has changed. Heating, cooling, changing phase, or changing composition in a finite specimen is different: only a compact region changes, while the asymptotic state remains the same.

Threshold matching therefore changes which degrees of freedom carry the Hessian; it does not imply that the stiffness vanishes when a laboratory thermometer approaches zero. Let $A_{2,E}^{\mathrm{body}}(T_{\mathrm{lab}})$ denote the matched stiffness of

a body at laboratory temperature $T_{\rm lab}$, $A_{2,E}^{\rm GS}$ its matched ground-state contribution, and $\delta A_{2,E}^{\rm low}(T_{\rm lab})$ its thermal or other low-energy correction. The superscript GS means ground state. The structural form required of a consistent low-energy effective-field-theory (EFT) continuation is

$$A_{2,E}^{\rm body}(T_{\rm lab})=A_{2,E}^{\rm GS}+\delta A_{2,E}^{\rm low}(T_{\rm lab}), \qquad \lim_{T_{\rm lab}\to 0}\delta A_{2,E}^{\rm low}=0. \tag{179}$$

Above the electroweak crossover those excitations are conveniently described by the unbroken Standard-Model fields used in Sec. III. As the state is continued downward through electroweak symmetry breaking, quantum chromodynamics (QCD) confinement, nuclear and atomic binding, and finally condensed-matter formation, heavy thermal populations become Boltzmann suppressed while their virtual, binding and ground-state effects are transferred into the matched lower-energy description. At ordinary temperatures the thermally active modes are material-dependent phonons, electrons, magnons, photons and other collective excitations sitting on top of the matched ground-state coefficient.

The continuity is therefore one of effective description rather than one of fixed particle content:

$$\begin{aligned}&\text{unbroken SM plasma} \to \text{broken SM} \to \text{hadrons and nuclei}\\ &\to \text{atoms and molecules} \to \text{condensed matter.}\end{aligned} \tag{180}$$

Equation (180) expresses continuity of effective description. Available observables fix the reference-state normalization but do not separately identify every low-energy threshold coefficient. The degrees of freedom carrying the Hessian therefore change across thresholds without introducing a different gravitational law at each transition.

The cross-state problem becomes especially clear for two separated compact bodies. Let $\mathcal{K}_{\rm stat}$ denote the full static inverse-response operator of the two-body configuration, $\mathcal{K}_\infty$ the operator of the common ambient branch, $\Delta\mathcal{K}_1$ and $\Delta\mathcal{K}_2$ the localized state-dependent modifications produced by the bodies, and B_i the finite spatial region occupied by body i. The operator symbol supp denotes support, the set on which a kernel is nonzero. For the two-point kernels used here, $\mathrm{supp}\Delta\mathcal{K}_i \subseteq B_i \times B_i$ means that $\Delta\mathcal{K}_i(\mathbf{x},\mathbf{x}')$ vanishes unless both spatial arguments lie inside body i. Then

$$\mathcal{K}_{\rm stat} = \mathcal{K}_\infty + \Delta\mathcal{K}_1 + \Delta\mathcal{K}_2, \qquad \mathrm{supp}\Delta\mathcal{K}_i \subseteq B_i \times B_i. \tag{181}$$

Let $\mathcal{G}(\mathbf{x},\mathbf{x}')$ denote the corresponding static Green function, defined as the inverse of $\mathcal{K}_{\rm stat}$, and let $\delta^{(3)}$ denote the three-dimensional Dirac delta distribution. It satisfies

$$\mathcal{K}_{\rm stat}\,\mathcal{G}(\mathbf{x},\mathbf{x}')=\delta^{(3)}(\mathbf{x}\text{-}\mathbf{x}'). \tag{182}$$

This formulation removes an ambiguity that arises if one tries to assign a separate state-labeled Newton constant to each body and then asks how two such constants should be averaged. The physical problem is instead an ordinary inhomogeneous Green-function problem. The bodies modify the same operator locally; outside both compact supports the response returns to the same asymptotic massless branch.

At distances large compared with the body sizes, let $A_{2,E}^{\infty}$ denote the TT stiffness of the common ambient branch and ∇^2 the spatial Laplacian. The ambient tensor operator has the leading static form

$$\mathcal{K}_\infty=A_{2,E}^{\infty}(\text{-}\nabla^2)+O(\nabla^4), \tag{183}$$

Let $r \equiv |\mathbf{x}-\mathbf{x}'|$ be the separation between the two spatial points. The Green function of the common ambient branch then contains

$$\mathcal{G}_\infty(r)=\frac{1}{4\pi A_{2,E}^{\infty} r}+\text{subleading terms.} \tag{184}$$

The expression is understood up to the tensor projector and source normalization already fixed in Secs. II–VI. Localized insertions can change scattering phases, finite-size structure, induced multipoles and short-distance response, but they cannot create a second asymptotic $1/r$ pole unless the altered state itself extends to infinity or introduces an additional massless degree of freedom.

The same conclusion follows from the zero-momentum stress form factor of a complete composite body. Translation invariance and conservation normalize the zero-momentum matrix element of the conserved stress to the body's total four-momentum; in the static limit, the monopole coupling is therefore fixed by the total energy. Let $M_i(0)$ be the reference mass of body i, let $\Delta U_i(T_{\rm lab})$ be its change in internal energy at laboratory temperature $T_{\rm lab}$, and let c be the local limiting signal speed. Heating or cooling then changes the mass-energy by

$$M_i(T_{\rm lab}) = M_i(0) + \frac{\Delta U_i(T_{\rm lab})}{c^2}. \tag{185}$$

The state change may alter finite-momentum form factors, stresses and multipole moments, but it does not assign the specimen a new independent long-range gravitational charge. Consequently, for two compact bodies sharing one ambient branch, define $G_\infty \equiv G_{\rm comp}[\rho_\infty]$ as the common ambient coupling of the asymptotic state ρ_∞. It is neither a state-independent fundamental constant nor a private property of either body. Let R_1, R_2 be characteristic radii of the two bodies, $V_{12}(r)$ their leading static interaction potential, and $G_{12}^{\rm monopole}$ the corresponding leading monopole coupling. Then

$$V_{12}(r) = -G_\infty \frac{M_1 M_2}{r} + O\left(\frac{R_1^2 + R_2^2}{r^3}\right), \qquad G_{12}^{\rm monopole} = G_\infty. \tag{186}$$

Equation (186) is reciprocal because both bodies couple through the same Green function. It also excludes the naïve inference that cooling a finite sample makes its gravitational coupling scale as $T_{\rm lab}^{-2}$. Temperature scaling of a homogeneous KMS branch concerns the asymptotic state; Eq. (186) governs compact laboratory bodies embedded in one common branch.

This result also sharpens what a genuine laboratory anomaly would have to look like. Ordinary phase changes, magnetic ordering, superconductivity, cavity loading or thermal preparation can certainly change internal stress correlations and finite-frequency susceptibilities. Such changes are not gravitational evidence by themselves. A gravitational modification would have to survive as a universal change in the common long-range response after ordinary electromagnetic, thermal, mechanical and chemical effects were removed. The more different the microscopic constructions of the compared standards, the more restrictive that universality test becomes.

The empirical infrared constraints are correspondingly powerful. Optical-clock experiments have resolved gravitational redshift across millimeter-scale height differences [47]. Within OQG this is not a new prediction distinct from general relativity; it requires that the emergent operational tensor act universally on matter-built clocks once the response has reached its settled infrared branch. The measurement is nevertheless conceptually important because it demonstrates directly that quantum matter standards resolve the gravitational comparison field at laboratory scale.

Universality of free fall is more restrictive still. Let g be a common reference acceleration and parameterize the measured accelerations of bodies a and b by $a_a = (1 + \beta_a)g$ and $a_b = (1 + \beta_b)g$, where β_a and β_b are fractional composition-dependent deviations. The Eötvös parameter is then, to leading order,

$$\eta_{ab} \equiv 2\frac{|a_a - a_b|}{|a_a + a_b|} \simeq |\beta_a - \beta_b|. \tag{187}$$

The final MICROSCOPE result found no titanium–platinum violation at approximately the 10^{-15} Eötvös level in the tested regime [48]. The collective infrared projection must therefore erase constituent-level differences—electronic, nuclear, QCD, binding, spin and material-response differences—to extraordinary accuracy. This is one

of the strongest empirical reasons for using the complete conserved Standard-Model stress response rather than identifying gravity with the damping or polarizability of one microscopic constituent.

Gravitational-wave observations impose a complementary tensor requirement. Direct detection establishes propagating gravitational degrees of freedom [49], while the association of GW170817 with gamma-ray burst (GRB) 170817A constrains the low-energy gravitational causal speed to track the electromagnetic causal speed extremely closely under the standard astrophysical interpretation [50]. Writing m_{TT}^{IR} for the infrared tensor mass, v_{TT}^{IR} for the tensor propagation speed, and N_{helicity} for the number of long-range tensor helicities, the infrared OQG branch must consequently satisfy

$$m_{TT}^{\mathrm{IR}}=0, \qquad v_{TT}^{\mathrm{IR}}=c, \qquad N_{\mathrm{helicity}}=2, \tag{188}$$

with no additional unsuppressed long-range scalar or vector gravitational polarization. These requirements are naturally aligned with the massless two-helicity WTDiff representation of Sec. VI, but they remain empirical conditions on any microscopic realization of that representation.

Precision-gravity observations constrain the infrared output while remaining largely insensitive to whether its successful tensor representation is fundamental or collective. Any microscopic matter-first realization must reproduce that tested long-wavelength structure. At the same time, state dependence of the microscopic susceptibility does not assign a private Newton constant to each finite specimen.

The bridge to macroscopic matter is therefore continuous and universal. Homogeneous branches may possess different state-dependent stiffnesses, but compact changes alter mass-energy, finite-size structure, multipoles, and short-range response within one ambient Green function. Its shared $1/r$ residue remains common to both bodies, while redshift, free-fall, and gravitational-wave observations enforce a massless, two-helicity, composition-independent infrared limit.

IX. OPERATIONAL EQUIVALENCE AT THE MEASURED ORDER

Operational equivalence is equality of the response data actually sampled by an experiment, not identity of microscopic ontology. The normalized influence functional separates the mean, retarded, and fluctuation kernels so that the required equality can be stated exactly.

The closed-time-path (CTP) formalism gives the reduced dynamics of a controlled matter probe after unobserved degrees of freedom have been traced out [38, 51–52]. The symbols "x" and "y" denote spacetime points. Let Q denote the controlled probe degrees of freedom. Let $X_A(x)$ denote the collective environmental operator coupled to that probe at the spacetime point x. For a gravitational channel, each capital collective label, such as A, abbreviates a symmetric tensor-index pair. The functional $J^A[Q](x)$ denotes the probe source history, including all coupling constants. With d^4x denoting the spacetime volume element, the interaction action S_{int} is defined by

$$S_{\mathrm{int}}[Q,X] = -\int d^4x\, J^A[Q](x) X_A(x). \tag{189}$$

The two CTP branches carry a forward source history J^+ and a backward source history J^-. Let ρ_E be the prepared density operator of the unobserved environment, let U_J be its evolution operator in the presence of the history J, and let Tr_E denote the trace over that environment. A dagger denotes the Hermitian adjoint. The normalized influence functional $\mathcal{F}$ is

$$\mathcal{F}[J^+,J^-] = \frac{\mathrm{Tr}_E\left(U_{J^+}\rho_E U_{J^-}^{\dagger}\right)}{\mathrm{Tr}_E\rho_E}, \qquad \mathcal{F}[J,J] = 1. \tag{190}$$

The normalization condition states that identical histories on the two branches have unit influence. Define the contour-average history J_r and contour-difference history J_a by

$$J_r = \frac{J^+ + J^-}{2}, \qquad J_a = J^+ - J^-. \tag{191}$$

The doubled contour is causal bookkeeping; it is neither a second physical geometry nor a backward-time physical trajectory. To state the second-order expansion, define the mean response $\overline{X}_A(x) \equiv \langle X_A(x) \rangle$, the centered operator $\delta X_A(x) \equiv X_A(x) - \overline{X}_A(x)$, the retarded collective response $\mathcal{D}^R_{AB}(x,y)$, and its symmetrized operational fluctuation kernel $N^{\mathrm{op}}_{AB}(x,y)$. The superscript "op" labels a quantity defined for the reduced probe. A centered dot denotes integration over repeated spacetime arguments together with contraction of repeated collective indices. The remainder $O(J^3)$ collects terms cubic and higher in the probe source histories. With these definitions, the cumulant expansion through second order is

$$\ln \mathcal{F} = -\frac{i}{\hbar} J_a \cdot \overline{X} - \frac{i}{\hbar} J_a \cdot \mathcal{D}^R \cdot J_r - \frac{1}{2\hbar^2} J_a \cdot N^{\mathrm{op}} \cdot J_a + O(J^3). \tag{192}$$

Let θ denote the Heaviside step function; x^0 and y^0 are the time coordinates of x and y. Square brackets denote an operator commutator, curly brackets an anticommutator, and angle brackets the expectation value in ρ_E. With the coupling sign fixed by Eq. (189), the response and fluctuation kernels are

$$\mathcal{D}^R_{AB}(x,y) = -\frac{i}{\hbar} \theta(x^0 - y^0) \langle [\delta X_A(x), \delta X_B(y)] \rangle, \tag{193}$$

$$N^{\mathrm{op}}_{AB}(x,y) = \frac{1}{2} \langle \{\delta X_A(x), \delta X_B(y)\} \rangle. \tag{194}$$

An ideal quasistatic Bose-Marletto-Vedral (BMV) protocol tests only part of Eq. (192). Its entangling phase is fixed by the retarded response in the static conserved-source sector. Visibility loss and correlated fluctuations additionally test N^{op}. Thus an apparatus-specific noise calculation is not a prerequisite for deriving the coherent entangling phase.

Here a reduced probe map means the evolution of the probe density operator after the environmental degrees of freedom have been traced out. A description is Gaussian on a source domain when connected cumulants above second order vanish there. These definitions give the following operational criterion.

Theorem 2 (operational equivalence at the measured order). Consider two causal descriptions coupled linearly to the same probe histories with identical preparation and boundary data. Equality of their mean and retarded responses on the tested source domain gives the same coherent probe phase through second order in the coupling. Equality of their symmetrized fluctuation kernels additionally gives the same reduced probe map through that order. If both descriptions are exactly Gaussian on the domain, equality of all three quantities makes their reduced maps identical for every allowed history.

The result follows directly from Eq. (192). Mean and retarded equality is sufficient for the coherent static phase, whereas equality of N^{op} is the additional condition for the complete Gaussian reduced map. These are distinct experimentally accessible statements.

Equality of the mean and retarded kernels therefore fixes the coherent probe phase through second order. Equality of the symmetrized noise kernel is the additional and logically separate condition for equality of the complete Gaussian reduced map.

X. GRAVITY-INDUCED ENTANGLEMENT IN THE INFRARED CHANNEL

The Einstein-equivalent conserved-source response has a direct finite-body observable: four branch-dependent interaction energies generate one locally invariant two-body phase, and that phase determines the ideal path entanglement.

A. Conserved-source Newtonian interaction

The Einstein-equivalent equation derived in Sec. VI.C has a definite weak, static, nonrelativistic limit. Let $\Phi_N(\mathbf{x})$ be the Newtonian potential, $\rho(\mathbf{x})$ the mass density, and ∇^2 the spatial Laplacian. For bodies with densities $\rho_1(\mathbf{x})$ and $\rho_2(\mathbf{y})$, let U_{12} be their interaction energy and d^3x, d^3y the spatial volume elements. Then

$$\nabla^2\Phi_N = 4\pi G\rho, \qquad U_{12} = -G\int d^3\mathbf{x}\, d^3\mathbf{y}\, \frac{\rho_1(\mathbf{x})\rho_2(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}. \tag{195}$$

For point masses m_1 and m_2 separated by the distance r, Eq. (195) reduces to $U_{12} = -Gm_1m_2/r$. This limit is neither a second microscopic derivation nor an exchange attributed to free radiative TT modes alone. It is the static limit of the Einstein-equivalent conserved-source completion already normalized by the Standard-Model TT stiffness.

The same fixed-measure source is exactly insensitive to a homogeneous identity displacement by Sec. VII, while retaining polarization, boundary effects, condensate differences, anomalies, phase transitions, and connected fluctuations. The entanglement calculation depends on the conserved-source interaction in Eq. (195); vacuum-shift blindness guarantees that an arbitrary energy zero cannot alter its coefficient.

B. Branch phases, concurrence, finite wave packets, and noise

The conserved-source response of Sec. VI assigns an interaction energy to each of the four pairings of two spatially superposed bodies. Each pairing accumulates a phase during the hold time. Contributions that can be assigned to either body alone are removable local phases; the remaining correlated combination is the quantity that can entangle the two path systems.

Consider two bodies of masses m_1 and m_2. Each body is prepared in two localized branches labeled L and R. During a quasistatic hold interval of duration τ, let r_{ij} be the distance between branch i of body 1 and branch j of body 2, with $i,j \in \{L,R\}$. Let U_{ij} be the interaction energy for that branch pair, let t denote time within the hold interval, and let ϕ_{ij} be the phase accumulated by the pair. Equation (195) gives

$$\phi_{ij} = -\frac{1}{\hbar}\int_0^\tau U_{ij}\, dt = \frac{Gm_1m_2\tau}{\hbar r_{ij}}. \tag{196}$$

The accumulated phase is set by the interaction-energy integral in Eq. (196). A closer branch pair therefore has a larger-magnitude interaction energy and accumulates a larger phase during the same hold time. No single phase is an entanglement signal; the observable depends on the pattern across all four pairings. If that pattern separated into a contribution from body 1 and a contribution from body 2, the final state would remain separable.

Some of the four phases merely describe independent phase shifts of the first or second body. Such one-body phases can be removed by operations acting on each body separately and therefore cannot be the source of two-body entanglement. The alternating sum in Eq. (197) cancels all such removable contributions and defines the locally invariant two-body phase Φ:

$$\Phi = \phi_{LL} + \phi_{RR} - \phi_{LR} - \phi_{RL}. \tag{197}$$

The alternating combination isolates the entangling content of the four phases. A zero value means that they contain no irreducibly joint contribution. A nonzero value means that the phase associated with one body's path

depends on the other body's path, so the final path state cannot in general be written as a product of two independent states.

Each body's left-right path choice forms a two-level quantum system, conventionally called a path qubit. In this context, the qubit is simply the pair of coherent path alternatives, not an additional electronic or computing device. Let $\mathcal{C}$ denote the concurrence, a dimensionless measure of entanglement between the two path systems. It ranges from zero for a separable state to one for a maximally entangled pure state. For initially equal, independent superpositions and negligible visibility loss, it is

$$\mathcal{C} = \left|\sin\frac{\Phi}{2}\right|. \tag{198}$$

For the ideal state, the concurrence vanishes when the correlated phase is an integer multiple of a full cycle, reaches unity at an odd multiple of a half cycle, and varies continuously between those limits. Entanglement is therefore inferred from the locally invariant phase combination, not from the mere presence of four branch phases.

Finite wave-packet width replaces the point-separation factor by a profile average. Let $f_{n,i}(\mathbf{x})$ be the normalized mass profile of body $n \in \{1,2\}$ in branch i, so that its integral over space is unity. The replacement is

$$\frac{1}{r_{ij}} \to \int d^3\mathbf{x}\, d^3\mathbf{y}\, \frac{f_{1,i}(\mathbf{x}) f_{2,j}(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}. \tag{199}$$

The profile average accounts for the finite extent of the two wave packets and changes the phase quantitatively without changing the logic of Eqs. (196)–(198). For a specified apparatus, its splitting, holding, and recombination trajectories enter as time-dependent source histories in the same phase functional.

The remaining distinction is between phase accumulation and loss of interference contrast. A density matrix records both probabilities and coherence. Its off-diagonal elements describe interference between histories: their complex argument carries relative phase, while their magnitude carries visibility. Environmental or intrinsic fluctuations can reduce that magnitude without changing the mean coherent phase.

To include fluctuations, label two many-branch histories by α and β. Let $\rho_{\alpha\beta}(t)$ be the corresponding off-diagonal element of the reduced probe density matrix at time t, with t_i and t_f denoting the initial and final times. Let $\Theta_{\alpha\beta}$ be the coherent relative phase fixed by the retarded compliance, and let the nonnegative exponent $\Gamma_{\alpha\beta}$ describe visibility loss fixed by the symmetrized operational fluctuations N^{op}. The general second-order form is

$$\rho_{\alpha\beta}(t_f) = \rho_{\alpha\beta}(t_i)\exp\left[i\Theta_{\alpha\beta} - \Gamma_{\alpha\beta}\right]. \tag{200}$$

The visibility exponent $\Gamma_{\alpha\beta}$ is fixed by the symmetrized fluctuation kernel of the prepared laboratory state and is therefore left as the state-dependent quantity defined in Eq. (200). The coherent prediction is nevertheless complete: Sec. VI fixes the infrared normalization and static interaction, Eqs. (196)–(198) give the four branch phases, their locally invariant combination, and the ideal concurrence. Equation (200) separates that phase from fluctuation-induced visibility loss.

C. Operational equivalence of the coherent entangling phase

The coherent phase depends only on the infrared conserved-source compliance, whereas visibility also depends on state-specific fluctuations. A phase can identify a unique microscopic description only if no distinct quantum construction realizes the same compliance.

To state this comparison, "IR" denotes the infrared domain, and "cons" denotes restriction to conserved source histories. The symbol $\mathcal{D}^R_{\mathrm{Einstein}}$ denotes the retarded conserved-source response represented by the Einstein equation, whereas $\mathcal{D}^R_{\mathrm{OQG}}$ denotes the compliance obtained by constitutive inversion of the Standard-Model stress response.

Theorem 3 (OQG equivalence for the BMV phase). Let the probe stresses be conserved on the domain of a quasistatic gravity-induced-entanglement protocol. On the massless, scalar-free, fixed-measure branch normalized in Sec. VI, the OQG compliance and Einstein-equivalent response agree throughout the infrared conserved-source domain. They therefore predict the same branch phases, invariant entangling phase, and ideal concurrence:

$$\mathcal{D}^R_{\mathrm{OQG}}\big|_{\mathrm{IR,cons}} = \mathcal{D}^R_{\mathrm{Einstein}}\big|_{\mathrm{IR,cons}} \quad \Rightarrow \quad \big(\phi_{ij}, \Phi, \mathcal{C}\big)_{\mathrm{OQG}} = \big(\phi_{ij}, \Phi, \mathcal{C}\big)_{\mathrm{Einstein}}. \tag{201}$$

The normalized Standard-Model CTP functional supplies the Ward-completed susceptibility Π^R; constitutive inversion gives D^R; the TT stiffness fixes C_W and G/c^4; and conservation with the Bianchi identity gives the Einstein-equivalent field equation. Its static limit produces Eq. (195), from which Eqs. (196)–(198) give the BMV observables. This completes the proof.

The theorem does not identify the microscopic descriptions. In a quantized-metric description, the tensor field may be treated as a fundamental subsystem. In OQG, the same infrared tensor relation represents the constitutive response of quantum Standard-Model matter. The measured phase depends on the common response, not on the ontological label assigned to its representation.

OQG is not a classical measurement-and-feedback model. Its retarded susceptibility originates in noncommuting Standard-Model stress operators, and the same normalized quantum functional contains connected fluctuations and higher stress responses. The absence of an independently fundamental metric does not make the resulting effective channel classical.

Equality of the complete Gaussian reduced maps would additionally require equality of N^{op} in the prepared state. That stronger condition is not required for Theorem 3, which concerns the coherent BMV phase and ideal entangling unitary.

D. What an entanglement observation establishes

Suppose initially independent probes become entangled after controlled nongravitational interactions have been excluded. Under the locality and subsystem assumptions of the tested model, a nonzero entanglement witness rules out a channel composed only of local operations and classical communication. In that precise sense, the observation establishes a nonclassical effective gravitational interaction channel.

That conclusion does not identify a unique microscopic carrier. Theorem 3 provides a constructive counterexample to the stronger inference: quantum Standard-Model stress response with an Einstein-equivalent constitutive representation produces the same coherent entangling phase without an independently fundamental metric Hilbert space. The experiment detects the quantum character of the interaction seen by matter, while the static phase alone does not decide whether its tensor representation is fundamental or collective.

This distinction does not weaken the experiment. It states exactly what the observable certifies. The result can exclude particular semiclassical, collapse, measurement-feedback, or hybrid models when their complete reduced dynamics fail to reproduce the observed entanglement and visibility [9–11]. It also tests gravitational phase accumulation while both source bodies occupy controlled quantum superpositions.

Table 5. Experimental observables and the response information they test. The retarded compliance $\mathcal{D}^R$ controls the coherent response; the symmetrized kernel N^{op} controls state-dependent fluctuations.

Experimental result	Response information tested	Inference supported
Static BMV phase	Infrared conserved-source $\mathcal{D}^R$	Coherent gravitational response
Certified entanglement	Phase plus preparation and locality assumptions	Nonclassical effective channel
Frequency-dependent phase	Retardation, poles, and spectral support	Causal propagation structure
Visibility and correlated noise	Symmetrized collective kernel N^{op}	State and fluctuation content
Connected three-source signal	Higher stress-response vertices	Departure from Gaussian equivalence

Table 5 separates experimental strength from ontological overreach. Each additional observable narrows the operational equivalence class, but no ontological label is measured independently of the response that gives the label physical content.

The derived phase and concurrence establish the observable consequence of the matter-first chain. Under the protocol assumptions, certified entanglement demonstrates a nonclassical effective gravitational channel; the static phase does not uniquely identify a fundamental metric as its microscopic carrier.

XI. SCOPE, ASSUMPTIONS, AND FALSIFIABILITY

The domain is a two-point conserved-source response on the stated locally analytic massless two-helicity branch, supplemented by an exact all-orders theorem for homogeneous identity shifts. Within that domain, the static matching, retarded kernel, constitutive inverse, WTDiff normalization, Einstein-equivalent equation, vacuum-shift identity, and coherent entangling phase form one closed response chain.

A. Declared numerical inputs

A numerical value of the total stiffness requires ξ_H, the two Yang–Mills lattice responses c_3^{NP} and c_2^{NP}, and the reference-state intercept. Each is defined by an independent physical prescription. None is an adjustable gravitational parameter, and every perturbative Standard-Model contribution through $\mathcal{O}(g^2)$ has already been assigned and matched.

The coherent quasistatic theorem does not require a numerical laboratory-state noise exponent. Equality of complete Gaussian reduced maps would additionally require the prepared-state symmetrized fluctuation kernel, while connected three- and higher-stress responses probe information outside the completed two-point domain.

B. Experimental discriminators

The static BMV phase samples the shared zero-frequency infrared response. Microscopic descriptions that agree on that response can be separated only by observables that probe additional parts of their normalized response functionals.

One possibility is retardation. The light-crossing time is the signal-propagation time across the separation of the two probes. Protocols operating on comparable time scales test the causal support of $\mathcal{D}^R$, rather than only its zero-frequency integral. Such a protocol can distinguish an instantaneous direct-interaction approximation from a causal response, although it cannot distinguish two descriptions having the same retarded kernel.

Joint phase-and-visibility spectroscopy supplies a second discriminator. The phase measures reactive response, while visibility loss and correlated fluctuations constrain N^{op}. Equilibrium response and noise obey a fluctuation-

dissipation relation; a driven state instead tests the CTP kernel of its actual preparation. The measured pair therefore probes state information not contained in the coherent phase alone.

A third possibility is higher connected response. Let J_1, J_2, and J_3 be three independently controlled source histories, and let $\Theta[J]$ denote the coherent phase functional evaluated on the combined history J. Define $\Delta\Theta_{123}$ as the part not reducible to one-source or pairwise phases. In the sums below, the index letters i and j label the three histories. The first sum counts each unordered pair once, and the second runs over the three individual histories:

$$\Delta\Theta_{123} = \Theta[J_1 + J_2 + J_3] - \sum_{i<j} \Theta\left[J_i + J_j\right] + \sum_{i} \Theta\left[J_i\right]. \tag{202}$$

After one-source terms are removed, $\Delta\Theta_{123}$ vanishes for a strictly Gaussian pairwise channel. A nonzero value isolates a connected three-source response and thus information not fixed by the two-point operational equivalence established above.

Additional propagating polarizations, a low-energy scalar pole, a nonzero graviton mass, or a mismatch between temporal and spatial residues would also falsify the minimal WTDiff completion used by OQG. Conversely, observation of only the shared two-helicity infrared response would not select between a fundamental and a composite representation of that response.

C. Falsifiability boundary

The minimal infrared completion is falsified by an unsuppressed scalar or vector pole, a nonzero tensor mass, a causal speed inconsistent with the electromagnetic limiting speed, unequal temporal and spatial residues incompatible with the selected branch, failure of Ward/contact completion, or lattice values inconsistent with the magnetostatic observables as defined. Source dependence of the fixed measure, failure of normalized CTP closure, or symmetry-preserving mixing of the identity into a nontrivial source functional would likewise invalidate the vacuum-shift theorem.

Within the stated domain, no additional perturbative coefficient remains undetermined. The external inputs set the numerical stiffness, while retardation, visibility, higher connected response, and additional polarizations test progressively smaller operational equivalence classes.

XII. CONCLUSION

Within the conserved-source infrared domain, the normalized stress functional of the interacting Standard Model determines a causal tensor response without introducing a fundamental metric degree of freedom. Ward and source-contact completion produce a positive physical TT spectrum, and the static shear susceptibility closes through $\mathcal{O}(g^2)$. Hard, screened, Higgs-contact, electric, local-curvature, and magnetostatic sectors combine into a finite stiffness; the only nonperturbative inputs are the two explicitly defined three-dimensional Yang–Mills observables.

Constitutive inversion converts the completed static stiffness into a $1/k^2$ long-range compliance. The finite-frequency retarded kernel separately identifies Landau and production continua and a simple underdamped resonance. On the stated locally analytic massless two-helicity branch, fixed-measure WTDiff provides a scalar-free tensor representation whose coefficient is fixed by the calculated compliance; for conserved sources, its field equation is Einstein equivalent. Geometry enters at this representational stage rather than as the microscopic starting point.

Because fixed-measure normalization precedes functional differentiation, a homogeneous identity displacement leaves the normalized CTP functional and every response derivative unchanged. Mean stress, contacts, retarded response, noise, higher vertices, stiffness, compliance, and the Einstein source coefficient are blind to an arbitrary common energy zero, while vacuum polarization, boundary effects, condensate differences, anomalies, phase

transitions, and the global cosmological integration datum remain physical. The radiative local-source vacuum catastrophe is therefore absent from this response channel without erasing measurable vacuum physics.

For two finite bodies in spatial superposition, the conserved-source interaction fixes all four branch phases and their locally invariant entangling combination, which determines the ideal concurrence. A verified entanglement witness then certifies a nonclassical effective gravitational channel under the protocol assumptions. Equality of mean and retarded responses fixes the coherent phase, whereas equality of the symmetrized noise kernel is additionally required for equality of the Gaussian reduced map. The static phase therefore does not distinguish a fundamental quantum metric from the collective matter-first realization.

The renormalized Higgs-curvature coupling, the two lattice observables, and the reference-state intercept determine the absolute stiffness. They are physical inputs, not omitted terms in the completed $\mathcal{O}(g^2)$ calculation. Ultraviolet completion, strong-field solutions, and response beyond the two-point domain are outside the claim. Retardation, phase-noise consistency, higher connected response, and additional polarizations provide experimental discriminators. Within these limits, the normalized Standard-Model stress response leads through constitutive inversion to Einstein-equivalent dynamics and a measurable entangling phase. It supplies a complete matter-first realization of a nonclassical gravitational channel without assuming a fundamental quantum spacetime.

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Conflict of Interest

The author declares no conflict of interest.

Author Contributions

Todd J. Desiato conceived the OQG framework, developed the physical interpretation and research program, directed and checked the theoretical synthesis, analyzed the results, prepared the manuscript, and approved its final form.

Data Availability

No new experimental datasets were generated for this theoretical study. The benchmark inputs, executable verification code, and generated audit outputs supporting the reported coefficients and response identities are provided in the accompanying Operational Quantum Gravity numerical reproducibility package [53]. Values for the two nonperturbative Yang–Mills observables defined in Sec. III.E are not presently available in the specified lattice scheme and are therefore retained as physical inputs.

Code Availability

The numerical assembly and verification code, benchmark inputs, tests, and machine-readable outputs are included in the Supplemental Material [53].

Use of Generative Artificial Intelligence

The author used OpenAI ChatGPT (GPT-5.6 Sol Max) and OpenAI Codex (GPT-5, September 2026 service version) for literature organization, symbolic and consistency cross-checking, manuscript assembly, generation and debugging of the accompanying verification code, drafting, and copyediting. The author supplied the scientific premises and source materials, constrained and directed each task, reviewed the cited sources, equations, code, and numerical outputs, revised the resulting text, and takes full responsibility for the content.

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